Accurate Odometry and Error Modelling for a Mobile Robot

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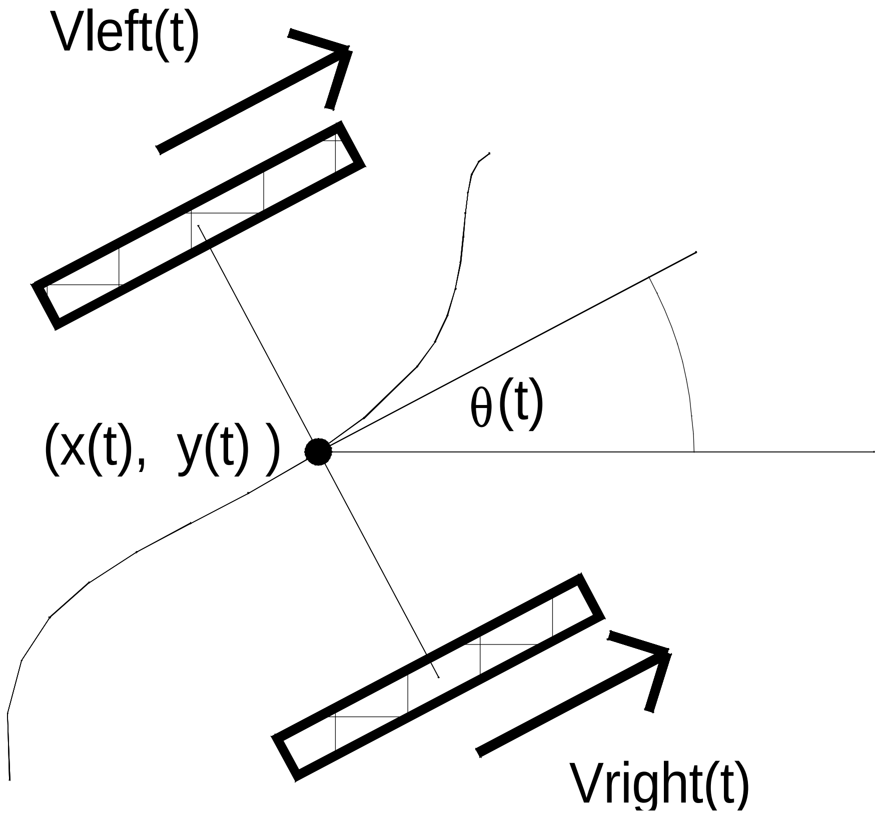
Introduction to Odometry



Given a two wheeled robot, odometry estimates position and orientation from left and right wheel velocities as a function of time.

$$x(t) = x(0) + \int_0^t \left(\frac{Vright(t) + Vleft(t)}{2} \right) \cos\theta(t)\, dt$$

$$y(t) = y(0) + \int_0^t \left(\frac{Vright(t) + Vleft(t)}{2} \right) \sin\theta(t)\, dt$$

$$\theta(t) = \theta(0) + \int_0^t \left(\frac{Vright(t) - Vleft(t)}{B} \right) dt$$

A discrete time implementation of these equations is

$$D_k = (d_R + d_L)/2$$

$$\mathbf{S}_k = \mathbf{f}\left(\mathbf{S}_{k-1}, \mathbf{u}_k\right) = \mathbf{S}_{k-1} + \begin{bmatrix} D_k \cos(q_{k-1} + a_k/2) \\ D_k \sin(q_{k-1} + a_k/2) \\ a_k \end{bmatrix}$$

State

$$\begin{bmatrix} x_k \\ y_k \\ q_k \end{bmatrix}$$

Input

$$\mathbf{u}_k = \begin{bmatrix} D_k \\ a_k \end{bmatrix}$$

Rotation

$$a_k = (d_R - d_L)/B$$

d_R, d_L are right and left wheel distances covered between time steps *k-1* and *k*.

State error covariance matrix, $\mathbf{Cov}(\mathbf{S}_k)$, is useful in:

- localisation data fusion with other sensors
- map building applications, eg Kalman Filter.

Propagation of $\mathbf{Cov}(\mathbf{S}_k)$ is often implemented with linearisation (assuming input errors independent from state errors):

Jacobian matrix of **f** w.r.t. **S**

(i,j) element $\left[\frac{¶ f_i}{¶ S_j}\right]$

$$\mathbf{Cov}(\mathbf{S}_k) = (\nabla_{\mathbf{S}}\mathbf{f})\ \ \mathbf{Cov}(\mathbf{S}_{k-1})\ \ (\nabla_{\mathbf{S}}\mathbf{f})^T$$
$$+(\nabla_{\mathbf{u}}\mathbf{f})\ \ \mathbf{Cov}(\mathbf{u}_k)\ \ (\nabla_{\mathbf{u}}\mathbf{f})^T \quad \textbf{(@@)}$$

"guessed" as $\begin{bmatrix} s_D^2 & 0 \\ 0 & s_a^2 \end{bmatrix}$

This popular model is **INCONSISTENT**!

