Joint Optimization Scheme and Sum Constellation Distribution for Multi-User Gaussian Multiple Access Channels with Finite Input Constellations

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Abstract—Phase adjusting and power adjusting input schemes of non-orthogonal *Gaussian multiple access channels* (GMAC) for two-user with finite input constellations have been studied. In this paper, the input scheme for multi-user is jointly optimized by both powers and phases. This scheme is deduced from the *constellation constrained* (CC) capacity of multi-user GMAC. Performance analysis indicates that the proposed input scheme outperforms previous input schemes for most SNR and achieves the largest enlargement of CC capacity region at high SNR values. Motivated by the properties of sum constellation's distance distribution, a procedure to efficiently obtain optimal multi-user input schemes is proposed for MPSK input constellations.

I. INTRODUCTION

Wireless communications, e.g. satellite communications and uplink of 5G communications, will be demanded for supporting a considerable number of users while achieving high spectral efficiency [1], [2]. In the last few decades, *non-orthogonal multiple access* (NO-MA) technology [3], [4], [5], [6] has drawn much attention since it provides better spectrum efficiency than orthogonal multiple access such as FDMA and TDMA. Towards this direction, a lot of researches have focused on evaluation of non-orthogonal multiple access channels' capacity regions and their effective input schemes [4], [5] of GMAC. In [7], the capacity achieving input scheme of GMAC is proved to be a continuous Gaussian distribution. However, the practical input whose constellations have finite size and uniform distribution are essentially different from the continuous Gaussian input. It has driven many researchers to study GMAC with finite input constellations [8], [9], [10].

In [8], J. Harshan, etc. deduced the expression of two-user GMAC's CC capacity region and provided an insight into the design of input schemes which maximize the capacity region. The further study shows that two approaches of input schemes, the *constellation rotation* (CR) scheme [9] and the *constellation power allocation* (CPA) scheme [10], are available in two-user case with identical input constellations. These schemes introduce differences between input constellations to eliminate the ambiguity of sum constellation. For the CR scheme, one appropriate angle of rotation is introduced between input constellations to construct unambiguous sum constellation. The CPA scheme varies the instantaneous power of two users and achieves similar performance as the CR scheme. Inspired by these results, researches have been extended to MIMO case including MIMO GMAC [11], [12], MIMO Gaussian

interference channel [13], [14], [15] and MIMO broadcast channel [16].

However, these two schemes only use powers or phases to achieve a better sum constellation distribution. Although these schemes can enlarge the system's CC capacity to upper-bound at high SNR, neither of them optimizes the distribution. Moreover, little researches have been done on GMAC's multi-user input schemes. In this paper, the joint optimization input scheme is proposed to maximally enlarge the multi-user GMAC's CC capacity. Firstly, CC capacity of multi-user GMAC is derived as a preliminary knowledge base for this scheme. Then, the joint optimization scheme which adjusts factors of both power and phase is proposed to achieve maximal enlargement of CC capacity. The simulation result of a three-user case is presented to show the performance of this scheme. Furthermore, we notice the multi-peak phenomenon that different input schemes achieve similar CC capacity at some SNR. This phenomenon motivates us to study the relationship between different input schemes' sum constellation distributions. We find that minimum distances of sum constellation have significant impact on the performance of CC capacity. A classification of sum constellation's distance pairs which is universally valid for arbitrary input schemes is also discovered based on some unique properties of sum constellation's distance distribution. This classification inspires us to propose an efficient algorithm for obtaining optimal multi-user input schemes.

The rest of this paper is organized below. In Section II, we introduce the signal model of multi-user GMAC and its sum constellation. Section III presents the expression of CC capacity for arbitrary multi-user case and the multi-user joint optimization scheme. In Section IV, we analyze the sum constellation distribution and give the procedure of providing optimal joint optimization schemes. Section V gives the conclusion of this paper.

