Collision Resistant Modulation

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Abstract

This paper proposes a new class of signal-space codes combined with time-frequency hopping for wireless mobile communication. These schemes are designed to operate over a Multiple Access Channel with slotted random access protocols, where transmission is impaired by collisions, fading and additive noise. Examples of construction of such signal sets are provided and numerical results, based on closed-form error probability analysis show that our schemes achieve a significant reduction of the information loss probability with respect to standard modulation formats. Unlike standard forward error correcting codes, our schemes do not reduce the spectral efficiency of transmission.

I. Introduction

Unregulated random access schemes are really appealing for their intrinsic simplicity. Their performance, however, is strongly limited by collisions i.e., multiple concurrent transmissions in the same slot, with consequent loss of the transmitted information. Due to collisions, the random access scheme performance degrades when the channel load increases beyond a certain threshold. As a consequence, the use of random access schemes is usually limited to transmissions on lightly loaded channels, such as packetized transmissions over control channels, while dynamic TDMA/FDMA allocation schemes are adopted for transferring data-packets.

Even if the use of random access schemes is limited to transmissions of only few control packets, collisions can heavily affect the system performance. Collisions of control packets, often lead to a delayed network answer to a resource allocation request, such as a new-call request, a hand-over request, or a talk-spurt restart. This can cause a significant degradation of quality for real time applications.

We consider a Gaussian or fading multiple access channel under a pure slotted ALOHA access protocol. Fundamental results assessing the channel capacity of an ALOHA system can be found in [1]. The performance of ALOHA in fading environment is analyzed in [2]. *Capture* is a simple technique to increase the system throughput in a wireless system, a comprehensive overview of which is given in [3]. Various combinations of ALOHA with frequency hopping and CDMA schemes with coding can be found in recent literature.

In this paper, we propose a combination of signal-space coding and time-frequency hopping. Code vectors (or equivalently, signals) are points in a D-dimensional real Euclidean space. The main idea is that, under certain conditions on the signal-space code and provided that signal points are transmitted so that their D components belong to different time-frequency slots, even if $k < D$ slots collide the signal can be correctly detected from its $D-k$ uncollided components. We define a class of signal-space codes called Collision Resistant Modulation (CRM) with the above property. We provide constructions of such signal sets and examples showing that a significant reduction of the information loss probability can be achieved by this technique.

The paper is organized as follows. Section II describes the traffic scenario and the access protocol; in Section III, CRM is defined and some examples of CRM are reported. In Section IV we provide error probability analysis and in Section V we show numerical results. Conclusions and directions for future work are summarized in Section VI.

II. The multiple access slotted collision channel

We consider a population of N_u users accessing N_f common subcarriers according to a slotted ALOHA access protocol. Time is subdivided in successive periods called *frames*. Each frame comprises N_t time slots for each channel. Thus, $N_s = N_t \times N_f$ time-frequency *slots* are available for transmissions in each frame. Furthermore we assume that each slot can carry L_s elementary signals for the transmission of L_s *real dimensions*. Then, according to the "$2WT$-Theorem" the total number of real dimensions is $N_s L_s \approx 2WT$, where T is the time duration of a frame and W is the overall system bandwidth. Note that the L_s real dimensions can be realized by further subdivision of each slot into smaller time and/or frequency units or even by orthogonal spreading sequences.

We suppose that each user fills $g = R_b/(\eta L_s)$ slots in each frame, where R_b [bits/frame] is the user transmission bit rate and η [bits/dim] is the spectral efficiency of the modulation scheme.

In each frame, the slots to be used for transmission are randomly selected by each user. This corresponds to providing each user with a pseudo-random time-frequency hopping code. Two options are available for subdividing long data packets into slots. *(i)* Small values of L_s imply the use of many slots to transmit a packet and result in a fast hopping scheme. *(ii)* Large values of L_s imply the use of fewer slots to transmit a packet and result in a slow hopping scheme.

