Electromagnetics
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713770615

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To cite this Article Daniele, Vito , Gilli, Marco and Viterbo, Emanuele(1990) 'Diffraction of a Plane Wave by a Strip Grating', Electromagnetics, 10: 3, 245 — 269
To link to this Article: DOI: 10.1080/02726349008908242
URL: http://dx.doi.org/10.1080/02726349008908242

DIFFRACTION OF A PLANE WAVE BY A STRIP GRATING

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ABSTRACT

The solution of a canonical problem regarding the diffraction of a plane electromagnetic wave incident, with an arbitrary angle, upon a strip grating formed by infinitely long thin metallic ribbons is found.
The formulation is given for any relation between the width of the ribbons and the width between the ribbons. The exact solution is obtained by using Wiener-Hopf techniques in the case of equal width of the ribbon and the spacing.
The numerical values of the transmission and reflection coefficients can be evaluated in a simple manner to any desired accuracy. Plots are given for different angles of incidence in a certain frequency range.
The closed form of the TE modes scattering matrix is also given.

This research was supported in part by ENEA under contract 552/88.

Electromagnetics 10:245-269, 1990

1. INTRODUCTION

The object of this paper is the analytical solution of a canonical problem using the Wiener-Hopf technique. We study the diffraction of a plane wave, with an arbitrary angle of incidence, upon an infinite strip grating, as shown in fig.1. The importance of solving such a canonical problem is that:

- the results give interesting information on similar structures;
- they may be used to verify the validity of numerical methods.

The main engineering applications of such a structure are to electromagnetic screening and dichroic surfaces; structures which are composed by cascading a single screen can also have interesting applications.

So far the problem has only been solved analytically in the case in which the width of the metal strips is equal to the width of the spacing between the strips.

The normally incident plane wave was first studied by Baldwin and Heins [3], but the results are not immediately usable for engineering applications. Another solution, proposed by Weinstein [4], gives more practical results.

The case of oblique incidence has been treated in [5] with an 'ad hoc' method.

The present work solves the problem of the oblique incidence by using Wiener-Hopf techniques, with a more general approach than [5] and [6] and obtains more accurate results in a larger range of frequencies.

A particular formulation of the problem is needed to obtain the Wiener-Hopf equation in the space-frequency domain. The solution of the Wiener-Hopf equation is carried out using a modified version of the classical Wiener-Hopf method [8]. In fact, the classical method would lead, in this case, to a function which does not vanish at infinity and cannot be a Fourier transform of the solution field. With this procedure, an analytical solution is obtained and reflection and transmission coefficients can be computed to any desired accuracy.

The TE-modes scattering matrix of the structure is also given in a closed form and can be a starting point to study more complex cascaded structures. The TM-modes scattering matrix can be obtained by using Babinet's principle [11].

A computer code has been developed to calculate the coefficients of the scattering matrix, from the analytical expression, in a simple and quick manner. Plots are shown for some incidence angles and in a certain range of frequencies, but very precise results are available for any incidence angle and without limits in the frequency range.

In this paper, we will use bold characters to indicate vectors and underlined bold characters to indicate matrices.

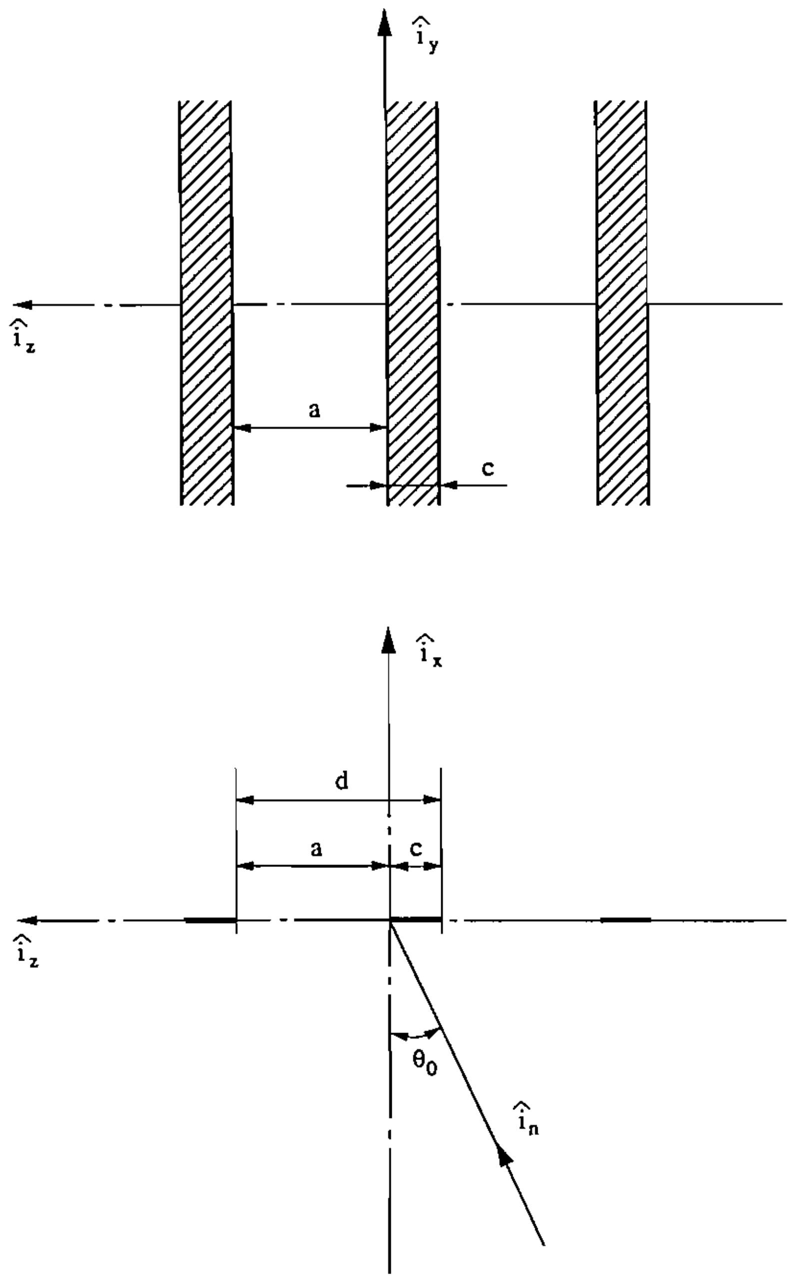

Figure 1: Geometry of the problem.

2. FORMULATION OF THE PROBLEM

Fig. 1 shows the geometry of the problem, where the structure is uniform in the y direction. Each conducting strip, of width c and zero thickness, is spaced periodically from its neighbours by a distance d = a + c.

The time dependence exp(jωt) is assumed and the incident plane wave is TE (the case of TM excitation is obtained by applying Babinet's principle in its rigorous version).

The incident plane wave is:

$$E^i(x,z) = E_o(\omega) i_y \exp[-jK_o(x \cos \theta_o + z \sin \theta_o)] \tag{1}$$

$$H^i(x,z) = Y\, i_n x E^i(x,z) \tag{2}$$

where $E_o(\omega)$ is the amplitude of the incident field, K_o the wave number, Y the admittance of free space; i_x and i_y are shown in Fig.1 and:

$$i_n = i_x \cos \theta_o + i_z \sin \theta_o \tag{3}$$

It is possible to write the incident field in the following manner:

$$E^i = E_1{}^i + E_2{}^i \tag{4}$$

$$H^i = H_1{}^i + H_2{}^i \tag{5}$$

where:

$(E_1{}^i, H_1{}^i)$ is half of the incident wave and its magnetic image;

$(E_2{}^i, H_2{}^i)$ is half of the incident wave and its electric image;

We can separate the main problem into two problems, where the incident field is either $(E_1{}^i, H_1{}^i)$ or $(E_2{}^i, H_2{}^i)$;

The first problem is shown in Fig. 2. A perfect magnetic conductor can be used instead of the apertures, because the tangential component of the incident magnetic field vanishes on the apertures and it is possible to show that also the tangential component of the total magnetic field is zero on the apertures.