II. MULTI-USER GMAC: SIGNAL MODEL AND SUM CONSTELLATION

A. Signal model of multi-user GMAC

The model of arbitrary multi-user GMAC is shown in Fig. 1. For the NO-MA scheme, these m users have to
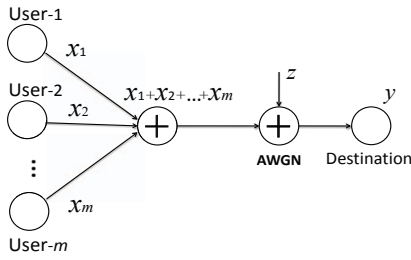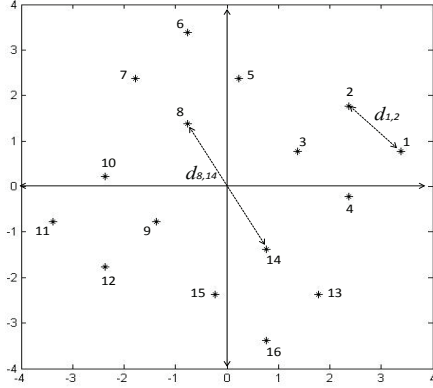
Fig. 1. m-user Gaussian MAC model.

Fig. 2.   The sum constellation of a two-user case.

transmit their respective information to a single destination at the same time and in the same frequency band. These m users are equipped with finite complex input signal sets $\mathcal{S}_1, \mathcal{S}_2, \cdots, \mathcal{S}_m$ whose size are $N_1, N_2, \cdots, N_m$ respectively. Moreover, considering the average power constraint P_i of each user, User-i transmits the symbol $\sqrt{P_i}x_i$ for $x_i \in \mathcal{S}_i$. When all users transmit symbols $x_1, x_2, \cdots, x_m$ from their own signal sets, the destination receives a symbol y:

$$y = \sqrt{P_1}x_1 + \sqrt{P_2}x_2 + \cdots + \sqrt{P_m}x_m + z, \qquad (1)$$

where $E[|x_i|^2] = 1$ and $z \sim \mathcal{CG}(0, \sigma^2)$. $\mathcal{CG}(\mu, \Gamma)$ is a circularly symmetric complex Gaussian random vector with mean μ and covariance matrix Γ. Note that input symbols are finite and chosen with uniform distribution while output is continuous.

B. Sum constellation of multi-user GMAC

The expression of transmitted symbol is shown in (1). When no noise is involved, destination y receives symbols chosen from a sum constellation

$$S_{\text{sum}} = \sqrt{P_1}S_1 + \sqrt{P_2}S_2 + \cdots + \sqrt{P_m}S_m. \qquad (2)$$

For example, the sum constellation of a two-user case is presented in Fig. 2. Both users apply MPSK signal sets and we assume that $M = 4$. The sum constellation has the following form

$$S_{\text{sum}} = S_1 + \gamma e^{j\theta}S_2, \qquad (3)$$

where $\gamma = 2.5$, $\theta = \frac{\pi}{10}$ and $S_1 = S_2 = e^{j\frac{k-1}{4}2\pi}$ with $k = 1, 2, 3$ and 4. Note that the number of points in this sum

constellation equals to M^2. This is the uniquely decodable group according to the definition in [8]. Unique decodability (UD) is one of the basic requirements of optimal input schemes to maximally enlarge the CC capacity.

Define pairwise distance $d_{i,j}$ as the 2-norm distance between point with label i and point with label j in complex plane. If S_{sum} has the form of (3), the pairwise distance $d_{1,2}$ and $d_{8,14}$ in Fig. 2 are

$$d_{1,2} = \sqrt{2},$$
$$d_{8,14} = |e^{j\frac{1}{4}2\pi} + \gamma e^{j\theta}e^{j\frac{3}{4}2\pi} - (e^{j\frac{3}{4}2\pi} + \gamma e^{j\theta}e^{j\frac{1}{4}2\pi})| \qquad (4)$$
$$= 4\gamma^2 + 4 - 8\gamma \cos\theta.$$

Different choices of γ and θ result in different distance distribution. As a result, proper parameters setting will provide input schemes with good distance distribution, and vice versa.