Outer coding and interleaving can further be applied across a certain number of frames to reduce the packet error rate to the desired level. As said before, we imagine that information is organized in packets. Packets are encoded by the outer code, so that, without loss of generality, an outer code word is produced for each packet. Outer code words are then interleaved and the interleaved encoded data sequence is partitioned into blocks, which are finally modulated and transmitted over the assigned sequence of time-frequency slots. The interleaving depth, i.e., the number of frames spanned by an outer code word, must be optimized by taking into account the maximum allowable decoding delay and the more or less bursty nature of transmission. On its own, the modulator can be seen as a signal-space encoder, where modulation acts as an inner code. In this paper we are mainly concerned with the optimization of this inner code for the collision channel with Gaussian noise and fading, typical of a wireless mobile environment.

In order to make the problem analytically tractable, we model the transmission process of each user in each slot, as a Bernoulli process with parameter $p = g/N_s = R_b/(2WT\eta)$ representing the probability that one user occupies one slot. This means that in each frame each user occupies a variable, binomially distributed, number of slots. This approximation does not appreciably affect the quality of results when the number of users exceeds some tens.

Under the above assumptions, the probability that a transmitted slot suffers from a collision is given by

$$P_c = 1 - (1-p)^{N_u - 1} \tag{1}$$

In the limit case for $N_s, N_u \to \infty$ (i.e. infinite users population) the number of transmitting users in each slot becomes a Poisson distributed random variable with average $G = \lim_{N_s, N_u \to \infty} g N_u / N_s$. As a consequence, the slot collision probability is

$$P_c = 1 - e^{-G} \tag{2}$$

The parameter G corresponds to the so called *offered channel traffic*. It can be verified that this second equation is applicable with a negligible error for $N_u > 20$ and will be used throughout our examples.

III. The Collision Resistant Modulation

The modulator signal set can be represented as a finite and discrete set of points in a Euclidean D-dimensional real space. We assume that modulation and time-frequency hopping are combined so that the D components $x_1, \ldots, x_D$ of the signal point $\mathbf{x}$ are transmitted over different time-frequency slots. Then, they will be affected by different collision events. Moreover, because of time and frequency selectivity, the components will be also affected by different fading coefficients. We model the collision channel with AWGN and fading as the on-off vector channel

$$\mathbf{y} = C(G\mathbf{x} + \mathbf{n}) \tag{3}$$

where $\mathbf{x} = (x_1, \ldots x_D)$ is a transmitted vector taken from the signal set (signal-space code) S of cardinality $|S|$, $\mathbf{n} = (n_1, \ldots n_D)$ is additive white Gaussian noise with i.i.d. components $n_i = \mathcal{N}(0, N_0/2)$, $\mathbf{y} = (y_1, \ldots y_D)$ is the received signal vector, $G = diag(g_1, \ldots g_D)$ represents the channel fading coefficients and $C = diag(c_1, \ldots c_D)$ represents the collision pattern. The elements c_i can be either 0 or 1, where $c_i = 0$ represents a collision on the i-th signal component. With the assumptions on the collision statistics made in Section II, c_i are Bernoulli i.i.d. random variables with $P_c = P(c_i = 0)$. The fading coefficients g_i are random variables whose joint statistics depends on the physical characteristics of the propagation channel, such as the multipath delay profile and the Doppler spectrum [7].

We assume that the receiver has perfect channel state information (CSI) on both the collisions and the fading, i.e., has a complete knowledge of C and G. In this case, the Maximum Likelihood (ML) detection is obtained by minimizing, over all $\mathbf{x} \in S$, the following metric

$$d_C^2(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{D} c_i (y_i - g_i x_i)^2 \ . \tag{4}$$

This detection criterium corresponds to the minimum distance criterium of the received point from the points of a signal set $S(C)$ in an $(D-k)$-dimensional Euclidean space, where $S(C)$ is the projection of S on the subspace generated by the $D-k$ axes corresponding to the non zero c_i's ($D - k = w_H(C)$, the Hamming weight of C and k is the number of collisions in the pattern C). According to this detection criterium, in order to avoid systematic errors in the presence of $k < D$ collisions, we require that the points in $S(C)$ are distinct, for all C with $w_H(C) > 0$. Then, we have the following

Definition — A *Collision Resistant Modulation* (CRM) is a signal-space code S with the property that any projection on any coordinate subspace has the same number of distinct points, i.e., $|S(C)| = |S|$ for all non-zero collision patterns C. □

Equivalently, the vectors in S must have all distinct coordinates or, more precisely, the Hamming distance between any pair of vectors in S must be D. A similar requirement is imposed in the design of *high diversity* signal constellations for the fading channel where the number of distinct components is called *modulation diversity* [4;5].

