Timing errors in distributed space-time communications

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***Abstract*— In this paper we study the effects of timing errors on distributed space-time codes for collaborative communications. We assume a half duplex, decode and forward scheme, where two relaying nodes are not perfectly synchronized in forwarding. We analyze 1) the impact of this asynchronism on the performance of an Alamouti and a Golden code scheme with different types of detectors, and 2) the sensitivity of above schemes to timing errors with optimum detectors, when perfect knowledge of timing errors is assumed.** [1]

I. Introduction

In a wireless communication channel, diversity techniques are commonly used to combat fading. Recently, in sensor network systems, there has been a growing interest in *co-operative diversity* techniques, where multiple terminals in a network cooperate to form a virtual antenna array in order to exploit spatial diversity in a distributed manner. The spatial diversity gain therefore can be obtained even when a local antenna array is not available. Such cooperative transmission protocols have been proposed in [1–10]. These protocols can be categorized into two principal classes: the amplify-and-forward (AF) scheme and the decode-and-forward (DF) scheme. In this paper, we focus on the DF scheme.

In all above schemes, the networking systems are assumed to have perfect *symbol level time-synchronization*. This can be reasonably assumed only when the spatial diversity is provided by an antenna array in one terminal. Cooperative diversity is asynchronous in nature as it is provided by different antennas in different terminals. The different distances between relays and destination may cause some timing errors, which should be considered in system design.

In [11], a method to achieve cooperative *delay diversity* by artificial delays between relays was proposed. At the destination receiver, a minimum mean square error (MMSE) estimator is used to exploit cooperative diversity. In [12], the authors developed a systematic construction of space-time trellis codes that achieve full cooperative diversity in *symbol asynchronous* (time delays are integer multiples of symbol period) cooperative networking for any number of relays.

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In [13], it is assumed that only *packet synchronization* is available. Asynchronous relays within a symbol period will result in small timing errors, which can reduce the benefits of *spatial diversity* provided by the relays. Performance analysis on the effect of timing errors was developed, where a simple Alamouti scheme [14] is employed with a baseband raise cosine pulse. Time reverse space-time codes and space-time coded orthogonal frequency division multiplexing (OFDM) were proposed to combat timing errors.

In our paper, we assume the same scenario as [13]. In contrast, we consider a baseband square pulse. This constitutes a simple worst case analysis with respect to a standard root raised cosine pulse. We first develop performance analysis on the effect of timing errors, where the Alamouti and the Golden code schemes [15] are used with different detection strategies. We further analyze the sensitivity of both schemes to timing errors, where a maximum likelihood detector (MLD) with knowledge of timing errors is assumed. It is shown that simulation results agree with the analysis.

The rest of the paper is organized as follows. Section II introduces a system model and performance analysis taking into account of the effects of timing errors of distributed space-time communication systems with Alamouti scheme. Following that, the sensitivity of the scheme to timing errors are analyzed. In Section III, we extend the above analysis to the Golden code. Finally, conclusions are drawn in Section IV.

II. Timing Errors in The Alamouti Scheme

In this Section, we consider a simple wireless network with two terminals and one destination. The two terminals are two relaying nodes cooperating in order to provide spatial diversity.

A. System Model

The system model is shown in Fig. 1. This model represents a system using a DF scheme for distributed space-time coding in half duplex mode. We assume that both terminals have successfully decoded a packet of symbols.

Under the above assumptions, we now focus on how the relaying nodes cooperatively forward the decoded symbols to the destination by using Alamouti scheme. We assume that the two relaying nodes are not fully synchronous and the time

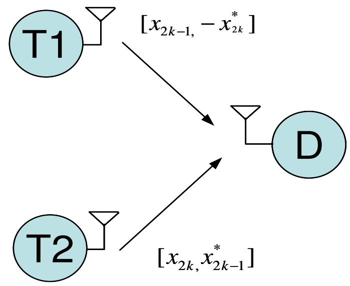

Fig. 1. System Model

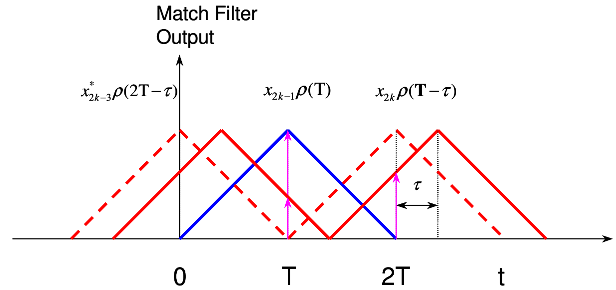

Fig. 2. Sampling at time nT in (2)