Here's why: Suppose $\mathbf{s}_{k-1} = [0 \quad 0 \quad 0]^T$ and $\mathbf{Cov}(\mathbf{S}_{k-1}) = \mathbf{0}$. Consider two scenarios:

$\mathbf{u}_{k0} = [D \quad 0]^T$ with $\mathbf{Cov}(\mathbf{u}_{k0}) = \begin{bmatrix} s_D^2 & 0 \\ 0 & s_a^2 \end{bmatrix}$.

From (@@) above

$$\mathbf{Cov}(\mathbf{S}_{k+1}) = \begin{bmatrix} s_D^2 & 0 & 0 \\ 0 & D^2 s_a & D s_a^2 \\ 0 & D s_a^2 & s_a^2 \end{bmatrix}$$

Same path, but two half distance steps:

$\mathbf{u}_k = [D/2 \quad 0]^T$ followed by $\mathbf{u}_{k+1} = [D/2 \quad 0]^T$

$$\mathbf{Cov}(\mathbf{u}_k) = \mathbf{Cov}(\mathbf{u}_{k+1}) = \begin{bmatrix} \sigma_D^2/2 & 0 \\ 0 & \sigma_a^2/2 \end{bmatrix}$$

(covariances add for independent **u** errors)

Applying (@@) twice,

$$\mathbf{Cov}(\mathbf{S}_{k+1}) = \begin{bmatrix} \sigma_D^2 & 0 & 0 \\ 0 & \frac{5}{8}D^2\sigma_a & \frac{3}{4}D\sigma_a^2 \\ 0 & \frac{3}{4}D\sigma_a^2 & \sigma_a^2 \end{bmatrix}$$

Covariance is inconsistent

The model is not physically based and does not accumulate errors correctly.

Analogous to ***lumped*** model of a transmission line versus ***distributed*** model.

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To overcome the problem, small time steps can be employed and covariance updated frequently.

=> computationally expensive and analogous to numerical integration.

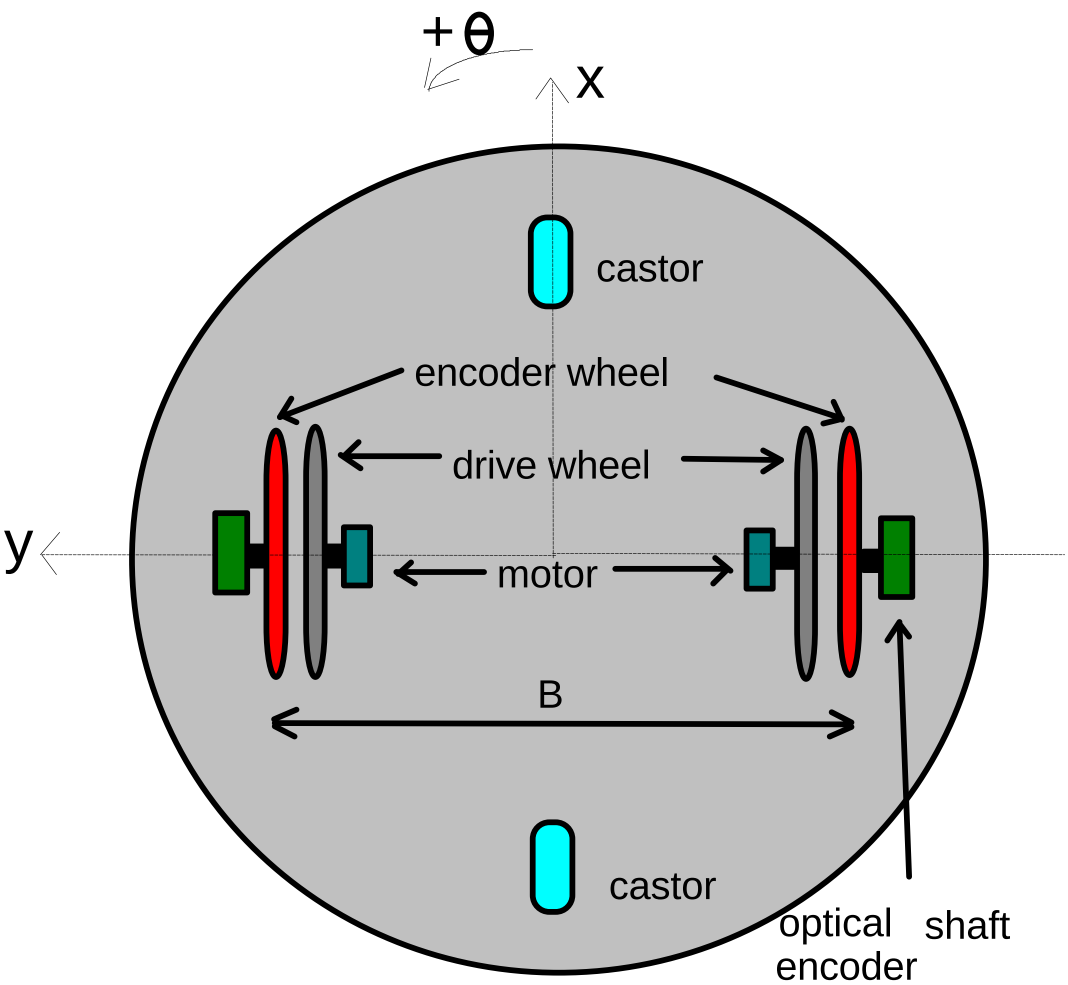
This paper develops a ***physically based*** approach and performs closed form odometry error ***integration*** for constant curvature paths.