Example 1 – A simple example is given in Figure 1, where both projections on the two coordinate axis have four distinct points. We note that this CRM can be viewed as block coded modulation obtained by concatenating a (2,1) repetition code with a 4-PAM. ◇

Example 2 – *Rotated cubic constellations* with maximum diversity introduced in [5] are CRMs. These constellations are obtained by applying a particular rotation matrix to the vertices of a D-dimensional hypercube in order to produce the maximum possible *modulation diversity* $L = D$. The two-dimensional case is shown in Figure 2. An obvious advantage of this type of CRM with respect to the one of Example 1 is a lower peak to average energy ratio. ◇

Let C_k denote any one of the $\binom{D}{k}$ collision patterns with k collided slots, then

$$P(C_k) = P_c^k (1-P_c)^{(D-k)} \quad \text{for} \ \ k = 0, \ldots D \tag{5}$$

If a CRM is used, the receiver is able to detect correctly the transmitted signal point even when all the dimensions but one are affected by collision, provided that the signal-to-noise ratio (SNR) is sufficiently large. However, the minimum distance of the projected signal set $S(C)$ is reduced by collisions, therefore the error performance in presence of noise degrades.

With the aim of characterizing good CRMs, let us consider the Union Bound on the *symbol error probability* (i.e., the average probability of choosing a signal $\widehat{\mathbf{x}} \neq \mathbf{x}$, when $\mathbf{x}$ was actually transmitted)

$$P(e) \leq \frac{1}{|S|} \sum_{\mathbf{x} \in S} \sum_{\mathbf{x} \neq \widehat{\mathbf{x}}} E_C[P(\mathbf{x} \to \widehat{\mathbf{x}}|C)] \tag{6}$$

where $P(\mathbf{x} \to \widehat{\mathbf{x}}|C)$ is the pairwise error probability averaged with respect to G and conditioned by C, and E_C denotes the expectation over all the possible 2^D collision patterns. $\{\mathbf{x} \to \widehat{\mathbf{x}}\}$ represents the event that the decoder chooses $\widehat{\mathbf{x}}$ when $\mathbf{x}$ was actually transmitted, as if $\mathbf{x}$ and $\widehat{\mathbf{x}}$ were the two only possible decoder outcomes. We refer to this as *pairwise error event.*

In the case of a pure AWGN channel, G is the identity matrix and we have

$$P(\mathbf{x} \to \widehat{\mathbf{x}}|C) = \frac{1}{2}\mathrm{erfc}\left(\frac{d_C(\mathbf{x}, \widehat{\mathbf{x}})}{2\sqrt{N_0}}\right) \tag{7}$$

where $d_C(\mathbf{x}, \widehat{\mathbf{x}})$, defined in (4), is the Euclidean distance between the projections $\mathbf{x}^{(C)}$ and $\widehat{\mathbf{x}}^{(C)}$ of the vectors $\mathbf{x}$ and $\widehat{\mathbf{x}}$ in the projected signal set $S(C)$. This indicates that good CRMs for the Gaussian channel should maximize the minimum distance $d_{C,min}$ in the projected constellations $S(C)$.

In the case of an independent Rayleigh fading channel, the g_i's are real i.i.d. Rayleigh random variables with unit second moment. From [6] we have

$$\begin{aligned} \mathrm{P}(\mathbf{x} \to \widehat{\mathbf{x}}|\mathrm{C}) &= \frac{1}{2} \sum_{i=1}^{D-k} \left(1 - \frac{\delta_{C,i}}{\sqrt{4N_0 + \delta_{C,i}^2}}\right) \prod_{\substack{j=1 \\ j \neq i}}^{D-k} \frac{\delta_{C,i}^2}{\delta_{C,i}^2 - \delta_{C,j}^2} \\ &\leq \frac{(4N_0)^{D-k}}{\prod_{i=1}^{D-k} \delta_{C,i}^2} \end{aligned} \tag{8}$$