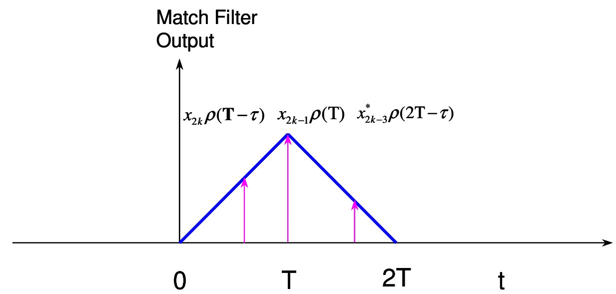

Fig. 3. A simplified illustration of Fig. 2

delay τ is less than one symbol duration T, i.e., $0 < \tau < T$. We also assume the destination is synchronous with one of the relaying nodes. We therefore analyze the effect of timing errors on the performance of such system, where the delay τ may be known or unknown at the receiver.

B. Effects of Timing Errors

Let $h_1(t)$ and $h_2(t)$ be the complex baseband channel impulse responses between the two relays and the destination. We assume the channel is flat fading, i.e., $h_i(t) = h_i\delta(t)$ where $h_i \sim \mathcal{N}_c(0,1)$. In the block fading model the channel changes from one packet to the next.

For simplicity, we consider a baseband square pulse $p(t)$ of duration T and amplitude one.[2] We consider packets of Alamouti codewords of length M and duration $2MT$. Looking at two consecutive transmitted Alamouti codewords

$$\begin{array}{l} \text{Relay 1} \\ \text{Relay 2} \end{array} \begin{bmatrix} x_{2k-3} & -x_{2k-2}^* \\ x_{2k-2} & x_{2k-3}^* \end{bmatrix} \begin{bmatrix} x_{2k-1} & -x_{2k}^* \\ x_{2k} & x_{2k-1}^* \end{bmatrix},$$

we can write the received signal as

$$\begin{aligned} r(t) = & \sum_{k=1}^{M} h_1(t) * [x_{2k-1}p(t-(2k-2)T) \\ & \quad -x_{2k}^* p(t-(2k-1)T)] \\ & + \sum_{k=1}^{M} h_2(t) * [x_{2k}p(t-(2k-2)T-\tau) \\ & \quad +x_{2k-1}^* p(t-(2k-1)T-\tau)] + z(t), \end{aligned} \tag{1}$$

where τ is the delay between the relays.

The receiver uses a matched filter (MF) $h_{MF}(t) = p(t)$ followed by a sampler at the instants nT. Define

$$\rho(t) = h_{MF}(t) * p(t) = \begin{cases} t & 0 \le t < T \\ 2T - t & T \le t < 2T \\ 0 & \text{elsewhere} \end{cases}$$

then sampling the output of the MF at time nT yields

$$\begin{aligned} y_{2k-1} &= h_1 x_{2k-1}\rho(T) + h_2 x_{2k}\rho(T-\tau) \\ &\quad + h_2 x_{2k-3}^*\rho(2T-\tau) + z_{2k-1} \\ y_{2k} &= -h_1 x_{2k}^*\rho(T) + h_2 x_{2k-1}^*\rho(T-\tau) \\ &\quad + h_2 x_{2k}\rho(2T-\tau) + z_{2k}. \end{aligned} \tag{2}$$

Fig. 2 depicts the sampling process at time nT with timing error τ in (2) yielding the output y_{2k-1}. Both signal pulses x_{2k} and x_{2k-3}^* (dashed lines) are shifted due to τ. This causes the interference term $x_{2k-3}^*\rho(2T-\tau)$ and signal x_{2k} with degraded amplitude, i.e., $x_{2k}\rho(T-\tau)$, in (2). A simplified illustration is given in 3. We now assume the receiver uses the standard Alamouti detection scheme and we get the decision variables (we set $k=1$ to simplify the notation)

$$\begin{aligned} \tilde{x}_1 &= h_1^* y_1 + h_2 y_2^* \\ \tilde{x}_2 &= h_2^* y_1 - h_1 y_2^* \end{aligned}$$