The second problem is shown in Fig.3. The computation of the scattered field is trivial in this case; we have:

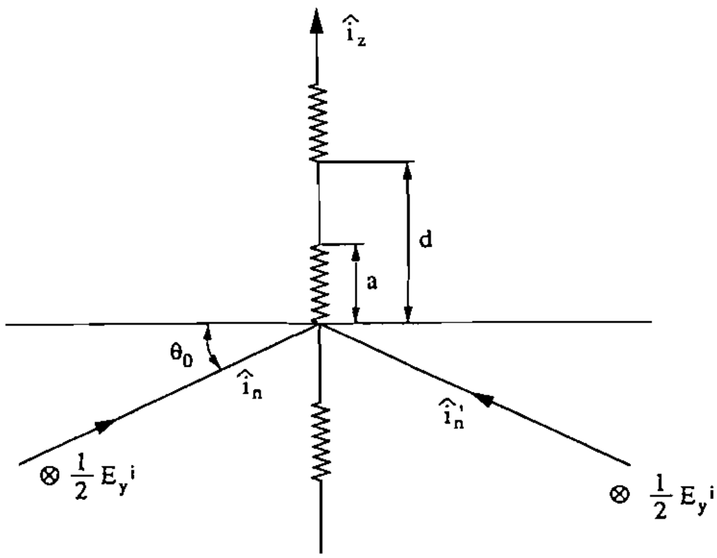

Figure 2: Structure equivalent to fig. 1 when $(E_1^i(r), H_1^i(r))$ incides

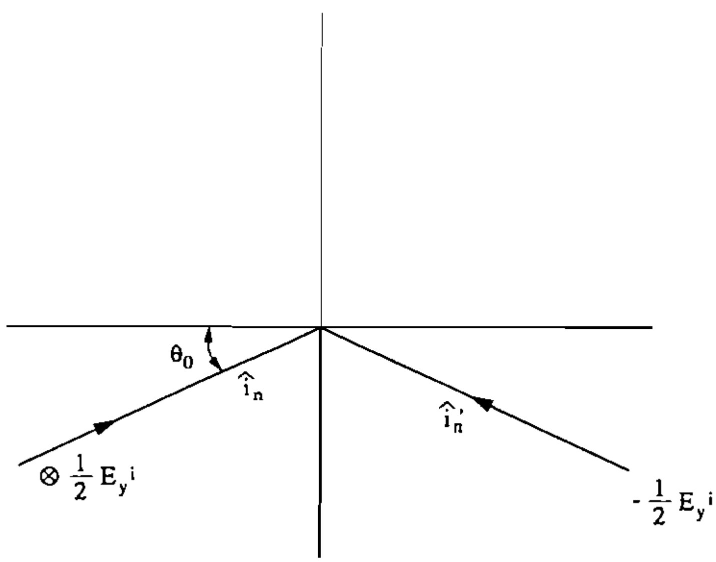

Figure 3: Structure equivalent to fig. 1 when $(E_2^i(r), H_2^i(r))$ incides

$$x<0 \quad \mathbf{E}_2^{s}(x,z) = -1/2\ E_o(\omega)\ \mathbf{i}_y \exp[-jK_o(-x \cos\theta_o + z \sin\theta_o)] \tag{6}$$

$$x>0 \quad \mathbf{E}_2^{s}(x,z) = 1/2\ E_o(\omega)\ \mathbf{i}_y \exp[-jK_o(x \cos\theta_o + z \sin\theta_o)] \tag{7}$$

The image wave propagation versor is indicated in Figs. 2 and 3 with $\mathbf{i}'_n$ and it is: $\mathbf{i}'_n = -\mathbf{i}_x \cos\theta_o + \mathbf{i}_z \sin\theta_o$

3. SOLUTION OF THE FIRST PROBLEM

Since the structure is periodic, it is convenient to use Floquet's theorem and the scattered field takes the form:

$$E_y^{s}(x,z) = \Sigma_{-\infty}^{\infty} R_n \exp(-j\tau_n z - j\alpha_n x) \tag{8}$$

where:

$$\tau_n = \eta + 2\pi n/d \qquad (n=0,\pm1,\pm2,\dots) \text{ with } \eta = K_o \sin(\theta_o) \tag{9}$$

$$\alpha_n = [K_o^2 - (\eta + 2\pi n/d)^2]^{1/2} \tag{10}$$

Every term in the infinite sum (8) is a TE mode of the structure.
In this section we assume $\theta_o \neq 0$; in section 5 we will discuss the case $\theta_o = 0$.
We propose an analytical method, based on the Wiener-Hopf technique, to determine the coefficients R_n of the scattered field.

We reduce the problem to that of semi-infinite wave guides in the $\mathbf{i}_z$ direction with particular boundary conditions.
- for $n\,d \leq z \leq n\,d + c$; perfect electric conductor;
- for $c + n\,d \leq z \leq (n+1)\,d$; perfect magnetic conductor;

where n is an integer;
We will consider a periodic transmission line along the $\mathbf{i}_z$ axis and use Marcuvitz-Schwinger's formalism [2].

The transversal fields are given by the following expressions:

$$\mathbf{E}_t(x,z) = \int_0^{\infty} V_\alpha(z)\ \mathbf{e}_\alpha(x)\ d\alpha \tag{11}$$

$$\mathbf{H}_t(x,z) = \int_0^{\infty} I_\alpha(z)\ \mathbf{h}_\alpha(x)\ d\alpha \tag{12}$$

where $V_\alpha(z)$ and $I_\alpha(z)$ are voltage and current in the transmission line and $\mathbf{e}_\alpha(x)$

and $\mathbf{h}_\alpha(x)$ are the eigenfunctions of the semi-infinite transversal section wave guide; more precisely:

- for perfect electric conductor boundary condition:

$$\mathbf{e}_\alpha = 2 \sin(\alpha x)\mathbf{i}_y \tag{13}$$
$$\mathbf{h}_\alpha = -2 \sin(\alpha x)\mathbf{i}_x \tag{14}$$

- for perfect magnetic conductor boundary condition :

$$\mathbf{e}_\alpha = - 2j \cos(\alpha x)\mathbf{i}_y \tag{15}$$
$$\mathbf{h}_\alpha = . 2j \cos(\alpha x)\mathbf{i}_x \tag{16}$$

We indicate with $F^{\$}(\alpha)$ the unilateral Fourier transform of the function $f(x)$:

$$F^{\$}(\alpha) = \int_0^\infty f(x)\, e^{j\alpha x}\, dx \tag{17}$$

By imposing the continuity of the field at z=0 and z=a and its semiperiodicity, we obtain the following Wiener-Hopf equation:

$$\underline{G}'(\alpha)\cdot\theta(\alpha) = \theta(-\alpha) \tag{18}$$

where

$$\theta(\alpha) = \begin{bmatrix} E_y^{\$}(z=0) \\ -H_x^{\$}(z=0) \\ E_y^{\$}(z=a) \\ -H_x^{\$}(z=a) \end{bmatrix} \tag{19}$$

$$\underline{G}'(\alpha) = (\underline{\mathbf{M}}_d - t\underline{1})^{-1} * \begin{bmatrix} t\underline{1}+\underline{\mathbf{M}}_d & -2t\underline{\mathbf{M}}_a \\ 2\underline{\mathbf{M}}_c & -t\underline{1}-\underline{\mathbf{M}}_d \end{bmatrix} \tag{20}$$

$$t = \exp(j\eta d) \quad , \tag{21}$$

$$\underline{\mathbf{M}}_\ell = \exp[j\tau\ell \begin{bmatrix} 0 & Z_\alpha \\ Y_\alpha & 0 \end{bmatrix}] \tag{22}$$

$$\tau = (K_0^2 - \alpha^2)^{1/2} \quad , \quad Z_\alpha = 1/Y_\alpha = \omega\mu/\tau \tag{23}$$

where the symbol * in (20) indicates the multiplication of the 2x2 matrix $(\underline{M}_d - t\underline{1})^{-1}$ by each 2x2 matrix of the 4x4 matrix; μ is the magnetic permeability of free space.