where $D - k = w_H(C)$ and $\delta_{C,i} = |x_i^{(C)} - \widehat{x}_i^{(C)}|$, for $i = 1, \ldots D - k$, are the componentwise distances between the projections $\mathbf{x}^{(C)}$ and $\widehat{\mathbf{x}}^{(C)}$ (ordering of components is irrelevant and we can always reorder the non-zero components of the projections so that they are indexed by $i = 1, \ldots, D-k$). The bound in (8) is the well known Chernoff bound which proves that the asymptotic performance is essentially determined by the *diversity* $D - k$ of the projected signal set $S(C)$ and by the so called *product distance* $d_P = \prod_{i=1}^{D-k} \delta_{C,i}$ between the projected points $\mathbf{x}^{(C)}$ and $\widehat{\mathbf{x}}^{(C)}$.

In this case the criterium for selecting good CRMs is not so sharp. However, a sensible approach is to maximize the minimum component distance $\delta_{C,min}$ of the projected signal sets $S(C)$. We note that since

$$d_C^2(\mathbf{x}, \widehat{\mathbf{x}}) = \sum_{i=1}^{D-k} \delta_{C,i}^2 \tag{9}$$

a good CRM for the fading channel would also be reasonably good for the Gaussian channel but not vice versa. In general, the design of optimal CRMs is strictly dependent on the type and statistic of the collision patterns and of the fading coefficients, as well as on the SNR range.

Example 2 (cont.) – *Rotated cubic constellations* introduced in [5] satisfy the requirement of maximum diversity and maximum minimum product distance $d_{P,min}$ and hence minimize the Chernoff bound in (8). Although these signal sets may not be optimal CRMs, they guarantee some reasonably large $\delta_{C,min}$'s. For example, for $D = 2$ (see Fig. 2) the best rotation angle which maximizes $\delta_{C,min}$ and hence the performance of the CRM on the Gaussian channel is $\alpha = \arctan(3) \approx 72°$. The rotation angle which maximizes $d_{P,min}$ is $\alpha = (1/2)\arctan(1/2)) + \pi/4 \approx 77°$ and gives the best performance of the CRM over the fading channel. In the first case $\delta_{C,min} = 0.63$ and in the second case $\delta_{C,min} = 0.46$. We note that the two one-dimensional projected signal sets are equivalent and $d_{C,min} = \delta_{C,min}$. In the general D-dimensional case, Algebraic Number Theory [4] can be used to find rotations which maximize $\delta_{C,min}$, whereas no tool has yet been devised to maximize the minimum Euclidean distance of all the projections $S(C)$. ◇

Example 3 – *Rotated hypercube.* For $D = 4$, we can use the following rotation matrix to rotate the 16 vertices of a hypercube $(\pm 1, \pm 1, \pm 1, \pm 1)$

$$R = \begin{pmatrix} 0.4857 & 0.7859 & -0.2012 & -0.3255 \\ -0.7859 & 0.4857 & 0.3255 & -0.2012 \\ 0.2012 & 0.3255 & 0.4857 & 0.7859 \\ -0.3255 & 0.2012 & -0.7859 & 0.4857 \end{pmatrix} \tag{10}$$

This was found in [5] to maximize the minimum product distance within a certain family of rotation matrices. It is interesting to note that not all the $(D-k)$-dimensional projected constellations $S(C)$ are equivalent, this can be seen by simply considering the minimum Euclidean distances $d_{C,min}$ of the projected constellations $S(C)$. The four 1-dimensional projections have $d_{C,min} = 0.051$, the six 2-dimensional projections have three different $d_{C,min}$'s, namely $0.765, 0.650$ and 0.453, while all the four 3-dimensional projections have $d_{C,min} = 1.237$. ◇

We conclude this paragraph with an upper bound to the bit error probability of a CRM. Let $b = \eta D$ be the number of bits per D-dimensional signal. Assume that the points of S are labeled by binary strings of b bits and denote by $\mathcal{L}(\mathbf{x})$ the label of the point $\mathbf{x}$. Then, the bit error probability can be upper bounded by

$$P_b(e) \leq \frac{1}{b} \frac{1}{|S|} \sum_{\mathbf{x} \in S} \sum_{\mathbf{x} \neq \widehat{\mathbf{x}}} d_H(\mathcal{L}(\mathbf{x}), \mathcal{L}(\widehat{\mathbf{x}})) E_C[P(\mathbf{x} \to \widehat{\mathbf{x}}|C)] \tag{11}$$

where $d_H(\mathcal{L}(\mathbf{x}), \mathcal{L}(\widehat{\mathbf{x}}))$ is the Hamming distance between the binary labels of the projections of $\mathbf{x}$ and $\widehat{\mathbf{x}}$.