where $\tilde{x}_1$ and $\tilde{x}_2$ are given in (3) and (4), respectively. In (3) and (4), we have

$$\begin{aligned} A &= \left[|h_1|^2\rho(T) + |h_2|^2\rho(T-\tau)\right] \\ B &= h_1^* h_2\left[\rho(T-\tau) - \rho(T)\right]x_2 \\ &\quad + \left[h_1^* h_2 x_{-1}^* + |h_2|^2 x_2^*\right]\rho(2T-\tau) \\ D &= -h_1 h_2^*\left[\rho(T-\tau) - \rho(T)\right]x_1 \\ &\quad + \left[|h_2|^2 x_{-1}^* - h_1 h_2^* x_2^*\right]\rho(2T-\tau) \\ w_1 &= h_1^* z_1 + h_2 z_2^* \\ w_2 &= h_1^* z_1 - h_2 z_2^* \end{aligned}$$

where $w_1, w_2 \sim \mathcal{N}_c(0, (|h_1|^2 + |h_2|^2)N_0)$.

For simplicity, let us consider a BPSK constellation $\mathcal{S}$ with $x_1, x_2 = \pm 1$, the symbol error probability using symbol by symbol detection, *without knowledge* of τ, can be evaluated

[2]Other baseband pulses, such as square root raised cosine pulse, can be used here but more intersymbol interference terms of relatively small values will appear.

$$\begin{aligned}
\tilde{x}_1 &= h_1^*\left[h_1x_1\rho(T)+h_2x_2\rho(T-\tau)+h_2x_{-1}^*\rho(2T-\tau)+z_1\right] \\
&\quad +h_2\left[-h_1x_2^*\rho(T)+h_2x_1^*\rho(T-\tau)+h_2x_2\rho(2T-\tau)+z_2\right]^* \\
&= |h_1|^2x_1\rho(T)+h_1^*h_2x_2\rho(T-\tau)+h_1^*h_2x_{-1}^*\rho(2T-\tau)+h_1^*z_1 \\
&\quad -h_1^*h_2x_2\rho(T)+|h_2|^2x_1\rho(T-\tau)+|h_2|^2x_2^*\rho(2T-\tau)+h_2z_2^* \\
&= \left[|h_1|^2\rho(T)+|h_2|^2\rho(T-\tau)\right]x_1+h_1^*h_2\left[\rho(T-\tau)-\rho(T)\right]x_2 \\
&\quad +\left[h_1^*h_2x_{-1}^*+|h_2|^2x_2^*\right]\rho(2T-\tau)+h_1^*z_1+h_2z_2^* \\
&= Ax_1+B+w_1,
\end{aligned} \tag{3}$$

$$\begin{aligned}
\tilde{x}_2 &= h_2^*\left[h_1x_1\rho(T)+h_2x_2\rho(T-\tau)+h_2x_{-1}^*\rho(2T-\tau)+z_1\right] \\
&\quad -h_1\left[-h_1x_2^*\rho(T)+h_2x_1^*\rho(T-\tau)+h_2x_2\rho(2T-\tau)+z_2\right]^* \\
&= h_1h_2^*x_1\rho(T)+|h_2|^2x_2\rho(T-\tau)+|h_2|^2x_{-1}^*\rho(2T-\tau)+h_2^*z_1 \\
&\quad |h_1|^2x_2\rho(T)-h_1h_2^*x_1\rho(T-\tau)-h_1^*h_2x_2^*\rho(2T-\tau)-h_1z_2^* \\
&= \left[|h_1|^2\rho(T)+|h_2|^2\rho(T-\tau)\right]x_2-h_1h_2^*\left[\rho(T-\tau)-\rho(T)\right]x_1 \\
&\quad +\left[|h_2|^2x_{-1}^*-h_1h_2^*x_2^*\right]\rho(2T-\tau)+h_1^*z_1-h_2z_2^* \\
&= Ax_2+D+w_2,
\end{aligned} \tag{4}$$