It is worth noting that matrices involved in $\underline{G}'$ commute because they are functions of the same matrix:

$$\begin{bmatrix} 0 & Z_\alpha \\ Y_\alpha & 0 \end{bmatrix}$$

The singularities of $\underline{G}'$ are only simple poles and, precisely, they are the zeros of the determinant of $\underline{M}_d - t\underline{1}$:

$$\det[\underline{M}_d - t\underline{1}] = 2\exp(j\eta d)\,[\cos(\eta d) - \cos(\tau d)] \tag{24}$$

In the complex plane these zeros are located at:

$$\alpha = \pm\,\alpha_n \quad (n=0,\pm1,\pm2,\ldots) \tag{25}$$

where :

$$\alpha_n = [K_o^2 - (\eta + 2\pi n/d)^2]^{1/2} \qquad (n = 0,\ \pm1,\ \pm2,\ldots) \tag{26}$$

We introduce a medium with small losses, to have

$K_o = K_o' - jK_o''$ where $K_o'' \ll K_o'$ and $K_o'' > 0$

and we choose the branch of the complex root with negative imaginary part; thus for $\alpha = \alpha_n$ we can write:

$$\tau = \tau_n = \eta + 2\pi n/d \qquad (n=0,\pm1,\pm2,\ldots) \tag{27}$$

and similarly the values of Z_α and Y_α corresponding to α_n are:

$$Y_n = Z_n^{-1} = \tau_n/(\omega\mu) \qquad (n=0,\pm1,\pm2,\ldots) \tag{28}$$

Remembering that $\theta(\alpha)$ is given by the sum of the unilateral Fourier transform of the incident and scattered field, it follows that:

$$\theta(\alpha) = \theta^i(\alpha) + \theta^s(\alpha) \tag{29}$$

and taking into account the expression of $\mathbf{E}_1^{\,i}$ we obtain the equation:

$$\underline{G}'(\alpha)\cdot\theta^s(\alpha) = \theta^s(-\alpha) - \underline{G}'(\alpha)\cdot\mathbf{f}(\alpha) + \mathbf{f}(-\alpha) \tag{30}$$

where

$$\mathbf{f}(\alpha) = j(1/2)\,E_o(\omega)\,\mathbf{q}^{(0)}\,\frac{1}{\alpha + \alpha_0} \tag{31}$$

and

$$q^{(0)} = \begin{bmatrix} 1 \\ Y_0 \\ \exp(-j\tau_0\, a) \\ Y_0\, \exp(-j\tau_0\, a) \end{bmatrix} \tag{32}$$

Equation (30) may be solved with a Wiener-Hopf procedure;
we factorize the matrix $\underline{G}'$ into two matrices $\underline{G}'_+$ and $\underline{G}'_-$, which are regular and without zeros for $\mathrm{Im}(\alpha)\geq 0$ and $\mathrm{Im}(\alpha)\leq 0$ respectively; then we separate in equation (30), the function regular in $\mathrm{Im}(\alpha)\geq 0$, from the function regular in $\mathrm{Im}(\alpha)\leq 0$:

$$\underline{G}'_+(\alpha)\cdot\theta^s(\alpha) + [\underline{G}'_+(\alpha)-\underline{G}'_+(-\alpha_0)]\cdot\mathbf{f}(\alpha) - [\underline{G}'_-(\alpha_0)]^{-1}\cdot\mathbf{f}(-\alpha)=$$

$$=[\underline{G}'_-(\alpha)]^{-1}\cdot\theta^s(-\alpha)-\underline{G}'_+(-\alpha_0)\cdot\mathbf{f}(\alpha)+\{[\underline{G}'_-(\alpha)]^{-1} - [\underline{G}'_-(\alpha_0)]^{-1}\}\cdot\mathbf{f}(\alpha)=$$

$$= \mathbf{w}(\alpha) \tag{33}$$

Since the two functions coincide on the real axis they are equivalent in all the complex plane to an entire function, that we have to determine.
The most important problem in this procedure is to factorize the matrix $\underline{G}'$; no general method exists in the literature for a matrix, like $\underline{G}'$, which is a transcendental function of α.
We will show that $\underline{G}'$ can be factorized using the procedure proposed in [1].
We will factorize the new matrix $\underline{G}(\alpha)$ defined in this way:

$$\underline{G}'(\alpha) = \underline{C}' \cdot \underline{G}(\alpha)\cdot \underline{C} \tag{34}$$

with

$$\underline{C}' = \begin{bmatrix} C\underline{1} & \underline{0} \\ \underline{0} & 1/C\underline{1} \end{bmatrix} \quad \text{and} \quad \underline{C} = \begin{bmatrix} 1/C\underline{1} & \underline{0} \\ \underline{0} & -C\underline{1} \end{bmatrix} \tag{35}$$

where

$$C = t^{1/4} = \exp(j\eta d/4) \tag{36}$$

That is

$$\underline{G}(\alpha) = (\underline{M}_d - t\underline{1})^{-1} * \begin{bmatrix} t\underline{1}+\underline{M}_d & 2t^{(1/2)}\underline{M}_a \\ 2t^{(1/2)}\underline{M}_c & t\underline{1}+\underline{M}_d \end{bmatrix} \tag{37}$$

The factorization is possible for different values of a and c, but in the case a≠c the determination of the entire function $w(\alpha)$ leads to a system that cannot be solved in a closed form. From now on we assume a=c=d/2.

To factorize $\underline{G}$ it is useful to make the following substitution:

$$\underline{\beta} = settch \frac{\underline{M}_d + \underline{1}t}{\underline{M}_d - \underline{1}t} = \ln\{\frac{\underline{M}_d + \underline{1}t}{\underline{M}_d - \underline{1}t} + [\frac{(\underline{M}_d + \underline{1}t)^2}{(\underline{M}_d - \underline{1}t)^2} - \underline{1}]\}^{1/2} =$$

$$= \ln [\frac{(\underline{M}_d)^{1/2} + \underline{1}t^{1/2}}{(\underline{M}_d)^{1/2} - \underline{1}t^{1/2}}] = \ln (\underline{Q}) \tag{38}$$

$\underline{G}(\alpha)$ can be rewritten in the following way:

$$\underline{G}(\alpha) = \begin{bmatrix} ch\, \underline{\beta} & sh\, \underline{\beta} \\ sh\, \underline{\beta} & ch\, \underline{\beta} \end{bmatrix} = \exp [\, \underline{\beta} * \begin{bmatrix} \underline{0} & \underline{1} \\ \underline{1} & \underline{0} \end{bmatrix}] \tag{39}$$

By the factorization of

$\underline{Q}(\alpha) = \underline{Q}_-(\alpha) \cdot \underline{Q}_+(\alpha);$ (40)

$\underline{\beta}$ will be decomposed in two parts:

$\underline{\beta} = \underline{\beta}_+ + \underline{\beta}_-$ where (41)

$\underline{\beta}_+ = \ln(\underline{Q}_+)$ and (42)

$\underline{\beta}_- = \ln(\underline{Q}_-)$ (43)

are regular respectively in the upper and lower planes.