Odometry error covariance can then be updated only when required or at the end of path segments.

Accuracy improved and computation saved

Robot Design and Modelling Assumptions

A new robot, called **Werrimbi,** has been designed for ***accurate odometry*** and ultrasonic sensor array experiments.



(Werrimbi - Australian aboriginal for bat.)

Features:

- Separate unloaded encoder wheels mounted on linear bearings - ***wheel slippage reduced***.
- Odometry wheels have narrow edged contact with floor - ***reduces wheel separation uncertainty B***.

Modelling Assumptions:

- Due to independent suspension, odometry errors on left wheel are assumed independent of the right wheel errors.
- Errors in one segment of travel are independent of the next segment.
- Errors assumed to be additive zero mean white noise.

The error ***variance*** of accumulated travel is the sum of each statistically independent variance per small length of travel. This implies error variance is proportional to distance travelled:

$$s_L^2 = k_L^2 \left| d_L \right| \qquad\qquad s_R^2 = k_R^2 \left| d_R \right|$$

Calibration of systematic odometry errors is performed using the University of Michigan bench mark test UMBmark:

Borenstein, J. and Feng, L. “UMBmark - A method for Measuring, Comparing, and Correcting Dead-reckoning Errors in Mobile Robots.”, Technical Report UM-MEAM-94-22, University of Michigan - also in IEEE Trans R&A Dec 1996.

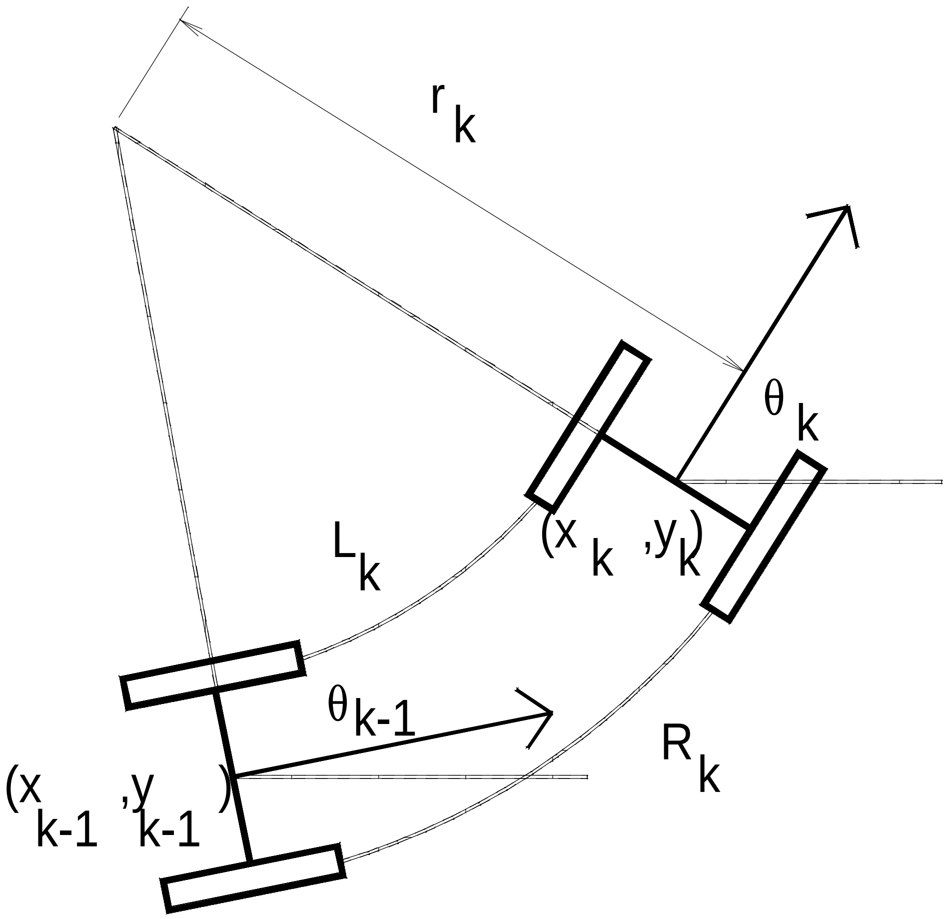
This derives correction factors for the wheel separation *B* and diameters based on mean odometry errors on 5 CW and 5 CCW traversals of a 4 m square.