Note that some pairwise distances are identical no matter what input scheme is applied. These distance groups are defined as *distance classes* [17]. For example, pairwise distance $d_{3,9}$ is always identical to $d_{8,14}$. Therefore, distance distribution of sum constellation can be classified into several classes. Based on this property, a classification of multi-user sum constellation's distance pairs is presented in Section IV.

III.   CC CAPACITY AND THE JOINT OPTIMIZATION SCHEME OF MULTI-USER GMAC

In this section, the expression of two-user CC capacity is generalized to arbitrary multi-user case based on UD. Then, the joint optimization scheme is proposed to maximize multi-user GMAC's CC capacity region based on this expression.

A. CC capacity of multi-user GMAC

Without loss of generality, the CC capacity of three-user GMAC is computed below to exemplify the generalization from two-user CC capacity to multi-user case. The CC capacity of three-user GMAC is referred as the mutual information between input and output of this Gaussian channel $I(\sqrt{P_1}x_1 + \sqrt{P_2}x_2 + \sqrt{P_3}x_3; y)$. Then,

$$\begin{aligned} &I(\sqrt{P_1}x_1 + \sqrt{P_2}x_2 + \sqrt{P_3}x_3; y) \\ &= I(\sqrt{P_1}x_1; y) + I(\sqrt{P_2}x_2 + \sqrt{P_3}x_3; y|\sqrt{P_1}x_1) \\ &= I(\sqrt{P_1}x_1; y) + I(\sqrt{P_2}x_2; y|\sqrt{P_1}x_1) \\ &\quad + I(\sqrt{P_3}x_3; y|\sqrt{P_1}x_1, \sqrt{P_2}x_2). \end{aligned} \qquad (5)$$

Towards deriving its CC capacity, each term of the expression above is computed. Considering each term of (5), where $H(x)$ represents the entropy of x,

$$I(\sqrt{P_1}x_1; y) = H(y) - H(y|\sqrt{P_1}x_1). \qquad (6)$$

The other two terms have similar expression:

$$\begin{aligned} &I(\sqrt{P_2}x_2; y|\sqrt{P_1}x_1) \\ &= H(y|\sqrt{P_1}x_1) - H(y|\sqrt{P_1}x_1, \sqrt{P_2}x_2), \end{aligned} \qquad (7)$$

and

$$\begin{aligned} &I(\sqrt{P_3}x_3; y|\sqrt{P_1}x_1, \sqrt{P_2}x_2) \\ &= H(y|\sqrt{P_1}x_1, \sqrt{P_2}x_2) - H(y|\sqrt{P_1}x_1, \sqrt{P_2}x_2, \sqrt{P_3}x_3). \end{aligned} \qquad (8)$$

Using (6), (7) and (8), $I(\sqrt{P_1}x_1 + \sqrt{P_2}x_2 + \sqrt{P_3}x_3; y)$ is given by

$$
\begin{aligned}
&I(\sqrt{P_1}x_1 + \sqrt{P_2}x_2 + \sqrt{P_3}x_3; y) \\
&= H(y) - H(y|\sqrt{P_1}x_1, \sqrt{P_2}x_2, \sqrt{P_3}x_3).
\end{aligned} \tag{9}
$$

Note that all terms of $I(\sqrt{P_2}x_2; y|\sqrt{P_1}x_1)$ have been eliminated.