It follows from the exponential form in (39):

$$\underline{G}_+(\alpha) = \begin{bmatrix} \text{ch } \underline{\beta}_+ & \text{sh } \underline{\beta}_+ \\ \text{sh } \underline{\beta}_+ & \text{ch } \underline{\beta}_+ \end{bmatrix} \tag{44}$$

and

$$\underline{G}_-(\alpha) = \begin{bmatrix} \text{ch } \underline{\beta}_- & \text{sh } \underline{\beta}_- \\ \text{sh } \underline{\beta}_- & \text{ch } \underline{\beta}_- \end{bmatrix} \tag{45}$$

We will show that $\underline{Q}(\alpha)$ can be factorized, by using the general method proposed in [1], after some algebraic manipulations.
From (38) a more convenient expression of $\underline{Q}(\alpha)$ can be obtained:

$$\underline{Q} = j\, m(\alpha) \exp\left[(1/2)t(\alpha) \begin{bmatrix} 0 & \omega\mu \\ \tau^2/(\omega\mu) & 0 \end{bmatrix}\right] \tag{46}$$

where:

$$m(\alpha) = \frac{[\cos(\tau d/2)+\cos(\eta d/2)]^{1/2}}{[\cos(\tau d/2)-\cos(\eta d/2)]^{1/2}} \tag{47}$$

$$t(\alpha) = \frac{1}{\tau} \ln \frac{\sin(\eta d/2)+\sin(\tau d/2)}{\sin(\eta d/2)-\sin(\tau d/2)} \tag{48}$$

As indicated in [1] the factorization of $\underline{Q}$ can be carried out by decomposing $t(\alpha)$ and factorizing $m(\alpha)$.
From expression (47) one can note that the function $m(\alpha)$ can be factorized by the use of Weierstrass' theorem [9]; we have introduced in the expression of $m_+(\alpha)$ the Gamma function to improve the convergence of the infinite product and we have obtained the very cumbersome expression:

$$m_+(\alpha) = \frac{[\cos(K_o d/2)+\cos(\eta d/2)]^{1/4}}{[\cos(K_o d/2)-\cos(\eta d/2)]^{1/4}} \{\frac{1-(\alpha/\alpha_1)}{1-(\alpha/\alpha_0)} \cdot$$

$$\frac{[(d/\lambda)\sin\theta_o+1]\sin[(\pi d/2\lambda)\sin\theta_o}{\sin[(\pi/2)((d/\lambda)\sin\theta_o+1)](d/\lambda)\sin\theta_o} \cdot \frac{\Gamma(-\frac{j\alpha d}{4\pi}-\frac{d\sin\theta_o}{2\lambda}+1)}{\Gamma(-\frac{j\alpha d}{4\pi}-\frac{d\sin\theta_o}{2\lambda}+\frac{1}{2})} \cdot$$

$$\cdot \frac{\Gamma(-\frac{j\alpha d}{4\pi}+\frac{d\sin\theta_o}{2\lambda}+1)}{\Gamma(-\frac{j\alpha d}{4\pi}+\frac{d\sin\theta_o}{2\lambda}+\frac{3}{2})} \cdot$$

$$\cdot \prod_{n=1}^{n=\infty} \{\frac{(1-\alpha/\alpha_{2n+1})(1-\alpha/\alpha_{-2n+1})\{1-j\alpha d/[2\pi(2n-(d/\lambda)\sin\theta_o)]\}}{(1-\alpha/\alpha_{2n})(1-\alpha/\alpha_{-2n})\{1-j\alpha d/[2\pi(2n+1+(d/\lambda)\sin\theta_o)]\}} \cdot$$

$$\cdot \frac{\{1-j\alpha d/[2\pi(+2n+(d/\lambda)\sin\theta_o)]\}}{\{1-j\alpha d/[2\pi(2n-1-(d/\lambda)\sin\theta_o)]\}}\}^{1/2} \qquad (49)$$

and

$$m_-(\alpha) = m_+(-\alpha) \qquad (50)$$

From expression (48), by using the small losses hypothesis, we note that $t(\alpha)$ is regular in a small strip, which contains the real axis, and within this strip $|t(\alpha)|$ vanishes, as $Re(\alpha) \to \infty$.

Then $t(\alpha)$ can be decomposed according to the formula given in [7]; only the the final results are reported here since their derivation is quite complex:

$$t_+(\alpha) = \sum_{n=-\infty}^{\infty} \frac{1}{\tau(\alpha)} \left\{ \ln \frac{\alpha - \alpha_{2n+1}}{\alpha - \alpha_{2n}} \right\} +$$

$$+ \sum_{n=-\infty}^{\infty} \frac{1}{\tau(\alpha)} \left\{ \ln \frac{(K_o+\tau_{2n+1})[-\alpha\alpha_{2n}+(K_o+\tau_{2n})(K_o+\tau(\alpha))]^2}{(K_o+\tau_{2n})[-\alpha\alpha_{2n+1}+(K_o+\tau_{2n+1})(K_o+\tau(\alpha))]^2} \right\} \qquad (51)$$

and

$$t_-(\alpha) = t_+(-\alpha); \qquad (52)$$

As a result of the factorization of $\underline{Q}(\alpha)$, and of (42) and (43), $\underline{G}_+(\alpha)$ and $\underline{G}_-(\alpha)$ are obtained from expressions (44) and (45).

The last step is to determine the entire function $\mathbf{w}(\alpha)$ in such a way that $\theta_s(\alpha)$ is the Fourier transform of a function and precisely:

$$\theta_s(\alpha) = O_{\alpha\to\infty}\ (1/\rho^{\varepsilon})\ ;\ \rho = |\ \alpha\ | \quad \text{and} \quad \varepsilon>0 \qquad (53)$$

The study of the behaviour at infinity of $\underline{G}_+(\alpha)$ shows that $\mathbf{w}(\alpha)$ has to be a constant and its expression is:

$$\mathbf{w}(\alpha) = \begin{bmatrix} 0 \\ w_2 \\ 0 \\ 0 \end{bmatrix} \qquad (54)$$

in which:

$$w_2 = \frac{-1}{\omega\mu}\ E_0(\omega)\ \exp[-j\pi/4 - \frac{1}{2}\ \tau(-\alpha_0)t_+(-\alpha_0)]\ \frac{1}{m_+(-\alpha_0)} \qquad (55)$$

Remembering that the only singularities of $\theta_s(\alpha)$ are simple poles in $\alpha = \alpha_n$, and that its behaviour at infinity is given by (53), Mittag-Leffler's expansion yields:

$$\theta_s(\alpha) = \sum_{-\infty}^{\infty} \frac{\mathbf{Y}_n}{\alpha - \alpha_n} \qquad (56)$$

where $\mathbf{Y}_n$ is the residue of $\theta_s(\alpha)$ at $\alpha = \alpha_n$.