IV. Error probability analysis

In this section we derive expressions for the symbol and the bit error probabilities of some examples of CRMs and non-CRMs, in the case of AWGN and independent Rayleigh fading channels. The offered channel traffic G is given by $gN_u/N_s = H/\eta$, where the parameter $H = N_u R_b/(L_s N_s)$ depends only on the system constants and not on the type of signal set used for transmission.

Since we are evaluating the performance of the inner coding scheme, without loss of generality we can consider the case of slots of length $L_s = 1$. In this case the terms "slot" and "dimension" can be interchanged. The results obtained here can be translated easily into packet error rates if we assume a sufficient interleaving depth between the inner and the outer code, so that errors in the demodulator output can be regarded as mutually independent by the outer decoder.

Let us consider D-dimensional CRMs of the type shown in Figures 1 (CRM1) and 2 (CRM2), with 2^D points and spectral efficiency $\eta = 1$ [bit/dim]. Each symbol carries D bits. We compare them with two common modulation schemes *(i)* 2-PAM, $\eta = 1$ [bit/dim] and *(ii)* 2^D-PAM, $\eta = D$ [bit/dim].

Note that when η is increased, as in case *(ii)*, each user reduces the number of occupied dimensions (slots), i.e., the offered channel traffic G. This in turn reduces the collision probabilities, and a performance improvement is expected. As we will show in this section, in the presence of fading and noise, this performance improvement cannot compensate the intrinsic lack of robustness of non-CRM, so that larger improvements can be obtained by CRMs.

In the following, we let $\gamma_b = E_b/N_0$ where E_b is the average energy per bit and N_0 is the one-sided noise spectral density. We assume that a complete collision produces a symbol error with probability 1 and a bit error with probability 1/2. As an example we consider the case $D = 4$.

2-PAM – Since $\eta = 1$, the slot collision probability is $P_c = 1 - e^{-H}$. The bit error probability is given by

$$P_b'(e) = Q\,(1 - P_c) + \frac{1}{2}P_c \tag{12}$$

where

$$Q = \frac{1}{2}\mathrm{erfc}(\sqrt{\gamma_b}) \quad \text{and} \quad Q = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}}\right) \tag{13}$$

for the Gaussian channel and for the fading channel, respectively. If we want to compare symbol error probabilities we need to consider the same number of bits per symbol. In this case, we need to consider blocks of four 2-PAM symbols, then we have

$$P'(e) = 1 - (1 - P_2)^4 \tag{14}$$

where $P_2 = Q(1 - P_c) + P_c$ and Q is the same as above. These symbols are taken from a 4-dimensional modulation with points $(\pm1, \pm1, \pm1, \pm1)$, i.e., a non rotated hypercube.

16-PAM – To fit a 16-PAM symbol we only need one dimension, thus the offered channel traffic is $G = H/4$ and the slot collision probability becomes $P_c = 1 - e^{-H/4}$.

The exact symbol error probability for the Gaussian channel can be computed in closed form as

$$P''(e) = \frac{15}{16}\,\mathrm{erfc}\left(\sqrt{12/255\,\gamma_b}\right)(1 - P_c) + P_c \tag{15}$$