Considering the m-user case in Fig. 1, its expression of CC capacity has m terms similar to (5). Based on detailed form of each term, only two terms are preserved in final expression. Therefore, the m-user CC capacity has the following form:

$$
\begin{aligned}
&I(\sqrt{P_1}x_1 + \sqrt{P_2}x_2 + \cdots + \sqrt{P_m}x_m; y) \\
&= H(y) - H(y|\sqrt{P_1}x_1, \sqrt{P_2}x_2, \cdots, \sqrt{P_m}x_m).
\end{aligned} \tag{10}
$$

Assume that input constellations are uniformly distributed. Thus, the probability density function $p(y)$ of y is

$$
\begin{aligned}
p(y) = \frac{1}{N_1 N_2 \cdots N_m} \sum_{i_1=0}^{N_1-1} \cdots \sum_{i_m=0}^{N_m-1} p(y|x_1 = x_1(i_1), \\
\cdots, x_m = x_m(i_m)),
\end{aligned} \tag{11}
$$

where $p(y|x_1 = x_1(i_1), \cdots, x_m = x_m(i_m))$ is given in (13) at the top of next page. Then,

$$
H(y) = -\int p(y) \log_2(p(y)) dy. \tag{12}
$$

For the second term of (10), the detailed value of two-user case is presented in [9]. For m-user case, the value is given in (14). Using (11)-(14), the achievable CC capacity of arbitrary multi-user is given by (15) where the expectation is with respect the distribution of z. Although similar computation for *multiple-input multiple-output* (MIMO) GMAC's CC capacity has been done in [11], (15) is firstly deduced for m-user NOMA case. Note that each absolute value of numerators in (15) is the pairwise distance of every two complex points of sum constellation.

B. The Joint optimization scheme for multi-user GMAC

The CC capacity of m-user GMAC is given by (15). Since $N_1, N_2, \cdots, N_m$ are constants, the value of (15) can be maximally enlarged if the input scheme which has optimal distance distribution of sum constellation is applied. We propose a method to maximize the CC capacity region of multi-user GMAC based on (15). This scheme is designed to adjust both power and phase of input constellations. It makes best use of complex plane and outperforms CPA schemes and CR schemes for most SNR.

We use $m-1$ variables $\alpha_1, \alpha_2, \cdots, \alpha_{m-1}$ to vary the square of relative amplitude of each input constellation. The constraints of these $m-1$ variables are:

$$
\begin{cases}
m - \sum_{i=1}^{m-1} \alpha_i > 0; \\
\alpha_1 > 0; \\
\cdots \\
\alpha_{m-1} > 0;
\end{cases} \tag{16}
$$

We also use $m-1$ phase factors $\theta_1, \theta_2, \cdots, \theta_{m-1}$ to rotate input constellations in order to further improve the distance distribution of sum constellation.

Note that direct adjustment of each input constellation's power will inevitably break their average power constraints P_i. Therefore, according to the definition introduced in [10], whole transmission time is regarded as the repetition of a time slot $\mathcal{T}$ and $\mathcal{T}$ is further divided into m parts in order to guarantee the average power constraint of each user. We define a variable $t \in [1, m]$ which denotes the instantaneous time of transmission. Let $\mathcal{T}_t = \{k * m + t | k = 0, 1, 2, \cdots\}$ denote the disjoint subsets of whole transmission time. At each time slot $\mathcal{T}_t$, we use $\alpha_{1,t}, \alpha_{2,t}, \cdots, \alpha_{m-1,t}$, which are the rearrangement of original $m-1$ power factors, to denote the specific power factors of all input constellations.

The practical implement of the m-user joint optimization scheme is presented below. At time slot $\mathcal{T}_t$, the transmitted symbol s_t has the following form:

$$
s_t = \sum_{i=1}^{m} \sqrt{\alpha_{i,t} P_i} x_i * e^{j\theta_{t-1}}. \tag{17}
$$

Remarks: We define $\alpha_m = m - \sum_{j=1}^{m-1} \alpha_j$ and $\theta_0 = 0$ in this paper for the brevity of m-user joint optimization scheme.

Following constraints of power factors are applied in order to guarantee the average power constraint P_i of each user:

$$
\sum_{t=1}^{m} \alpha_{i,t} = m. \tag{18}
$$

In this paper, amplitude factors of each instantaneous transmitted symbol are deduced from cyclic permutation of prior symbol's factors:

$$
\begin{cases}
\alpha_{i,1} = \alpha_i, \\
\alpha_{i,t} = \alpha_{i+1,t-1} \text{ for } i \leq m-1 \text{ and } t \geq 2, \\
\alpha_{m,t} = \alpha_{1,t-1} \text{ for } t \geq 2,
\end{cases} \tag{19}
$$

where $i \in \{1, 2, \cdots, m\}$ and $t \in \{1, 2, \cdots, m\}$. Phase factor of each input constellation remains constant.