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as

$$\begin{aligned}
P_\tau(e) &= E[P(e|h_1,h_2,x_{-1},x_2)] \\
&= E\left[\frac{1}{2}P(\Re(\tilde{x}_1)>0|x_1=-1)\right. \\
&\quad \left.+\frac{1}{2}P(\Re(\tilde{x}_1)<0|x_1=+1)\right] \\
&= \frac{1}{4}E\left[\operatorname{erfc}\left(\frac{A-\Re(B)}{\sqrt{(|h_1|^2+|h_2|^2)N_0}}\right)\right. \\
&\quad \left.+\operatorname{erfc}\left(\frac{A+\Re(B)}{\sqrt{(|h_1|^2+|h_2|^2)N_0}}\right)\right]
\end{aligned} \tag{5}$$

where $P(e|h_1,h_2,x_{-1},x_2)$ is the conditional error probability and $E[\cdot]$ is the average over h_1,h_2,x_{-1},x_2. Note that the same result can be obtained using $\tilde{x}_2$. The bit error rate (BER) is shown in Fig. 4.

If τ *is known at the receiver*, a decision feedback equalizer (DFE) can be used to cancel the interference from previous symbol x_{-1}. A slightly improved performance can be observed in Fig. 5.

In Figs. 4 and 5, we notice that an error floor for large E_b/N_0 appears when $\tau/T \geq 0.5$. This is given by the probability that $E[P(\Re(B)>A|h_1,h_2,x_{-1},x_2),w_1=0]$, which can be shown to vanish for $\tau/T<0.5$.

C. Sensitivity of the Alamouti Scheme to Timing Errors

Assuming τ is known at the receiver, using the ideal DFE, an MLD is employed based on minimizing

$$\begin{aligned}
&\min_{x_1,x_2\in\mathcal{S}}\left\{|y_1-h_1x_1\rho(T)-h_2x_2\rho(T-\tau)|^2\right. \\
&\quad \left.+|y_2+h_1x_2^*\rho(T)-h_2x_1^*\rho(T-\tau)-h_2x_2\rho(2T-\tau)|^2\right\}.
\end{aligned} \tag{6}$$

over all pairs of $x_1,x_2\in\mathcal{S}$.

We thus can rewrite (2) as

$$\mathbf{Y}_k = \mathbf{H}_d\times(\mathbf{D}\times\mathbf{X}_k+\mathbf{X'}_k)+\mathbf{Z}_k \tag{7}$$

where

$$\mathbf{Y}_k=[y_{2k-1} \quad y_{2k}]^T$$

$$\mathbf{H}_d=[h_1 \quad h_2]$$

$$\mathbf{D}=\begin{bmatrix}\rho(T) & 0\\ 0 & \rho(T-\tau)\end{bmatrix}$$

$$\mathbf{X}_k=\begin{bmatrix}x_{2k-1} & -x_{2k}^*\\ x_{2k} & x_{2k-1}^*\end{bmatrix}$$

$$\mathbf{X'}_k=\begin{bmatrix}0 & 0\\ x_{2k}\rho(2T-\tau) & 0\end{bmatrix}$$

and

$$\mathbf{Z}_k=[z_{2k-1} \quad z_{2k}]^T$$

If $\tau\approx 0$, we have $\rho(2T-\tau)\approx 0$, yielding $\mathbf{X'}_k\approx\mathbf{0}$, where $\mathbf{0}$ is a 2×2 zero matrix. In such a case, diversity and coding gains of the system depend on the determinant of $\mathbf{D}$ solely, i.e., $\det(\mathbf{D})=1-\tau/T$. Since $\det(\mathbf{D})$ is less than 1, we can see that a coding gain loss appears without affecting the diversity gain.

If τ is large, we assume $\rho(T-\tau)\approx 0$ and $\rho(2T-\tau)\approx 1$. Consequently, a large τ yields $\det\mathbf{D}\approx 0$, which causes both rank and determinant loss. Simulation results confirming the above analysis are shown in Fig. 6. In particular, up to $\tau/T=0.5$, the performance curves maintain the same diversity.