The New Non-systematic Error Model

- Robot path treated as consisting of *k* small segments. Propagation of error covariance is performed *k* times to obtain the error covariance of the final state.

- A closed form solution is presented, as k approaches infinity, for a general circular arc path.

- Special cases of straight line and on-spot turn paths are obtained by suitably taking limits.



Over an infinitesimal time increment, the speed of the wheels can be assumed constant => path has constant radius of curvature:

$$r_k = \frac{B}{2}\left(\frac{L_k + R_k}{L_k - R_k}\right)$$

$$\mathbf{S}_k = \mathbf{S}_{k-1} + \begin{bmatrix} r_k\left[\sin q_{k-1} - \sin\left(q_{k-1} + \frac{R_k - L_k}{B}\right)\right] \\ r_k\left[\cos\left(q_{k-1} + \frac{R_k - L_k}{B}\right) - \cos q_{k-1}\right] \\ \frac{R_k - L_k}{B} \end{bmatrix}$$

$$= \mathbf{f}\left(\mathbf{S}_{k-1}, \mathbf{u}_k\right)$$

where $\mathbf{u}_k = \left[L_k \quad R_k\right]^T$.

The expression for covariance propagation can be recursively expanded

$$\mathbf{Cov}(\mathbf{S}_k) = \nabla_{\mathbf{S}_k}\mathbf{f}\ \ \mathbf{Cov}(\mathbf{S}_{k-1})\ \ \nabla_{\mathbf{S}_k}\mathbf{f}^T + \nabla_{\mathbf{u}_k}\mathbf{f}\ \ \mathbf{Cov}(\mathbf{u}_k)\ \nabla_{\mathbf{u}_k}\mathbf{f}^T$$

$$= \left(\prod_{i=1}^{k} \nabla_{\mathbf{S}_i}\mathbf{f}\right)\mathbf{Cov}(\mathbf{S}_0)\left(\prod_{i=1}^{k} \nabla_{\mathbf{S}_i}\mathbf{f}\right)^T$$

$$+ \sum_{i=1}^{k}\left\{\left(\prod_{j=i+1}^{k} \nabla_{\mathbf{S}_j}\mathbf{f}\right)\mathbf{Cov}(\mathbf{u}_k)\left(\prod_{j=i+1}^{k} \nabla_{\mathbf{S}_j}\mathbf{f}\right)^T\right\}$$

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Let L_i , R_i denote the small increments in wheel turn at the ith segment.

For circular arc motion, $L = kL_i$, $R = kR_i$.

propagation of state

$$\mathbf{Cov}(\mathbf{S}_k)$$

$$= \begin{bmatrix} 1 & 0 & r(\cos q_0 - \cos q_k) \\ 0 & 1 & r(\sin q_0 - \sin q_k) \\ 0 & 0 & 1 \end{bmatrix} \mathbf{Cov}(\mathbf{S}_0) \begin{bmatrix} 1 & 0 & r(\cos q_0 - \cos q_k) \\ 0 & 1 & r(\sin q_0 - \sin q_k) \\ 0 & 0 & 1 \end{bmatrix}^T$$

$$+ \sum_{i=1}^{k} \left\{ \left(\prod_{j=i+1}^{k} \nabla_{\mathbf{s}_j} \mathbf{f} \right) \begin{bmatrix} k_L^2 |L_i| & 0 \\ 0 & k_R^2 |R_i| \end{bmatrix} \left(\prod_{j=i+1}^{k} \nabla_{\mathbf{s}_j} \mathbf{f} \right)^T \right\}$$

defined as

As $k \to \infty$ (and considerable integration!)

$$Cov(U_k)_{1,1} = \frac{-2rc_1}{(L-R)^2}(\sin q_0 - \sin q_k)\cos q_k + \left(\frac{r}{B}\cos q_k\right)^2 c_2 - \frac{Bc_3}{4(L-R)^3}\left[2(q_k - q_0) - \sin(2q_0) + \sin(2q_k)\right]$$

$$Cov(U_k)_{2,2} = \frac{-2rc_1}{(L-R)^2}(\cos q_k - \cos q_0)\sin q_k + \left(\frac{r}{B}\sin q_k\right)^2 c_2 - \frac{Bc_3}{4(L-R)^3}\left[2(q_k - q_0) + \sin(2q_0) - \sin(2q_k)\right]$$

$$Cov(U_k)_{3,3} = \frac{c_2}{B^2}$$

$$Cov(U_k)_{1,2} = \frac{-rc_1}{(L-R)^2}(\cos(2q_k) - \cos(q_0 + q_k)) + \frac{c_2}{2}\left(\frac{r}{B}\right)^2 \sin(2q_k) - \frac{Bc_3}{4(L-R)^3}\left[\cos(2q_0) - \cos(2q_k)\right]$$