To search for the optimal variables that maximally enlarge the m-user CC capacity region, the specific expression of capacity is

$$
\frac{1}{m} \sum_{t=1}^{m} I(\sum_{i=1}^{m} \sqrt{\alpha_{i,t} P_i} x_i * e^{j\theta_{t-1}}; y). \tag{20}
$$

Both power factors and phase factors are introduced to make the best use of whole complex plane. If appropriate variables are selected, the value of CC capacity will be increased. Maximal multi-user CC capacity can be achieved via exhaustive search of all possible input schemes. However, it may result in overwhelming computational complexity and numerical instability.

C. Performance analysis and simulation results

The optimal joint optimization scheme at different SNR values and corresponding maximal CC capacity is presented in this subsection. The performance analysis shows that the capacity can reach the theoretical upper-bound at high SNR values if appropriate joint optimization scheme is applied. Results of CPA scheme are also presented for comparison. Without loss of generality, we provide numerical results of

$$p(y|x_1 = x_1(i_1), \cdots, x_m = x_m(i_m))$$
$$= \frac{1}{\sqrt{\pi}\sigma} exp\left(-\frac{|y - x_1(i_1) - x_2(i_2) - \cdots - x_m(i_m)|^2}{\sigma^2}\right) \tag{13}$$

$$H(y|\sqrt{P_1}x_1, \sqrt{P_2}x_2, \cdots, \sqrt{P_m}x_m)$$
$$= -\frac{1}{N_1 N_2 \cdots N_m} \sum_{i_1=0}^{N_1-1} \cdots \sum_{i_m=0}^{N_m-1} \int p(y|x_1 = x_1(i_1), \cdots, x_m = x_m(i_m)) \log_2 p(y|x_1 = x_1(i_1), \cdots, x_m = x_m(i_m)) dy \tag{14}$$

$$I(\sqrt{P_1}x_1 + \sqrt{P_2}x_2 + \cdots + \sqrt{P_m}x_m; y)$$
$$= \log_2(N_1 N_2 \cdots N_m) -$$
$$\frac{1}{N_1 N_2 \cdots N_m} \sum_{i_1=0}^{N_1-1} \cdots \sum_{i_m=0}^{N_m-1} E\left[\log_2\left[\frac{\sum_{j_1=0}^{N_1-1} \cdots \sum_{j_m=0}^{N_m-1} exp(-|\sqrt{P_1}(x_1(i_1) - x_1(j_1)) + \cdots + (\sqrt{P_m}(x_m(i_m) - x_m(j_m)) + z|^2/\sigma^2)}{exp(-|z|^2/\sigma^2)}\right]\right] \tag{15}$$

Fig. 3. CC capacity of QPSK constellation with identical average power constraint.

three-user GMAC with finite input constellations. We assume identical average power constraint, QPSK input constellations and $\sigma^2 = 1$ for all users. Thus, $P_1 = P_2 = P_3$ and $SNR = P_1$. The optimal α_1, α_2 and θ_1, θ_2 that maximally enlarge the three-user CC capacity region are obtained as follows. The values of α_1 and α_2 are varied in steps of 0.01 in their search area. For θ_1 and θ_2, search steps are $1°$.

In Fig. 3, the CC capacity of direct sum schemes, optimal CPA schemes and optimal joint optimization schemes at different SNR are compared. The optimal joint optimization schemes are obtained by the procedure presented in Section IV-B. In Fig. 3, the optimal joint optimization input schemes outperform the corresponding CPA schemes for most SNR. For the performance of corresponding CR schemes, previous research showed that they achieve similar CC capacity as CPA schemes when no random phase offsets are introduced [10]. Note that the direct sum of all users' input constellations results in ambiguity of the sum constellation. Therefore, its capacity cannot exceed 3.62bits/symbol even at high SNR values.