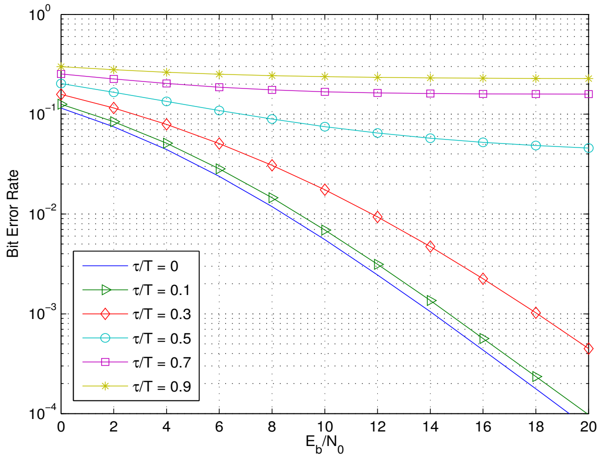

Fig. 4. Impact of time delay on bit error rate for BPSK Alamouti scheme using symbol by symbol Alamouti detection

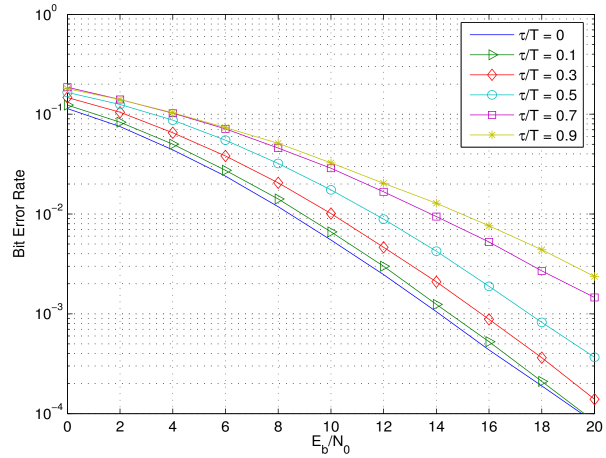

Fig. 6. Impact of time delay on bit error rate for BPSK Alamouti scheme using ML detection with ideal DFE

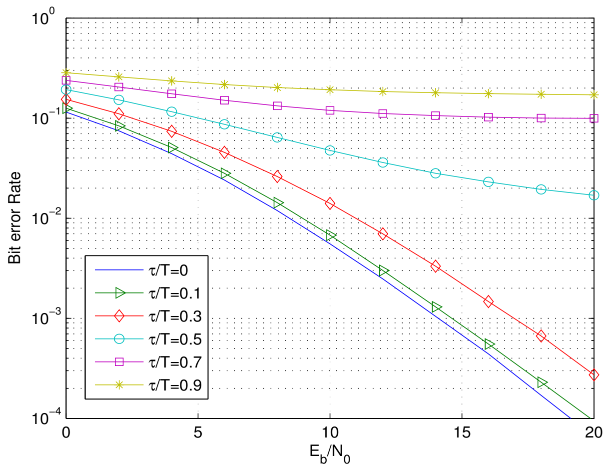

Fig. 5. Impact of time delay on bit error rate for BPSK Alamouti scheme using symbol by symbol Alamouti detection with ideal DFE

III. Timing Errors on The Golden Code Scheme

In this Section, we consider the same system model as above. We assume that two relaying nodes with single antenna cooperate by using the Golden code scheme to reach a two-antenna receiver. Each relay transmits four QAM information symbols using half Golden codeword as illustrated below. The delay τ may be known or unknown at the receiver. We analyze the effect of timing errors and the corresponding sensitivity to timing errors respectively.

A. Effects of Timing Errors

Let $H(t) = \{h_{i,j}(t)\}$, $i, j = 1, 2$, be the complex baseband channel impulse responses between two relays and the two destination antennas. We assume the channel is flat fading i.e. $h_{i,j}(t) = h_{i,j}\delta(t)$ where $h_{i,j} \sim \mathcal{N}_c(0, 1)$. In the block fading model the channel changes from one packet to the next.

We consider packets of length M Golden codewords with duration $2MT$. Assuming two consecutive Golden codewords are sent

$$\begin{matrix} \text{Relay 1} \\ \text{Relay 2} \end{matrix} \begin{bmatrix} x_{4k-7} & x_{4k-5} \\ x_{4k-6} & x_{4k-4} \end{bmatrix} \begin{bmatrix} x_{4k-3} & x_{4k-1} \\ x_{4k-2} & x_{4k} \end{bmatrix}$$

the Golden codewords are encoded as

$$\frac{1}{\sqrt{5}} \begin{bmatrix} \alpha\,(a + b\theta) & \alpha\,(c + d\theta) \\ i\bar{\alpha}\,(c + d\bar{\theta}) & \bar{\alpha}\,(a + b\bar{\theta}) \end{bmatrix}, \tag{8}$$

