New approach for transmission over fading channels

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Abstract – **The usual approach to improve transmission performance over fading channels is to increase the system diversity. This usually entails an increase in power or bandwidth. The type of diversity we propose does not present these drawbacks. The diversity is embedded in the signal constellation and it directly depends on the Hamming distance between the constellaton vectors. We also analyze additional coding schemes to apply to high modulation diversity constellations.**

Index Terms – **Modulation diversity, lattices, fading channels.**

1 Introduction

New lattice constellations matched to both Gaussian and fading channels have been published recently [1]. These lattices were built using algebraic number theory tools. They were found to be efficient on Rayleigh fading channels because they exhibit a very high diversity order $(8, 12, 16, \ldots)$. The same lattices have good performance on Gaussian channels due to their high density (asymptotic gain of 3.0, 4.5 or 6.0 dB). The study of these lattices to a new diversity technique that improves the performance on the Rayleigh fading channel without adding any redundancy and with no loss in performance on the Gaussian channel. This technique is simply described as a multidimensional rotation which increases the diversity order of the signal constellation.

A high dimensional space rotation produces a high diversity order. When the diversity grows to infinity, the Rayleigh channel acts like a Gaussian channel [8]. Figure 1 shows the pairwise error probability $P(\mathbf{x} \to \mathbf{y})$ on a Rayleigh channel versus the signal-to-noise ratio. The two lattice points $\mathbf{x}$ and $\mathbf{y}$ have L distinct components, i.e. the Hamming distance between the two points is $d_H(\mathbf{x}, \mathbf{y}) = L$. This creates a *modulation diversity* equal to L [1]. It is seen in Figure 1 that for L greater than 12, the performance is less than 2dB away from the Gaussian channel curve.

A brief summary of the algebraic construction of lattices is given in Figure 2. A special algebraic embedding σ called *canonical embedding* [3] is applied to an integral ideal I. The embedding σ converts the ideal I into a lattice $\Lambda_{n,L}$ in the n-dimensional real space $\mathbf{R}^n$. The ideal is a subset of the ring of integers O_K of a number filed $K = \mathbf{Q}(\theta)$ generated by a primitive element θ. The $\Lambda_{n,L}$ lattice rank is n and its diversity order is equal to L. The diversity of the algebraic lattice is related to the roots of the minimal polynomial of the primitive element. Thus, two important families of algebraic lattices are obtained when the roots are all real or all complex.

By appropriately selecting the number field and the ideal, it is possible to build the densest known lattices listed in Fig. 2 [2] with a high diversity order. Practically, the modulation diversity of these dense lattices is a consequence of a multidimensional rotation produced by the canonical embedding. Hence, we can also construct multidimensional rotated cubic $\mathbf{Z}^n$ lattices (rotated QAM constellation) by directly computing the canonical embedding in a totally real or a totally complex number field [7].

Section 2 describes how to compute some high diversity multidimensional rotations from totally complex number fields, and gives the exact expression of the rotation matrix. Section 3 compares the rotated lattices with trellis coded schemes designed for the fading channels. Finally, we propose in Section 4 some coding techniques to be combined with rotated constellations in order to outperform the AWGN channel.

In the sequel, we assume that the transmitter uses a quadrature amplitude modulation. The bidimensional QAM constellation associated to the modulation is viewed as a finite subset extracted from the integer lattice $\mathbf{Z}^2$. A point $\mathbf{p}$ in the real n-dimensional space is built by grouping $n/2$ bidimensional QAM symbols.

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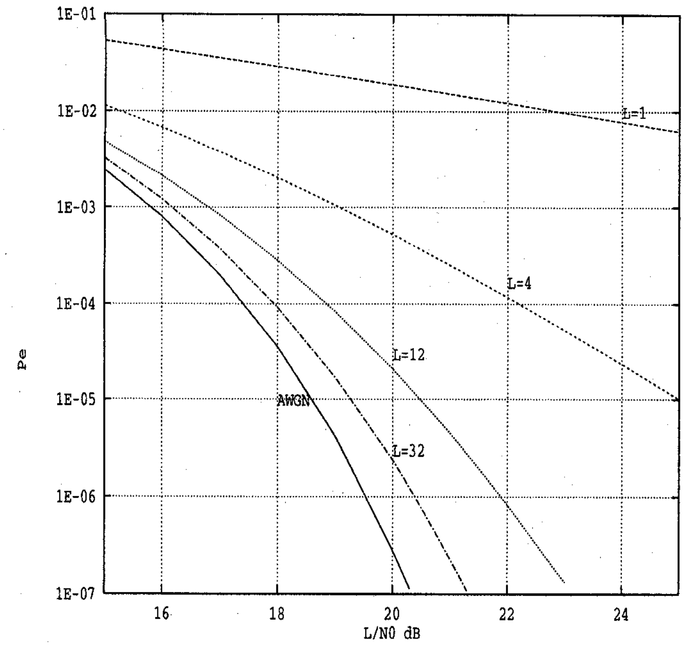

Figure 1: Pairwise error probability

This point $\mathbf{p}$ is rotated and its components are interleaved before being transmitted over the fading channel. The transmitted vector is $\mathbf{x} = \mathbf{R}\mathbf{p}$ where $\mathbf{R}$ is an $n \times n$ rotation matrix, $\mathbf{R}^t = \mathbf{R}^{-1}$). We assume that the Rayleigh channel coefficients are independent and perfectly known by the receiver.