The search area of α_1 and α_2 has been given in (16).

However, for identical input constellations, there are $m!$ groups (m is the number of users in Fig. 1) of α_1 and α_2 which provide the same value of optimal CC capacity. Therefore, following extra constraints are introduced to exclude redundant solutions of α_1 and α_2 groups:

$$\begin{cases} \alpha_1 \leq 3 - \alpha_1 - \alpha_2; \\ \alpha_2 \leq 3 - \alpha_1 - \alpha_2; \\ \alpha_1 < \alpha_2. \end{cases}$$

Above constraints are combined with (16) and the final search area is

$$\begin{cases} \alpha_1 > 0; \\ \alpha_1 \leq \alpha_2; \\ \alpha_2 \leq 3 - \alpha_1 - \alpha_2. \end{cases} \tag{21}$$

Similarly, the rotation angles should be restricted within $[0°, \frac{360°}{M})$ for MPSK input constellations.

In Table I, the values of two variables α_1 and α_2 which obtain optimal CPA schemes at different SNR are listed. Note that in this table, values of α_1 and α_2 don't change

TABLE I. VALUES OF α_1 AND α_2 OF OPTIMAL CPA SCHEMES AT DIFFERENT SNR

SNR in dB	α_1 for QPSK	α_2 for QPSK
0	1	1
2	1.23	0.54
4	0.98	0.45
6	0.97	0.38
8	0.97	0.35
10	0.96	0.33
12	0.6	0.16
14	0.59	0.15
16	0.59	0.15
18	0.58	0.15
20	0.57	0.14

continuously between $SNR = 10dB$ and $SNR = 12dB$. Indeed, there are two peaks of CC capacity values in reduced search area when input power is larger than 10dB. Therefore, for some $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$, they provide a suboptimal CPA scheme.

As is mentioned before, the sum constellation of a input scheme can be regarded as a set of points on a two-dimensional complex plane for multi-user GMAC. Therefore, the distance distribution of this sum constellation has direct impact on CC capacity.

TABLE II. SEVERAL DISTANCE CLASSES AND LABELS OF SUM CONSTELLATIONS

	Distance Pairs	Label Pairs
class 1	$(1,5)$	$(1,1)-(2,1)$
	$(2,6)$	$(1,2)-(2,2)$
	$(3,7)$	$(1,3)-(2,3)$
	$(4,8)$	$(1,4)-(2,4)$
class 2	$(1,5)$	$(1,1)-(2,1)$
	$(6,10)$	$(2,2)-(3,2)$
	$(11,15)$	$(3,3)-(4,3)$
	$(16,4)$	$(4,4)-(1,4)$
class 3	$(1,11)$	$(1,1)-(3,3)$
	$(6,16)$	$(2,2)-(4,4)$

IV. DISTANCE DISTRIBUTION OF SUM CONSTELLATION

In [17], S. Kundu, etc. noticed that some pairwise distances of two-user GMAC's sum constellation have identical expression. These distances are identical no matter what input scheme is applied and the set of these distances is called distance class. This result is the basis of optimizing fading MAC's distance distribution.

The multi-peak phenomenon drives us to study the distance distribution of sum constellation. We discover that there are two and only two types of distance classes. For m-user GMAC, it is proved that the concept of classification is valid for arbitrary input schemes. This result provides an insight into the design of multi-user input schemes for both GMAC and fading MAC. In this section, we will focus on GMAC for the sake of simplicity and leave its application for fading MAC as future research. As is mentioned in Section III, minimum distances have a direct impact on multi-user GMAC's CC capacity. Combining the classification of distance pairs and importance of minimum distances, we propose an efficient algorithm to obtain optimal joint optimization schemes.

A. Classification of sum constellation's distance pairs

In this subsection, we present the classification of sum constellation's distance pairs which is universally valid for arbitrary input schemes. Some unique properties of sum constellation's distance distribution are also presented for preliminary knowledge of this classification.