Let us define

$$P_2(x) = \frac{1}{2}\left(1 - \frac{x}{\sqrt{4N_0 + x^2}}\right) \tag{16}$$

then the pairwise error probability with fading is $P_2(d_0)$ where $d_0 = 4/\sqrt{85}$ is the minimum distance of the 16-PAM with E_b normalized to 1. The exact symbol error probability for the fading channel is

$$P''(e) = \frac{30}{16}P_2(d_0)(1 - P_c) + P_c\ . \tag{17}$$

Assuming Gray labeling of the signal points, the bit error probability can be exactly computed as

$$\begin{aligned} P_b(e) \;=\; & \frac{1}{32}[16Q_1 + 15Q_3 + Q_7 + 2Q_9 \\ & +Q_{11} - 8Q_{13} - 7Q_{15} + 7Q_{17} \\ & +6Q_{19} - Q_{21} + Q_{25} - Q_{29}] \end{aligned} \tag{18}$$

where

$$Q_n = \frac{1}{2}\mathrm{erfc}(nd_0\sqrt{\gamma_b}/2) \quad \text{and} \quad Q_n = P_2(nd_0) \tag{19}$$

for the Gaussian and for the fading channel respectively. Taking into account the collisions we get

$$P_b''(e) = P_b(e)(1 - P_c) + \frac{1}{2}P_c\ . \tag{20}$$

CRM1 – This is a simple block coded modulation which compensates for the rate decrease due to the (4,1) repetition code by a signal set expansion. For $D = 4$, it corresponds to a 16-PAM placed along the main diagonal of a 4-dimensional hypercube. Let $d_0 = 4/\sqrt{85}$ be the minimum distance of this 16-PAM, then all the $(4-k)$-dimensional projected constellations (for $k = 0, \ldots 3$) are equivalent to a 16-PAM with minimum distance $d_k = d_0\sqrt{(4-k)/4}$. For the Gaussian channel, the conditional error probability given any collision pattern with k collisions is

$$P(e|C_k) = \frac{15}{16}\,\mathrm{erfc}\left(\sqrt{\frac{4-k}{85}\,\gamma_b}\right) \tag{21}$$

Since the componentwise distances are all equal to d_3 we apply the result from [6, Remark 3] and we get

$$\mathrm{P(e|C_k)} = \frac{30}{16}\left[1 - \left(1 + \frac{4}{d_3^2\gamma_b}\right)^{(k-7/2)} \cdot \sum_{i=0}^{3-k}\binom{7/2-k}{i}\left(\frac{4}{d_3^2\gamma_b}\right)^{i}\right] \tag{22}$$

Since $\eta = 1$, the slot collision probability is $P_c = 1 - e^{-H}$, and the exact symbol error probability can finally be computed as

$$P^{(1)}(e) = \sum_{k=0}^{4}\binom{D}{k}P(e|C_k)P(C_k)\ . \tag{23}$$

	$P(e)$	$P_b(e)$
2-PAM	$4H$	$H/2$
16-PAM	$H/4$	$H/8$
CRM1	H^4	$H^4/2$
CRM2	H^4	$H^4/2$

Table 1. Approximate values of the error floors

where $P(C_k)$ is given by (5). The same equation can be used for the bit error probability $P_b(e)$ if we replace $P(e|C_k)$ with

$$P_b(e|C_k) = \frac{1}{32}[16Q_1^k + 15Q_3^k + Q_7 + 2Q_9^k + Q_{11}^k - 8Q_{13}^k - 7Q_{15}^k + 7Q_{17}^k + 6Q_{19}^k - Q_{21}^k + Q_{25}^k - Q_{29}^k] \quad (24)$$

where for the Gaussian channel

$$Q_n^k = \frac{1}{2}\,\mathrm{erfc}\left(n\sqrt{\frac{4-k}{85}\gamma_b}\right) \quad (25)$$

and for the fading channel

$$Q_n^k = \frac{1}{2}\left[1 - \left(1 + \frac{4}{(nd_3)^2\gamma_b}\right)^{(k-7/2)} \cdot \sum_{i=0}^{3-k}\binom{7/2-k}{i}\left(\frac{4}{(nd_3)^2\gamma_b}\right)^i\right]. \quad (26)$$

CRM2 – Let us consider the 4-dimensional rotated hypercube of Example 3. In this case, exact analytical error rates cannot be computed and we resort to the general upper bounds given in (6) and (11).

V. Results

Figures from 3 to 6 show the symbol error probabilities of the four schemes analyzed in the previous section, for $H = 0.1$ and $H = 0.01$ and for the Gaussian and the Rayleigh fading channel. A similar behavior was found for bit error probabilities.