where $a, b, c, d \in \mathbb{Z}[i]$ are the Q-QAM information symbols, $\theta = 1 - \bar{\theta} = \frac{1+\sqrt{5}}{2}$, $\alpha = 1 + i - i\theta$, $\bar{\alpha} = 1 + i(1 - \bar{\theta})$, and the factor $\frac{1}{\sqrt{5}}$ is used to normalize energy [15]. This code has full rate, full rank and non-vanishing determinant ($\delta_{\min} = 1/5$) for increasing constellation size Q [15]. We can write the two received signals as

$$\begin{aligned} r_1(t) = &\sum_{k=1}^{M} h_{11}(t) * [x_{4k-3}p(t - (2k-2)T) \\ &\quad + x_{4k-1}p(t - (2k-1)T)] \\ &+ \sum_{k=1}^{M} h_{12}(t) * [x_{4k-2}p(t - (2k-2)T - \tau) \\ &\quad + x_{4k}p(t - (2k-1)T - \tau)] + z_1(t), \end{aligned} \tag{9}$$

$$\begin{aligned} r_2(t) = &\sum_{k=1}^{M} h_{21}(t) * [x_{4k-3}p(t - (2k-2)T) \\ &\quad + x_{4k-1}p(t - (2k-1)T)] \\ &+ \sum_{k=1}^{M} h_{22}(t) * [x_{4k-2}p(t - (2k-2)T - \tau) \\ &\quad + x_{4k}p(t - (2k-1)T - \tau)] + z_2(t). \end{aligned} \tag{10}$$

We assume we use the same matched filter as in the Alamouti scheme for each receive antenna. Then sampling both outputs of the MF at time nT yields four samples

$$\begin{aligned}y_{4k-3} &= y_1((2k-1)T)\\ &= h_{11}x_{4k-3}\rho(T)+h_{12}x_{4k-2}\rho(T-\tau)\\ &\quad +h_{12}x_{4k-4}\rho(2T-\tau)+z_{4k-3},\\ y_{4k-1} &= y_1(2kT)\\ &= h_{11}x_{4k-1}\rho(T)+h_{12}x_{4k}\rho(T-\tau)\\ &\quad +h_{12}x_{4k-2}\rho(2T-\tau)+z_{4k-1}\\ y_{4k-2} &= y_2((2k-1)T)\\ &= h_{21}x_{4k-3}\rho(T)+h_{22}x_{4k-2}\rho(T-\tau)\\ &\quad +h_{22}x_{4k-4}\rho(2T-\tau)+z_{4k-2},\\ y_{4k} &= y_2(2kT)\\ &= h_{21}x_{4k-1}\rho(T)+h_{22}x_{4k}\rho(T-\tau)\\ &\quad +h_{22}x_{4k-2}\rho(2T-\tau)+z_{4k}.\end{aligned}$$

Defining

$$\begin{aligned}\mathbf{y} &= [y_{4k-3}, y_{4k-1}, y_{4k-2}, y_{4k}]^T,\\ \mathbf{x} &= [x_{4k-3}, x_{4k-1}, x_{4k-2}, x_{4k}]^T,\\ \mathbf{z} &= [z_{4k-3}, z_{4k-1}, z_{4k-2}, z_{4k}]^T,\end{aligned}$$

we write $\mathbf{y}=\mathbf{H}\mathbf{x}+\mathbf{z}$, where

$$\mathbf{H}=\begin{bmatrix} h_{11}\rho(T) & 0 & h_{12}\rho(T-\tau) & 0\\ 0 & h_{11}\rho(T) & h_{12}\rho(2T-\tau) & h_{12}\rho(T-\tau)\\ h_{21}\rho(T) & 0 & h_{22}\rho(T-\tau) & 0\\ 0 & h_{21}\rho(T) & h_{22}\rho(2T-\tau) & h_{22}\rho(T-\tau)\end{bmatrix}$$

Separating real and imaginary parts yields

$$\begin{bmatrix}\Re(\mathbf{y})\\ \Im(\mathbf{y})\end{bmatrix}=\begin{bmatrix}\Re(\mathbf{H}) & -\Im(\mathbf{H})\\ \Im(\mathbf{H}) & \Re(\mathbf{H})\end{bmatrix}\times\begin{bmatrix}\Re(\mathbf{x})\\ \Im(\mathbf{x})\end{bmatrix}+\begin{bmatrix}\Re(\mathbf{z})\\ \Im(\mathbf{z})\end{bmatrix}$$