Without loss of generality, we label the complex points of each user's original input constellation of MPSK from 1 to M counterclockwise and each point's label is mapped to its original expression: $e^{j\frac{k-1}{M}2\pi}$. The complex points of m-users' sum constellation are denoted by 1-by-m matrices. For example, $(2,4)$ and $(3,2)$ denote point 8 and point 10 respectively in Fig. 2.

As is mentioned in Section II-C, some pairwise distances have same expression of distance for all input schemes. This concept is firstly proposed in [17]. For the sum constellation of Fig. 2, several distance classes as well as their components' labels are listed in Table II. Note that distance pair $(1,5)$ belongs to both distance classes.

Label pairs within the same class have some unique properties. For m-user sum constellation with MPSK input constellations, let $\leftrightarrow (\delta_1, \delta_2, \cdots, \delta_m)$ denotes horizontal difference between two components of one label pair. Let $\updownarrow (\delta_1, \delta_2, \cdots, \delta_m)$ denotes vertical difference between two label pairs. The operation here is similar to modulo-m addition

while reminders range from 1 to m. Define $*$ as an arbitrary integer within $[0, M-1]$. For instance, class 2 in Table II has following properties: $\leftrightarrow (1,0)$ and $\updownarrow (1,1)$.

Definition 1. *Type I Distance Classes: Difference between two components of one label pair is $\leftrightarrow (0,0,\cdots,*,\cdots,0)$ (only one position has nonzero integer). Difference between adjacent label pairs is $\updownarrow (*,*,\cdots,*)$.*

Theorem 1. *Members of each Type I distance class remain constant for arbitrary amplitude factor γ and phase factor θ.*

Proof: According to the definition of Type I classes, we assume that one label pair is $(l_{a,1}, l_{a,2}, \cdots, l_{a,i}, \cdots, l_{a,m}) - (l_{b,1}, l_{b,2}, \cdots, l_{b,i}+p, \cdots, l_{b,m})$ and its adjacent label pair is $(l_{a,1}+c_1, l_{a,2}+c_2, \cdots, l_{a,i}+c_i, \cdots, l_{a,m}+c_m) - (l_{b,1}+c_1, l_{b,2}+c_2, \cdots, l_{b,i}+p+c_i, \cdots, l_{b,m}+c_m)$. Since each point's label can be mapped to its original expression, the pairwise distance $d_{a,b}$ of the first label pair is

$$d_{a,b} = |\gamma_i e^{j\theta_i} e^{j\frac{k_i-1+p}{M}2\pi} - \gamma_i e^{j\theta_i} e^{j\frac{k_i-1}{M}2\pi}|.$$

Likewise, second label pair's pairwise distance $d_{a+c,b+c}$ is

$$d_{a+c,b+c} = |\gamma_i e^{j\theta_i} e^{j\frac{k_i-1+p+c_i}{M}2\pi} - \gamma_i e^{j\theta_i} e^{j\frac{k_i-1+c_i}{M}2\pi}|$$
$$= |e^{j\frac{c_i}{M}2\pi} * (\gamma_i e^{j\theta_i} e^{j\frac{k_i-1+p}{M}2\pi} - \gamma_i e^{j\theta_i} e^{j\frac{k_i-1}{M}2\pi})|.$$

Phase factor $e^{j\frac{c_i}{M}2\pi}$ has no influence on following term's modulus. Therefore, $d_{a,b} = d_{a+c,b+c}$ for any input schemes. ∎

Definition 2. *Type II Distance Classes: Difference between adjacent label pairs is $\updownarrow (1,1,\cdots,1)$. Difference between two components of one label pair is $\leftrightarrow (*,*,\cdots,*)$.*

If different input schemes are applied, it can be proved that members of each Type II distance class are unchanged. The proof of this conclusion is similar to Type I's and is omitted here.

Remarks: If one distance class belongs to both Type I and Type II, it is classified into Type II classes for simplicity.