$$Cov(U_k)_{1,3} = \frac{c_1}{(L-R)^2}(\sin q_0 - \sin q_k) - \frac{rc_2}{B^2}\cos q_k$$

$$Cov(U_k)_{2,3} = \frac{c_1}{(L-R)^2}(\cos q_k - \cos q_0) - \frac{rc_2}{B^2}\sin q_k$$

where

$$c_1 = k_R^2|R|L + k_L^2 R|L|$$

$$c_2 = k_R^2|R| + k_L^2|L|$$

$$c_3 = k_R^2 L^2|R| + k_L^2 R^2|L|$$

- These equations remove the need to incrementally update the covariance matrix in small time steps.
- As closed form expressions, they are applicable to any circular arc motion with constant radius of curvature.

Straight Line Path:

Limiting case when $L,R \to D$ and $\theta_k \to \theta_0$.

Contrary to popular assumptions, our model predicts:

- error variance perpendicular to motion is proportional $\mathbf{D}^3$,
- variance in the direction of motion is only proportional to **D**.

Rotation about the centre of wheel axle is another special case easily handled.

This odometry error covariance model is ***consistent*** unlike the models presented in the literature.

> Error covariance propagation in multiple parts generates exactly the same result as a single part.

The model is computationally ***inexpensive***:

> Covariance updates can be performed at the end of paths.
>
> Paths with varying radius of curvature need to be broken up into shorter paths with approximately constant curvature.

The model is physically based:

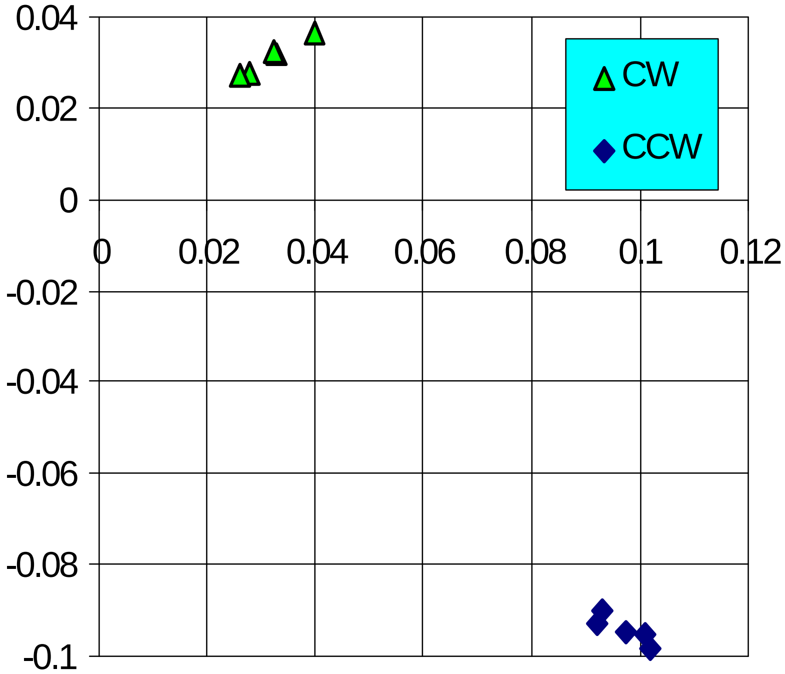
> Just two wheel error parameters are required for calibrating the error model: k_L and k_R
>
> These parameters are determined by the floor surface, wheel traction and wheel imperfections.