The error floor (i.e., the asymptotic error probability value for $E_b/N_0 \to \infty$) coincides with the values of the complete collision probability which are summarized in Table 1. These approximate expressions are valid for small values of H (e.g. $H < 0.2$). Both CRMs improve significantly the error floors and for lower SNRs they outperform the 2-PAM and the 16-PAM. The gain on the error floor of CRM is significant for $H < 0.3$ and gradually disappears as $H \to 1$.

The rotated hypercube CRM2 remarkably improves over the repetition block-coded modulation CRM1 in the approximate E_b/N_0 range 5-25 dB for both Gaussian and fading channels. Depending on the application and on the outer coding scheme, symbol or bit error probability should be regarded as the relevant figure of merit. Suppose that an outer coding scheme (e.g., a Reed-Solomon code) with sufficient interleaving depth is applied and that, in order to obtain the required residual error probability at the output of the outer encoder, the target bit error probability at the demodulator output is 10^{-3} (this appears to be a realistic value for wireless communication, at least as speech transmission is concerned [7]). We see that CRM2 provides a coding gain of 4 dB (AWGN) and 2.5 dB (Rayleigh) with respect to CRM1 for high load ($H = 0.1$) and of 14 dB (AWGN) and 9 dB (Rayleigh) with respect to CRM1 for low load ($H = 0.01$). On the other hand, the two one-dimensional PAM modulations do not reach error probability 10^{-3} even for infinitely large E_b/N_0, since their error floor is higher.

VI. Conclusions

In this paper, we proposed a signal-space coding scheme combined with time-frequency hopping for transmission over the multiple access channel with slotted random access, Gaussian noise and fading. We defined a class of signal-space codes named Collision Resistant Modulations (CRM) and showed that a significant reduction of the information loss probability can be achieved by using a CRM scheme instead of standard modulation formats for both Gaussian and Rayleigh fading channel.

The idea of applying standard coding schemes for reducing the negative impact of collisions on the overall system performance is not new. However, all the proposed schemes trade this gain for a bandwidth expansion, typical of forward error correcting codes. CRM schemes, on the contrary, require only a marginal increase in the modulator complexity, but do not imply any loss of spectral efficiency.

The application we have in mind is the control channel of some wireless PCS or LAN, where traffic is highly bursty, the load is typically low but the loss or delay of control packets may have deleterious effects on the network protocol and reduce the overall throughput. The use of CRM is not strictly limited to pure ALOHA. Every MAC protocol in which some packets are transmitted in a random access fashion and are subject to collisions can benefit from the introduction of CRM. Future work will study the impact of CRM on the performance of more elaborate wireless protocols like PRMA (Packet Reservation Multiple access) and DQRUMA (Distributed Queueing Request Update Multiple Access), and will investigate the concatenation of CRM with outer codes for data transmission in wireless LANs.

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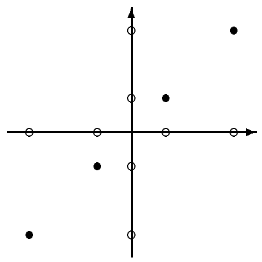

Fig. 1. Example of CRM for $D = 2$ (CRM1)

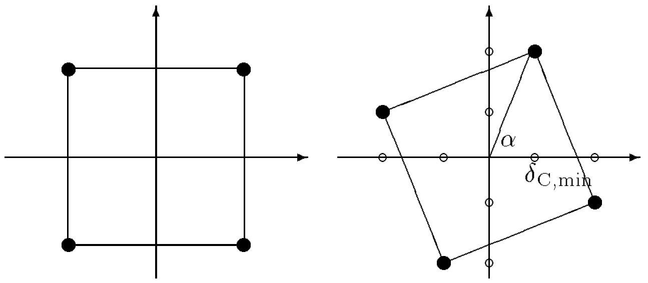

Fig. 2. Example of CRM for $D = 2$ obtained by rotation of a 4-PSK (CRM2)