The evaluation of the coefficients Y_n can be performed by means of (33), by noting that $[\underline{G}'_+(\alpha)]$ has no zeros in the upper half plane and that the pole at $\alpha=\alpha_0$ is a first order pole; in fact by putting:

$$[\underline{G}'_+(\alpha)]^{-1} = \frac{\underline{R}'(\alpha_0)}{\alpha - \alpha_0} + \underline{H}'_0(\alpha), \tag{57}$$

where $\underline{R}'(\alpha_0)$ is the residue of $[\underline{G}'_+(\alpha)]^{-1}$ at α_0 and $\underline{H}'_0(\alpha)$ regular at $\alpha = \alpha_0$, one obtains : $\underline{R}'(\alpha_0)\cdot [\underline{G}'_-(\alpha_0)]^{-1} \cdot q^{(0)} = 0$.

The computation of Y_n leads to the following results:

$$Y_n = \frac{E_o(\omega)\ \exp[-j(\pi/4)-(1/2)\tau(\alpha_0)t_+(-\alpha_0)]\ f(\alpha_n)}{m_+(-\alpha_0)} \left[(-1)^n\ j\ \frac{(1+Z_nY_0)}{(\alpha_n + \alpha_0)} - \frac{Z_n}{\omega\mu}\right] q^{(n)} \tag{58}$$

$$q^{(n)} = \begin{bmatrix} 1 \\ Y_n \\ (-1)^n \exp(-j\eta d/2) \\ (-1)^n Y_n \exp(-j\eta d/2) \end{bmatrix} \tag{59}$$

where:

$$f(\alpha_n) = \lim_{\alpha\to\alpha n} \frac{(\alpha-\alpha_n)\ m_+(\alpha)\ \exp(j\pi/4)\ \exp(t_+\tau/2)}{4} \quad \text{for n even}$$

$$f(\alpha_n) = \lim_{\alpha\to\alpha n} \frac{(\alpha-\alpha_n)\ [m_+(\alpha)]^{-1}\ \exp(-j\pi/4)\ \exp(-t_+\tau/2)}{4} \quad \text{for n odd} \tag{60}$$

It is worth noting that the limit can be evaluated in an analytical way because $m_+(\alpha)\ \exp(t_+\tau/2)$ contains the term $[\alpha-\alpha_n]^{-1}$ for n even, while $[m_+(\alpha)]^{-1}$ $\exp(-t_+\tau/2)$ contains the same term for n odd.

By the use of (19) and (58) we obtain the following expression for the scattered field in the problem of Fig.2 when a field of the type $(E_1^{\,i}, H_1^{\,i})$ incides:

$$E_{1y}{}^{s}(x,z) = \sum_{-\infty}^{\infty} R_n \exp(-j\tau_n z - j\alpha_n x) \tag{61}$$

$$-H_{1x}{}^{s}(x,z) = \sum_{-\infty}^{\infty} Y_n R_n \exp(-j\tau_n z - j\alpha_n x) \tag{62}$$

where:

$$\tau_n = \eta + 2\pi n/d \qquad (n=0,\pm1,\pm2,\dots) \tag{63}$$

$$\alpha_n = [K_o{}^2 - (\eta + 2\pi n/d)^2]^{1/2} \qquad (\mathrm{Im}\ \alpha_n \le 0) \tag{64}$$

and R_n are coefficients defined by :

$$R_n = \frac{E_o(\omega) \exp[-j(\pi/4)-(1/2)\tau(\alpha_0)t_+(-\alpha_0)]\ f(\alpha_n)}{m_+(-\alpha_0)} \left[(-1)^n \frac{(1+Z_n Y_0)}{(\alpha_n + \alpha_0)} + j\frac{Z_n}{\omega\mu}\right] \tag{65}$$

4. THE TE MODES SCATTERING MATRIX

By combining the results of the problem of fig.2 and the problem of fig.3 we will determine the coefficients of the TE modes scattering matrix in its non symmetric form [see 10]; in particular we will write the scattering matrix in the form:

$$\underline{S} = \begin{bmatrix} \underline{\Gamma} & \underline{T} \\ \underline{T} & \underline{\Gamma} \end{bmatrix} \tag{66}$$

where $\underline{\Gamma}$ is the reflection matrix and $\underline{T}$ is the transmission matrix; since the screen is infinitely thin it can be shown that:

$$\underline{T} = \underline{1} + \underline{\Gamma} \tag{67}$$

The TM modes scattering matrix can be obtained by the use of Babinet's principle [11]; in fact the dual source of a TM incident plane wave is a TE plane wave and the complementary screen of the structure shown in fig.1 (with a=c) is the same screen, shifted by d/2 along the i_z axis.
The following equations (68), (69) and (70) give the analytical form of $\underline{\Gamma}$ for TE modes; the second subscript indicates the incident mode, while the first indicates the scattered one. $\underline{S}$ can be obtained by the use of (67).

(for even values of n-m, except zero)

$$\Gamma_{nm} = \frac{\exp[-j(\pi/4) -(1/2)\tau(\alpha_m)t_+(-\alpha_m)]\ f(\alpha_n)}{m_+(-\alpha_m)} \left[\frac{(1+Z_nY_m)}{(\alpha_n + \alpha_m)} + j \frac{Z_n}{\omega\mu} \right] \tag{68}$$

(for odd values of n-m)

$$\Gamma_{nm} = \frac{\exp[-j(\pi/4) -(1/2)\tau(\alpha_m)t_+(-\alpha_m)]\ f(\alpha_n)}{m_+(-\alpha_m)} \left[-\frac{(1+Z_nY_m)}{(\alpha_n + \alpha_m)} + j \frac{Z_n}{\omega\mu} \right] \tag{69}$$

(for n=m) :

$$\Gamma_{nn} = \frac{\exp[-j(\pi/4) -(1/2)\tau(\alpha_n)t_+(-\alpha_n)]\ f(\alpha_n)}{m_+(-\alpha_n)} \left[\frac{1}{\alpha_n} + j \frac{Z_n}{\omega\mu} \right] - \frac{1}{2} \tag{70}$$

where $f(\alpha_n)$ is defined by (60);

5. NORMAL INCIDENCE CASE

In the case of normal incidence ($\Theta_o = 0$) the same method used in section 2 can be applied to study the problem of fig.2.
Some simplifications occur in the expressions (47) and (48) of $t(\alpha)$ and $m(\alpha)$; $t(\alpha)$ becomes $-1/\tau$ and its decomposition is well known [7].
We do not discuss the steps to obtain the exact expression of the transmission and reflection coefficients; the only problem consists in the right evaluation of the limit conditions, after having changed the expression of $t(\alpha)$.
We will report the analytical form of the TE modes reflection matrix, because the transmission matrix can be obtained by (67):

(for even values of n-m, except zero)

$$\Gamma_{nm} = \frac{\exp(-j\pi/4)}{m_+(-\alpha_m)}\ f(\alpha_n)\ \frac{\alpha_m}{\alpha_m^{\ 2} - \alpha_n^{\ 2}} \tag{71}$$

(for odd values of n-m)

$$\Gamma_{nm} = \frac{-j\ \ \exp(-j\pi/4)}{m_+(-\alpha_m)}\ f(\alpha_n)\ \frac{\alpha_m}{(\alpha_n - \alpha_m)}\ \tau_n \tag{72}$$