Note that in [15] the Golden code can be identified for decoding purposes with the rotated lattice $\mathbf{R}\mathbb{Z}^8=\{\mathbf{x}=\mathbf{R}\mathbf{u}\}$, where

$$\mathbf{u} = [\Re(a),\,\Re(b),\,\Re(c),\,\Re(d),\,\Im(a),\,\Im(b),\,\Im(c),\,\Im(d)]^T$$

is an 8 dimension integer lattice and

$$\mathbf{R}=\frac{1}{\sqrt{5}}\begin{bmatrix} 1 & \theta & 0 & 0 & -\bar{\theta} & 1 & 0 & 0\\ 0 & 0 & 1 & \theta & 0 & 0 & -\bar{\theta} & 1\\ 0 & 0 & -\theta & 1 & 0 & 0 & -1 & -\bar{\theta}\\ 1 & \bar{\theta} & 0 & 0 & -\theta & 1 & 0 & 0\\ \bar{\theta} & -1 & 0 & 0 & 1 & \theta & 0 & 0\\ 0 & 0 & \bar{\theta} & -1 & 0 & 0 & 1 & \theta\\ 0 & 0 & 1 & \bar{\theta} & 0 & 0 & -\theta & 1\\ \theta & -1 & 0 & 0 & 1 & \bar{\theta} & 0 & 0\end{bmatrix} \quad (11)$$

is a rotation matrix preserving the shape of the QAM information symbols a, b, c, d. Eq. (11) can be written as

$$\mathcal{Y}=\mathcal{H}\mathbf{u}+\mathcal{Z} \quad (12)$$

where

$$\mathcal{H}=\begin{bmatrix}\Re(\mathbf{H}) & -\Im(\mathbf{H})\\ \Im(\mathbf{H}) & \Re(\mathbf{H})\end{bmatrix}\times\mathbf{R}, \quad (13)$$

and

$$\mathcal{Y}=\begin{bmatrix}\Re(\mathbf{y})\\ \Im(\mathbf{y})\end{bmatrix}, \qquad \mathcal{Z}=\begin{bmatrix}\Re(\mathbf{z})\\ \Im(\mathbf{z})\end{bmatrix}. \quad (14)$$

DFE is used to cancel the interference term from the previous codeword. If τ is known at the receiver, a significant performance improvement can be obtained by using MLD (see Fig. 7). The MLD is obtained by a lattice decoder in order to find $\mathbf{u}$ such that

$$\min_{\mathbf{u}\in\mathbb{Z}^8}\|\mathcal{Y}-\mathcal{H}\mathbf{u}\|^2. \quad (15)$$

If τ is unknown at the receiver, let us define

$$\tilde{\mathbf{H}}=\begin{bmatrix} h_{11} & 0 & h_{12} & 0\\ 0 & h_{11} & 0 & h_{12}\\ h_{21} & 0 & h_{22} & 0\\ 0 & h_{21} & 0 & h_{22}\end{bmatrix} \quad (16)$$

and

$$\tilde{\mathcal{H}}=\begin{bmatrix}\Re(\tilde{\mathbf{H}}) & -\Im(\tilde{\mathbf{H}})\\ \Im(\tilde{\mathbf{H}}) & \Re(\tilde{\mathbf{H}})\end{bmatrix}\times\mathbf{R}, \quad (17)$$

The MLD is obtained by a lattice decoder in order to find $\mathbf{u}$ such that

$$\min_{\mathbf{u}\in\mathbb{Z}^8}\left\|\mathcal{Y}-\tilde{\mathcal{H}}\mathbf{u}\right\|^2. \quad (18)$$

The performance can be observed in Fig. 8. We note that severe error floor appears even for small values of τ/T.