Implementation and Results

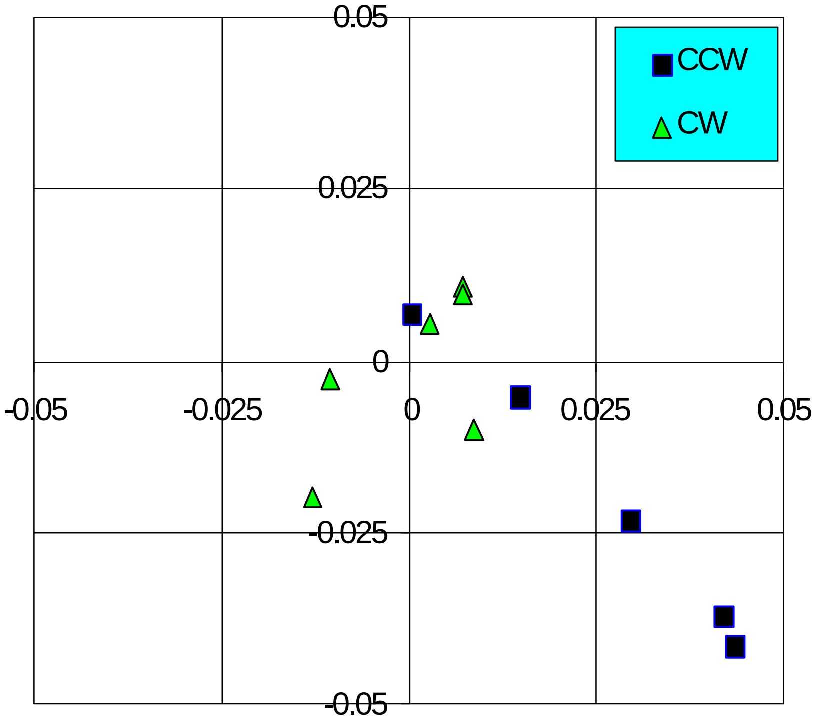
Systematic Errors

Wheel encoder measurements were used to calculate the perceived final state of the robot.

A sonar array on the robot was to used to estimate robot position and orientation by sensing reference walls.



BEFORE calibration (metres).



After Calibration (UMBmark in metres)

Key results before and after calibration

	Before	After
$x_{c.g.,CCW}$ (mm)	97	-26
$y_{c.g.,CCW}$ (mm)	-94	16
$x_{c.g.,CW}$ (mm)	32	1.5
$y_{c.g.,CW}$ (mm)	31	11
$E_{max,syst}$ (mm)	135	30

Comparison of dead-reckoning accuracy and cost. The first four sets of figures are obtained from [Borenstein and Feng]

Vehicle	$\mathbf{E_{max,syst}}$	**Cost (US$)**
TRC LabMate	average 27	10K
Cybermotion K2A	63	<50K
CLAPPER	22	30K
Andros with Trailer	74	?
Werrimbi	30	~4K

Computation of Non-systematic Error Parameters

After calibration, Werrimbi:

- sensed two reference walls with sonar
- moved forwards 10 metres
- moved backward 10 metres
- re-sensed the two reference walls

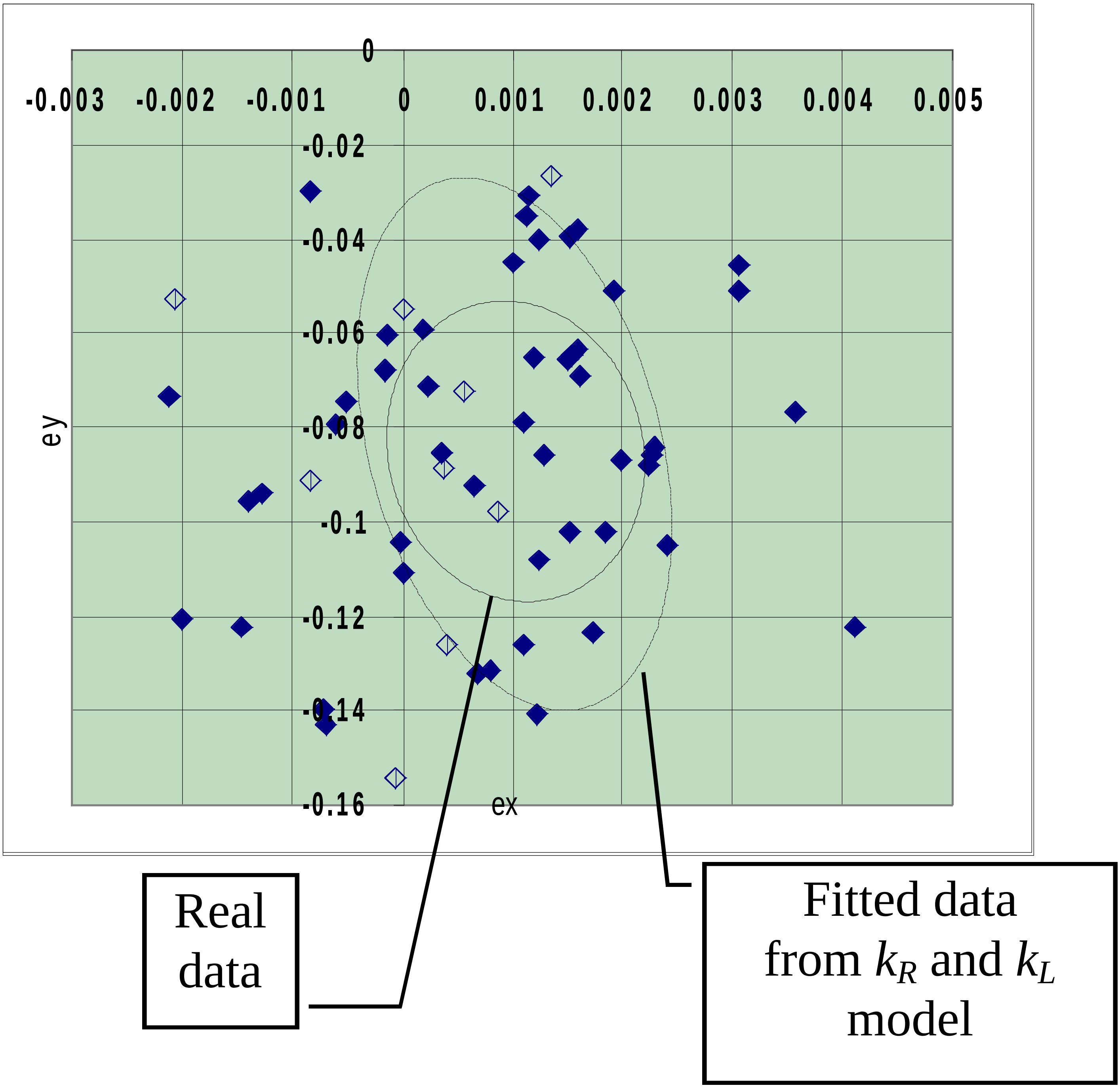
Position estimation from sonar sensing and odometry reading was compared.