(for n=m)

$$\Gamma_{nn} = \frac{f(\alpha_n)\ \exp(j\pi/4)}{m_+(-\alpha_n)} \ \frac{\pi\ \omega\ \mu}{4\ \alpha_n} - \frac{1}{2} \qquad (73)$$

where :

$$f(\alpha_n) = \frac{j\tau_n \exp(j\pi/4)}{2[j\tau_n + (K_o+\alpha_n)]^{1/2}\ [j\tau_n - (K_o+\alpha_n)]^{1/2}} \lim_{\alpha\to\alpha n} (\alpha-\alpha_n)\ m_+(\alpha)$$

(for n even) (74)

$$f(\alpha_n) = \frac{j\tau_n \exp(-j\pi/4)}{2[j\tau_n + (K_o+\alpha_n)]^{1/2}\ [j\tau_n - (K_o+\alpha_n)]^{1/2}} \lim_{\alpha\to\alpha n} (\alpha-\alpha_n)\ [m_+(\alpha)]^{-1}$$

(for n odd) (75)

$$f(\alpha_n) = \lim_{\alpha\to\alpha n} \frac{\exp(j\pi/4)\ m_+(\alpha)\ (\alpha-\alpha_n)^{1/2}}{2} \qquad \text{(for n=0)} \qquad (76)$$

6. NUMERICAL RESULTS

A simple computer code has been developed to calculate reflection and transmission coefficients from the analytical expression and to plot them for different values of the incidence angle and of the ratio d/λ .
Comparison with the results in [5] shows a very good agreement at low frequencies while at higher frequencies our results are more accurate.

We now report some examples of the convergence of the numerical value of the transmission coefficient of the fundamental mode:

-For Θ = 30 and $d/\lambda = 1$
considering the first 100 and 200 terms in the infinite sum (51) and infinite product (49) the results change from:
T_1 = 0.416692 + j 0.235711 to
T_2 = 0.416691 + j 0.235711
-For Θ = 30 and $d/\lambda = 5$
considering the first 100 and 200 terms in the infinite sum (51) and infinite product (49) the results change from:
T_1 = 0.492674 + j 0.0475643 to
T_2 = 0.492678 + j 0.0475640

For comparison the transmission coefficient of the third mode is reported.

-For $\Theta = 30$ and $d/\lambda = 20$
considering the first 100 and 200 terms in the infinite sum (51) and infinite product (49) the results change from:
$T_1 = -0.216098 + j\ 0.00393715$ to
$T_2 = -0.216091 + j\ 0.00393720$

Plots are shown (in Figs. 4 to 7) for the following incidence angles:
10, 30, 50, 70
for modes from -2 to +2 and for a range of d/λ from 0 to 8.

7. CONCLUSIONS

In this paper we have studied the diffraction of a plane wave, with an arbitrary angle of incidence, on an infinite strip grating formed by strips and gaps of equal width.
We have obtained an exact solution by using the Wiener-Hopf technique; the analytical expression for the TE modes scattering matrix, in terms of infinite series has been derived. Particular cases have been considered and very accurate results have been obtained.
By using Babinet's principle [11], the scattering matrix for TM modes can be derived from the TE matrix.
These results are valid in a large range of frequencies ; they can be used to study more complicated structures, obtained by cascading two or more simple parallel screens, placed at a finite distance and arbitrarily shifted.
This work can be developed in two directions:
- to estabilish the properties of the scattering matrix of a zero thickness metallic screen (a work in this sense is under publication);
- to study the screen shown in fig. 1 with $a \neq c$; in fact this case can be obtained by considering two coincident shifted screens with $a=c$.

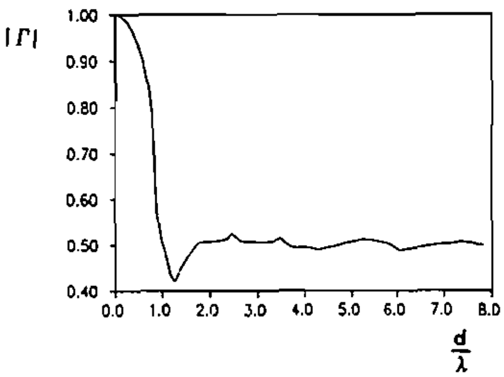

Figure 4.0: Plot of the modulus of the reflection coefficient of mode 0, when the field $E^i(r)$ incides (see 1), with Θ_o = 10.

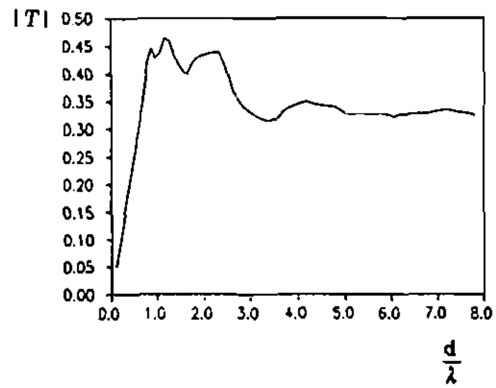

Figure 4.2: Plot of the modulus of the transmission coefficient of mode 1, when the field $E^i(r)$ incides (see 1), with Θ_o = 10.

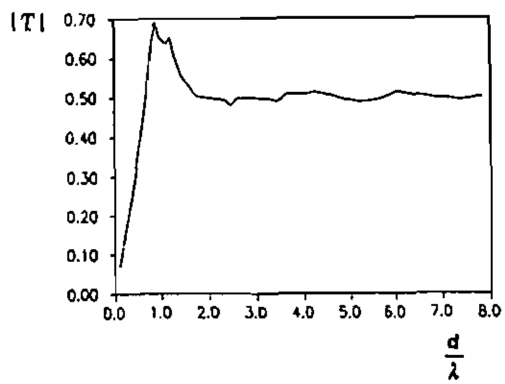

Figure 4.1: Plot of the modulus of the transmission coefficient of mode 0, when the field $E^i(r)$ incides (see 1), with Θ_o = 10.

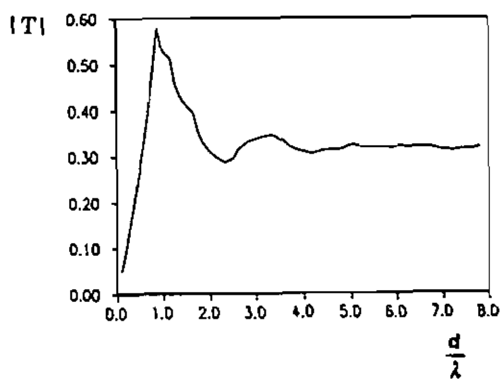

Figure 4.3: Plot of the modulus of the transmission coefficient of mode -1, when the field $E^i(r)$ incides (see 1), with Θ_o = 10.

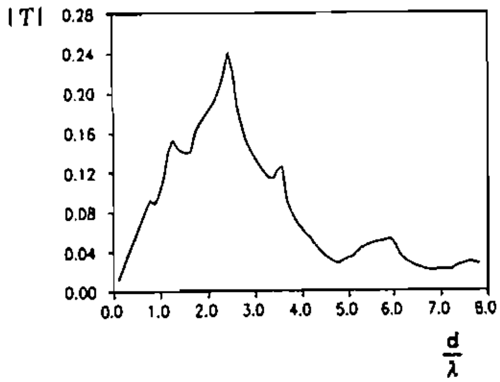

Figure 4.4: Plot of the modulus of the transmission coefficient of mode 2, when the field $E^i(r)$ incides (see 1), with θ_o = 10.