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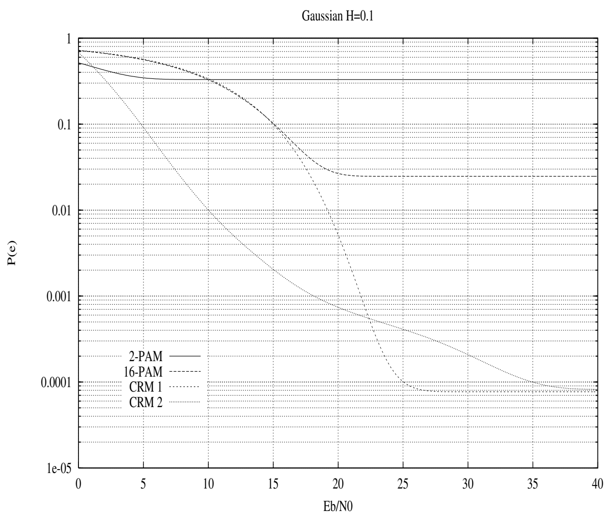

Fig. 3. Symbol error probability – Gaussian channel $H = 0.1$

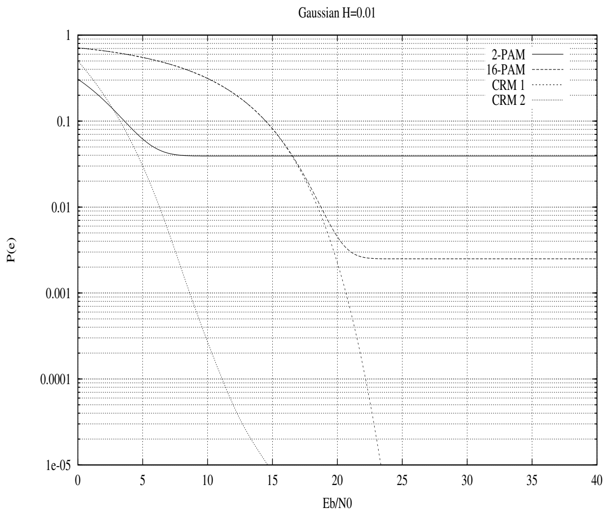

Fig. 4. Symbol error probability – Gaussian channel $H = 0.01$

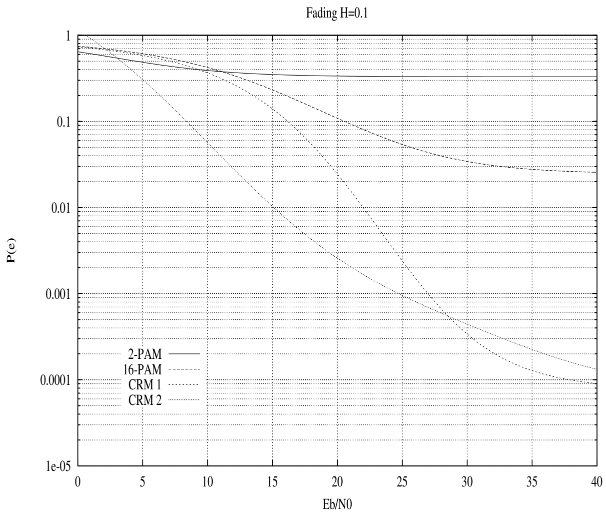

Fig. 5. Symbol error probability – Fading channel $H = 0.1$

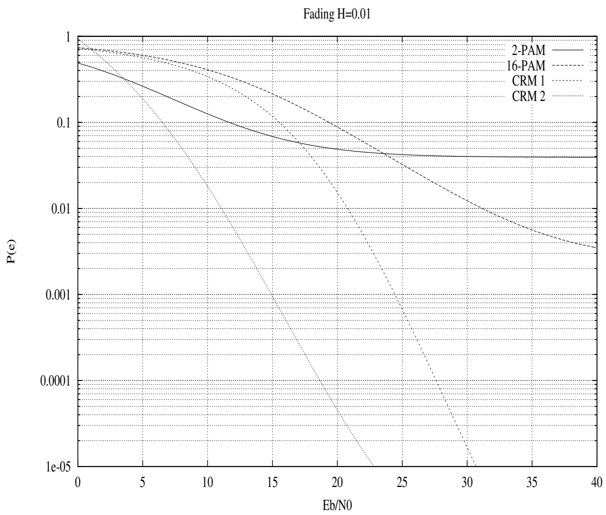

Fig. 6. Symbol error probability – Fading channel $H = 0.01$