The process was repeated 60 times in a 5 hour duration.

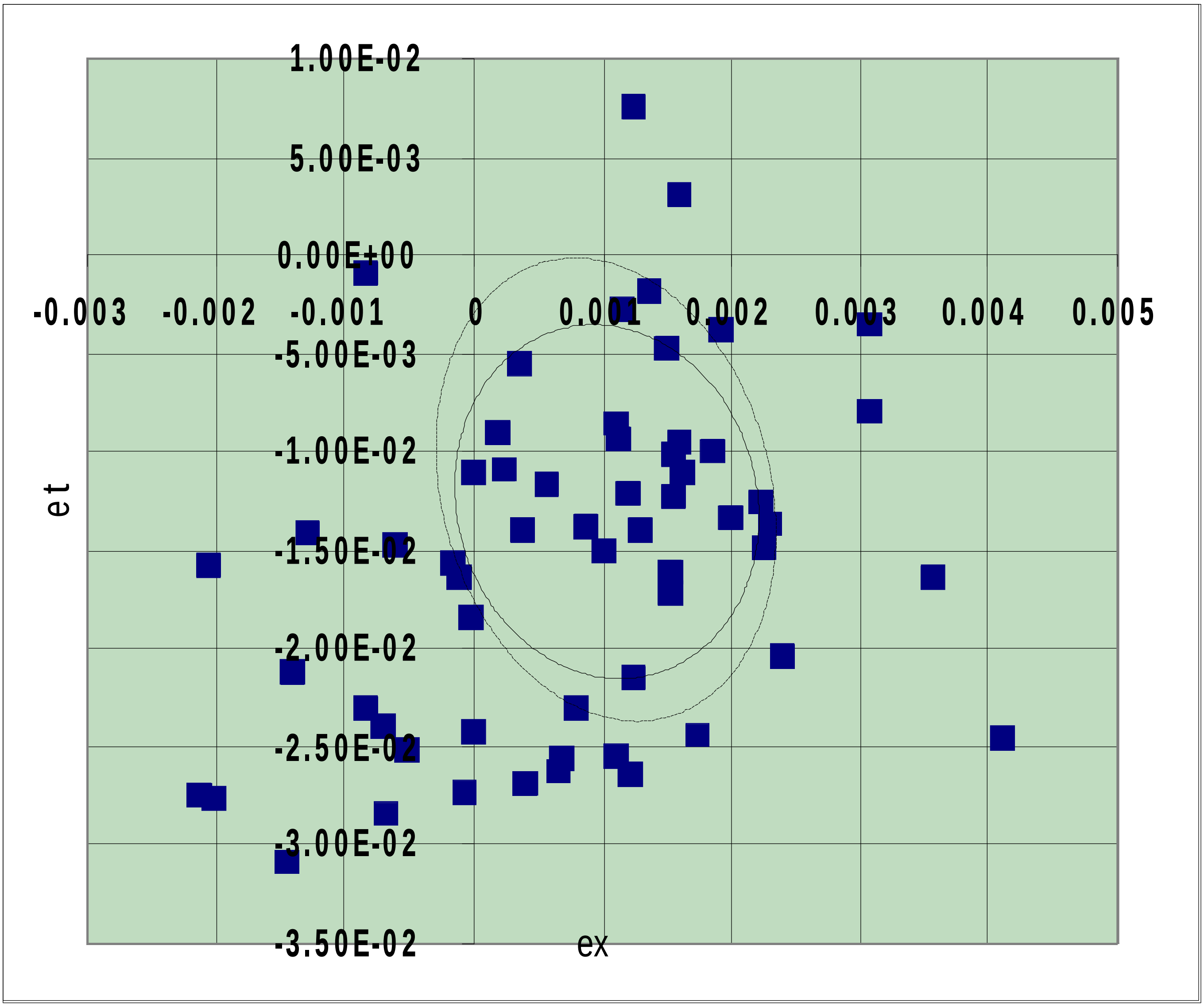
Errors in x, y and θ are plotted. Fitted values of k_L=0.00040m$^{1/2}$, k_R=0.00058m$^{1/}$ are shown with their error ellipses.