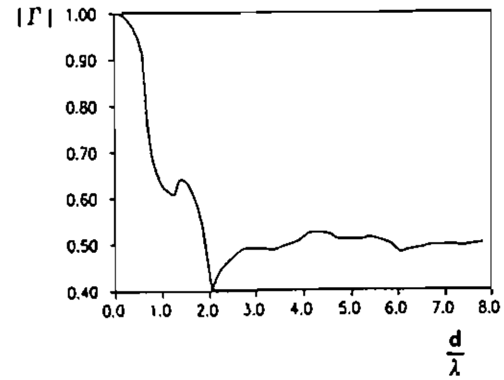

Figure 5.0: Plot of the modulus of the reflection coefficient of mode 0, when the field $E^i(r)$ incides (see 1), with θ_o = 30.

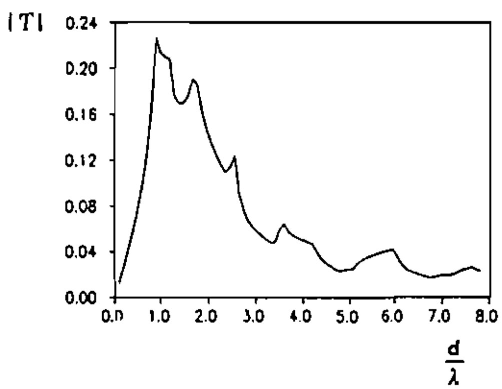

Figure 4.5: Plot of the modulus of the transmission coefficient of mode -2, when the field $E^i(r)$ incides (see 1), with θ_o = 10.

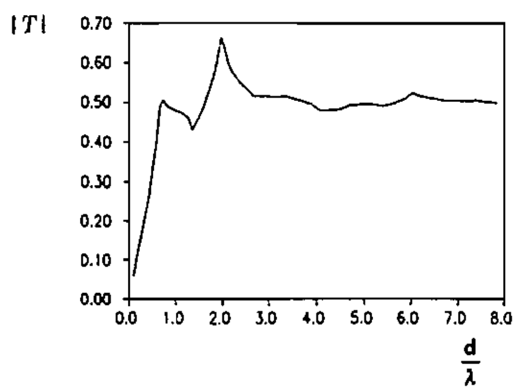

Figure 5.1: Plot of the modulus of the transmission coefficient of mode 0, when the field $E^i(r)$ incides (see 1), with θ_o = 30.

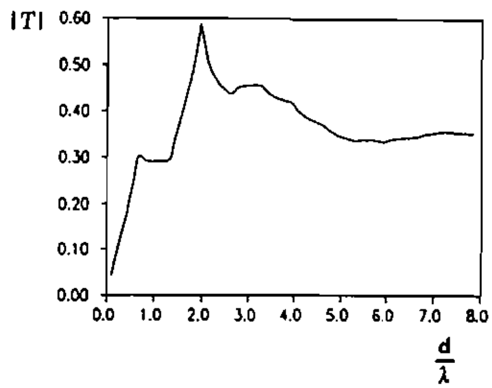

Figure 5.2: Plot of the modulus of the transmission coefficient of mode 1, when the field $E^i(r)$ incides (see 1), with Θ_o = 30.

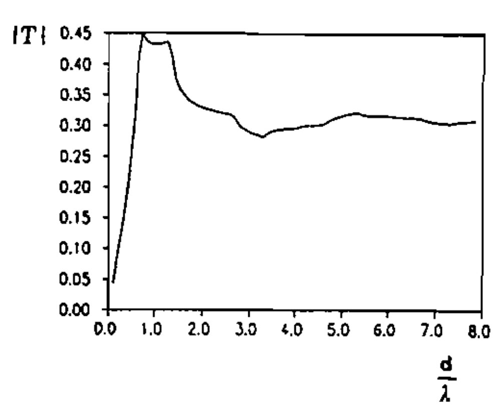

Figure 5.3: Plot of the modulus of the transmission coefficient of mode -1, when the field $E^i(r)$ incides (see 1), with Θ_o = 30.

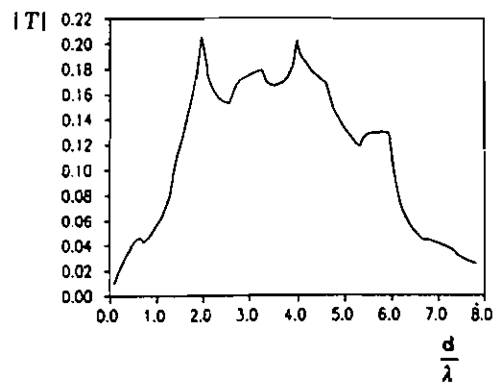

Figure 5.4: Plot of the modulus of the transmission coefficient of mode 2, when the field $E^i(r)$ incides (see 1), with Θ_o = 30.

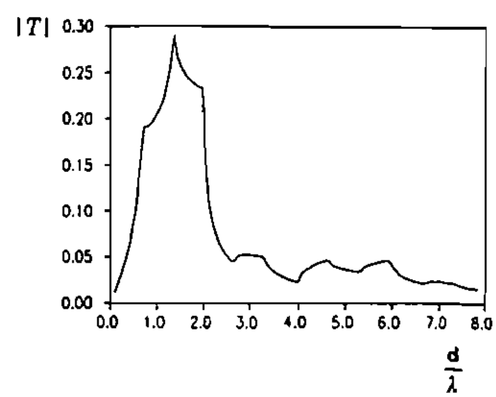

Figure 5.5: Plot of the modulus of the transmission coefficient of mode -2, when the field $E^i(r)$ incides (see 1), with Θ_o = 30.

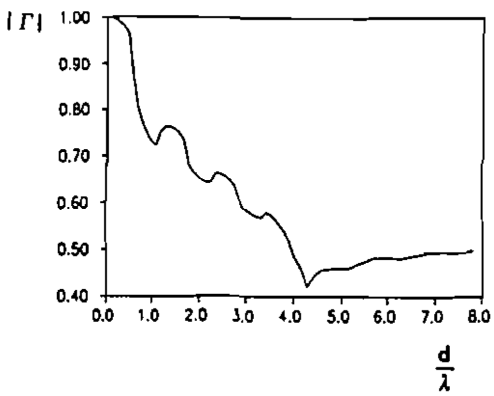

Figure 6.0: Plot of the modulus of the reflection coefficient of mode 0, when the field $E^i(r)$ incides (see 1), with $\Theta_o = 50$.

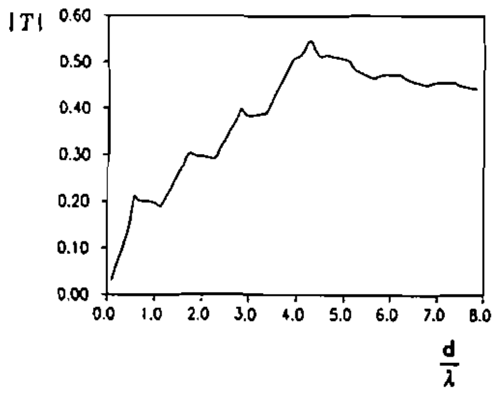

Figure 6.2: Plot of the modulus of the transmission coefficient of mode 1, when the field $E^i(r)$ incides (see 1), with $\Theta_o = 50$.

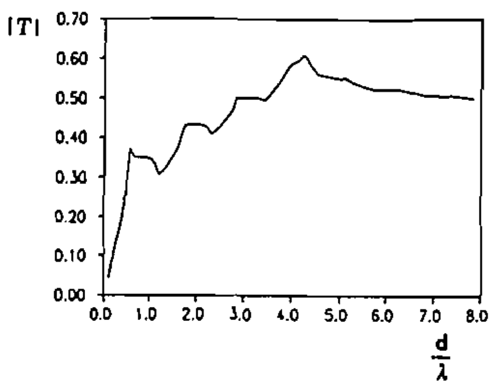

Figure 6.1: Plot of the modulus of the transmission coefficient of mode 0, when the field $E^i(r)$ incides (see 1), with $\Theta_o = 50$.