B. Sensitivity of the Golden Code to Timing Errors

Assuming τ is known at the receiver, using the ideal DFE, we have

$$\mathbf{H}=\mathbf{H}_d\times\mathbf{D}$$

where

$$\mathbf{H}_d=\begin{bmatrix} h_{11} & 0 & h_{12} & 0\\ 0 & h_{11} & h_{12}\cdot\frac{\rho(2T-\tau)}{\rho(T-\tau)} & h_{12}\\ h_{21} & 0 & h_{22} & 0\\ 0 & h_{21} & h_{22}\cdot\frac{\rho(2T-\tau)}{\rho(T-\tau)} & h_{22}\end{bmatrix}$$

and

$$\mathbf{D}=\begin{bmatrix}\rho(T) & 0 & 0 & 0\\ 0 & \rho(T) & 0 & 0\\ 0 & 0 & \rho(T-\tau) & 0\\ 0 & 0 & 0 & \rho(T-\tau)\end{bmatrix}$$

If $\tau\approx 0$, we have

$$\frac{\rho(2T-\tau)}{\rho(T-\tau)}\approx 0$$

and $\mathbf{H}_d\cong\tilde{\mathbf{H}}$ in (16). Consequently, degradation of the determinant of $\mathbf{D}$, i.e., $\det(\mathbf{D})=(1-\tau/T)^2$, fully affects the performance. For small τ, there is a small coding gain loss without diversity gain loss, since $\det(\mathbf{D})$ is less than 1. For large τ, we observe $\det(\mathbf{D})\approx 0$, which is equivalent to loosing rank thereby reducing diversity, as illustrated in Fig. 7.

Note that the degradation of $\det(\mathbf{D})=(1-\tau/T)^2$ in the Golden code scheme is more sensitive to timing errors τ, when compared to that of $\det(\mathbf{D})=(1-\tau/T)$ in the Alamouti scheme.

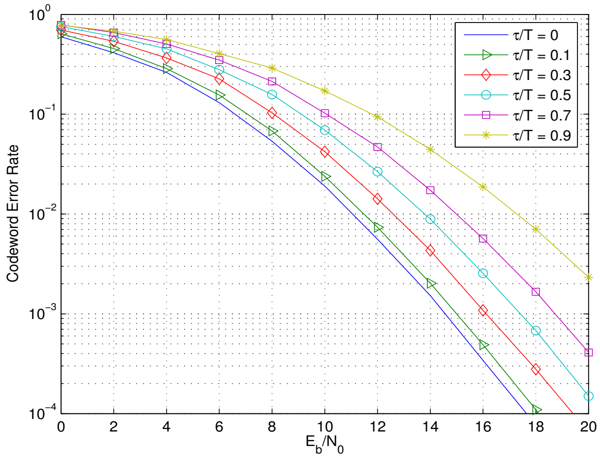

Fig. 7. Impact of time delay on codeword error rate for QPSK Golden code with knowledge of τ

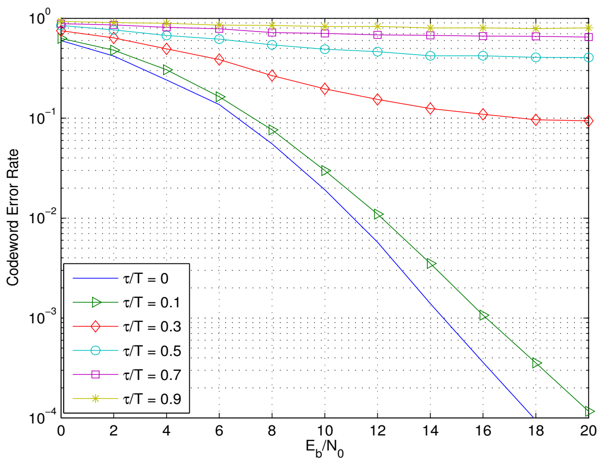

Fig. 8. Impact of time delay on codeword error rate for QPSK Golden code without knowledge of τ

IV. Conclusions

In this paper we considered distributed space-time codes for collaborative communications. We assumed a half duplex, decode and forward scheme, where two relaying nodes are not perfectly synchronized in forwarding and the delay is limited to one symbol period. We analyzed the effect of this asynchronism on the performance of an Alamouti and a Golden code scheme with different types of detectors. We also analyzed the sensitivity of both schemes to timing errors, when optimum detectors are assumed with knowledge of τ. In all cases, timing error τ is always a critical issue. With knowledge of τ, the degradation of determinant $\det(\mathbf{D})$ always affects the performance. This suggests us to use large determinant space-time block codes to compensate the performance loss due to the delay.

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