The solid ellipses belong to the real data whereas the dashed ellipses are generated with the k_R and k_L obtained by trial and error.

.Y errors against X errors



θ errors against X errors



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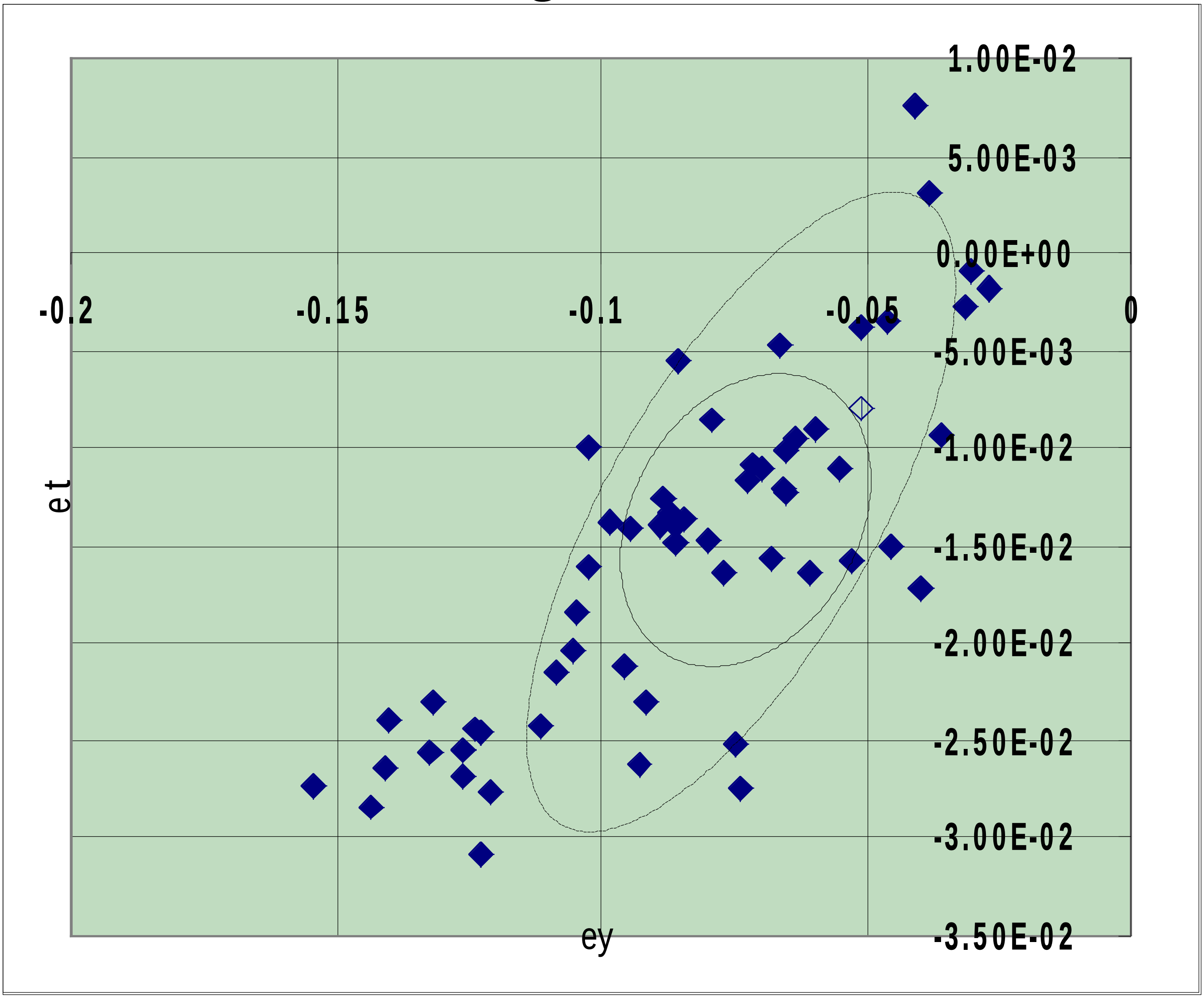
θ errors against Y errors



Conclusions

An accurate low cost odometry system has been presented comparable to the best reported system.

- A new first order odometry error model has been derived to consistently propagate error covariance following an arc, straight line or turn on the spot path. More complex paths can be approximated by these segments.

- Considerable computation can be saved using the model since covariance is not be updated on each odometry position update.

- The model fits into a Kalman filter framework and its applied to sonar map building in another paper at this conference.

- The model cannot account for unexpected errors such as hitting a bump on the floor. External referencing or redundant odometry can detect such errors.

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