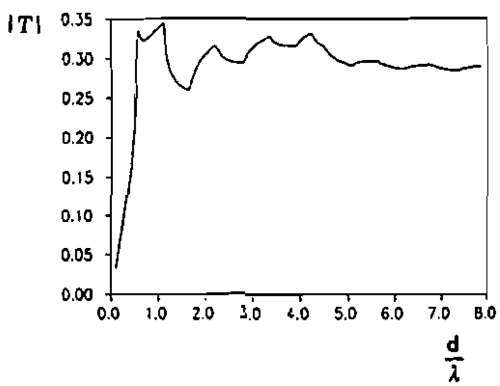

Figure 6.3: Plot of the modulus of the transmission coefficient of mode -1, when the field $E^i(r)$ incides (see 1), with $\Theta_o = 50$.

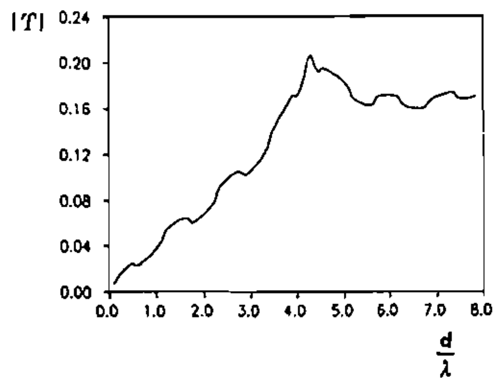

Figure 6.4: Plot of the modulus of the transmission coefficient of mode 2, when the field $E^i(r)$ incides (see 1), with $\theta_o = 50$.

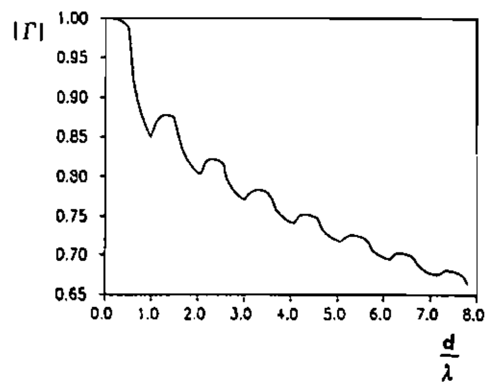

Figure 7.0: Plot of the modulus of the reflection coefficient of mode 0, when the field $E^i(r)$ incides (see 1), with $\theta_o = 70$.

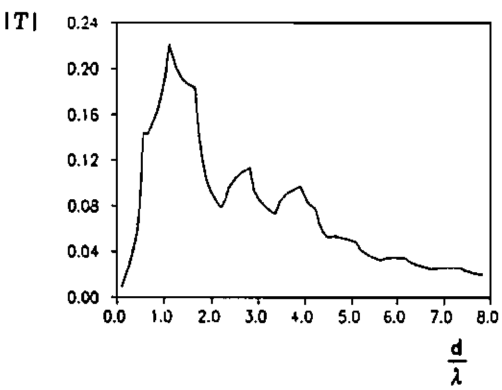

Figure 6.5: Plot of the modulus of the transmission coefficient of mode -2, when the field $E^i(r)$ incides (see 1), with $\theta_o = 50$.

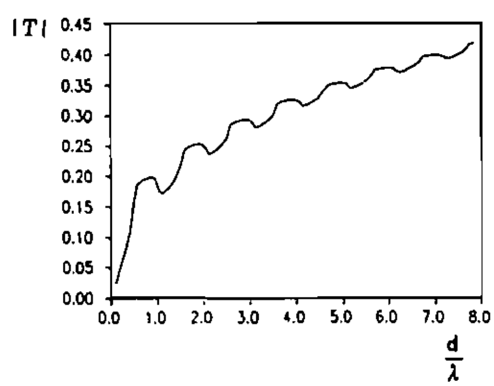

Figure 7.1: Plot of the modulus of the transmission coefficient of mode 0, when the field $E^i(r)$ incides (see 1), with $\theta_o = 70$.

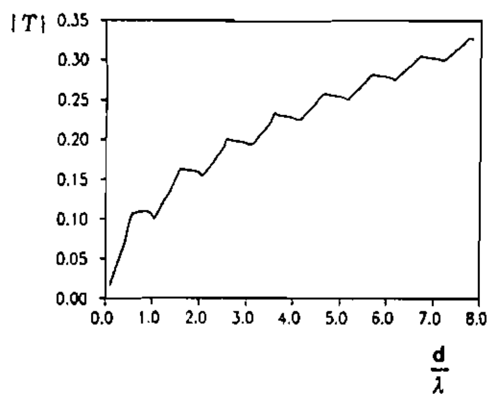

Figure 7.2: Plot of the modulus of the transmission coefficient of mode 1, when the field $E^i(r)$ incides (see 1), with Θ_o = 70.

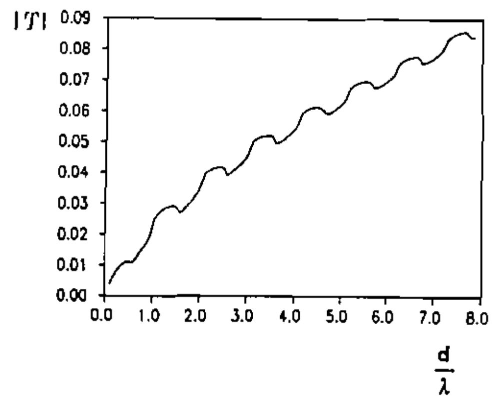

Figure 7.4: Plot of the modulus of the transmission coefficient of mode 2, when the field $E^i(r)$ incides (see 1), with Θ_o = 70.

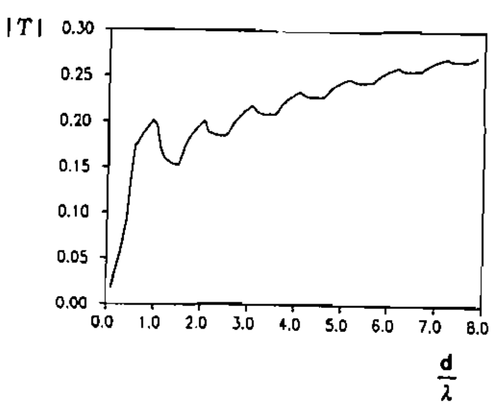

Figure 7.3: Plot of the modulus of the transmission coefficient of mode -1, when the field $E^i(r)$ incides (see 1), with Θ_o = 70.

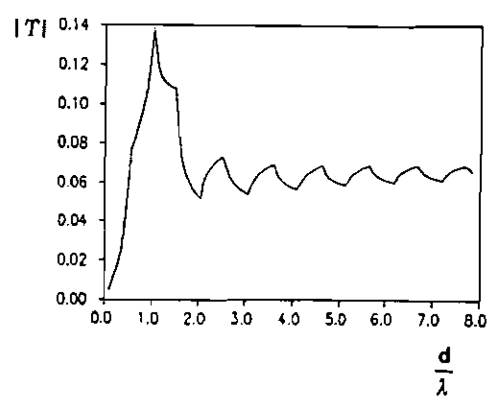

Figure 7.5: Plot of the modulus of the transmission coefficient of mode -2, when the field $E^i(r)$ incides (see 1), with Θ_o = 70.

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