Simulation results show that all distance classes of multi-user sum constellation are classified into one of these two types for arbitrary input schemes. Recall the fact that members of either Type I or Type II distance classes are invariant when different input schemes are applied. Therefore, for a certain input scheme, if distance pairs which share identical pairwise distances are regarded as one class, this classification is available for arbitrary input schemes. Note that classes with different expression of pairwise distances may have the same distances under certain input schemes. As a result, classification should be initialized with appropriate input schemes.

B. Procedure to obtain optimal joint optimization input schemes

In this subsection, we present the procedure of calculating CC capacity for MPSK based on classification of sum constellation and minimum distances, which has been used in Section III-C.

Recall the expression of CC capacity (15). Each absolute value of this expression is a pairwise distance of two corresponding complex points of sum constellation. Therefore,

the value of CC capacity is mainly affected by small pairwise distances. Our main approach to reduce computational complexity is to prevent redundant pairwise distances from getting involved in the calculation of CC capacity. Moreover, we use the deterministic metric [10] of (15) to eliminate the expectation of z at high SNR.

In this algorithm, the first step is to classify all distance pairs according to their pairwise distances. Recall the properties of Type I and Type II distance classes in Section IV-A. Distance pairs can be divided into classes whose members are invariant for arbitrary input schemes. Therefore, an initialization of classification is conducted at the beginning of this procedure. One characteristic distance pair is chosen from each class after initialization. Our approach is to iteratively pick one input scheme and test whether its corresponding classification is valid for other input schemes until a correct one is found.

The next part of algorithm is to use minimum pairwise distances to calculate the approximation of each joint optimization scheme's CC capacity. Since accurate CC capacity of (15) cannot be obtained without all distances getting involved in the calculation of CC capacity, this approximation is only used to select the optimal input scheme. For each input scheme, minimum distances are obtained by sorting all characteristic distance pairs, each of which represents a class of distance pairs. Only several characteristic distance pairs whose pairwise distances are small are used in further calculation of CC capacity. Therefore, bubble sort can effectively find these characteristic distance pairs. An upper bound (denoted by n_{bound}) of numbers of pairwise distances is set to exclude redundant distance pairs and restrict number of iteration of bubble sort. Our simulation results indicate that n_{bound} which scales to number of sum constellation's total complex points is effective to yield good joint optimization schemes for most SNR. Then, those chosen classes of pairwise distances are used in the calculation of CC capacity's approximation. A input scheme will be recorded as the optimal input scheme if this scheme produces maximal approximation of CC capacity. Finally, the maximal CC capacity of m-user GMAC is provided by the recorded optimal input scheme.

To summarize, the procedure is listed in Algorithm 1

Algorithm 1 Algorithm of obtaining optimal joint optimization input schemes

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1: : Initialize the classification of all distance pairs by their pairwise distances. Distance pairs are divided into classes whose members are invariant for arbitrary input schemes.

2: : A distance pair is chosen from each class as characteristic distance pair. Record the characteristic pairwise distance and all members of each class.

3: : The approximation of each input scheme's CC capacity is calculated based on minimum pairwise distances.

4: : Compare all approximations and select the scheme which has maximal approximation as the optimal joint optimization scheme. Use this optimal input scheme to provide maximal CC capacity at current SNR.

V. CONCLUSION AND FUTURE WORK

In this paper, we have proposed the multi-user joint optimization input scheme and the procedure of obtaining optimal input schemes for GMAC. The performance analysis shows that optimal joint optimization input schemes outperform the corresponding schemes which adjust only one-dimensional variables for most SNR. The investigation of the joint optimization scheme for other channels including interference channel, fading channel and broadcast channel, etc. and for other modulations is left for future work. For the study of distance classes, previous research is dedicated to devise the modulation scheme which improves the error performance of transmission in fading MAC. Our research presents more details of distance classes. We show that this concept can be generalized to multi-user case and is valid for arbitrary input schemes. These results will shed light on fast construction of distance classes and the design of modulation schemes for multi-user fading MAC.

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