2 High Diversity Rotations

The effect of the rotation matrix $\mathbf{R}$ is to spread the same information on different space axes. Thus, when deep fading occurs on one of the n axes, the information is still extracted from unfaded axes. The probabilty that a deep fading occurs at the same time on the n components is almost zero.

The most difficult problem is the search of a rotation matrix that guarantees the diversity order for any size of the QAM constellation, i.e. for any number of bits per symbol. This problem can be simplified if we look at a rotation of dimension n and diversity order L as the generator matrix of the rotated integer lattice $\mathbf{Z}_{n,L} = \mathbf{R}\mathbf{Z}^n$. The diversity is guaranteed if the minimum Hamming distance between the lattice points is equal to L [1]. One possible solution [7] is to build the lattice $\mathbf{Z}_{n,L}$ by applying a complex canonical embedding to the ring of integers in a cyclotomic number field.

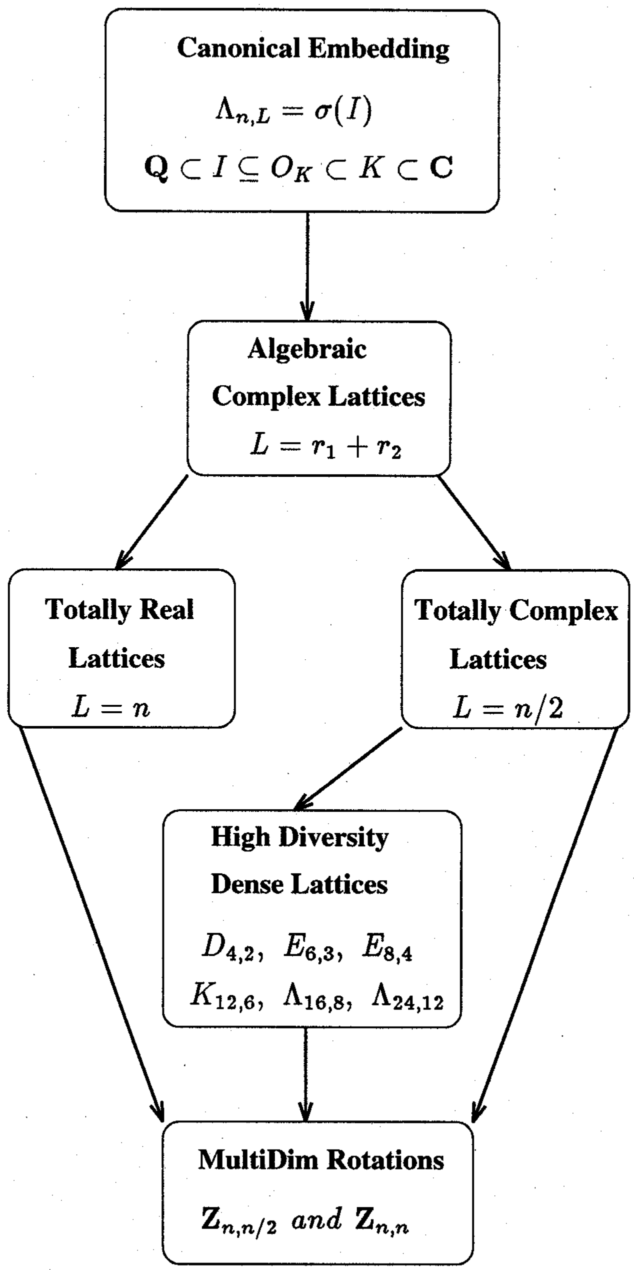

Figure 2: Algebraic Lattices for fading channels.

Let us define the canonical embedding and see how to compute the rotations. First, the number field $K = \mathbf{Q}(\theta)$ is defined by a minimal polynomial $\mu_\theta(x)$ of degree n whose roots are $\theta = \theta_1, \theta_2, ..., \theta_n$. It has been shown [1] that the diversity of a lattice derived from K is $L = r_1 + r_2$ where r_1 is the number of real roots and $2r_2$ is the number of complex roots ($r_1 + 2r_2 = n$). We restrict our search to totally complex fields generated by $\theta = e^{2j\pi/N}$ (cyclotomic fields) where the lattice dimension and the root order are related by the Euler function, $n = \phi(N)$. In this case, $r_1 = 0$ and $2r_2 = n$ and so the lattice diversity is equal to half of the dimension, $L = n/2$. The canonical embedding in the field described above is defined by n isomorphisms $\sigma_i : K \to \mathbf{C}, i = 1...n$. Each isomorphism σ_i associates a distinct root to the generator element, $\sigma_i(\theta) = \theta_i$. If we apply the canonical embedding to the set $1, \theta, \theta^2, ..., \theta^{n/2-1}$ using $n/2$ isomorphisms out of n, we obtain an $n/2 \times n/2$ complex matrix

$$R = \begin{pmatrix} \sigma_1(1) & \sigma_2(1) & \cdots & \sigma_{n/2}(1) \\ \sigma_1(\theta) & \sigma_2(\theta) & \cdots & \sigma_{n/2}(\theta) \\ \vdots & \vdots & & \vdots \\ \sigma_1(\theta^{n/2-1}) & \sigma_2(\theta^{n/2-1}) & \cdots & \sigma_{n/2}(\theta^{n/2-1}) \end{pmatrix} \quad (1)$$

The matrix written above in complex form is the generator matrix of the lattice $\mathbf{Z}_{n,n/2}$ under some conditions. The roots must be chosen in the following order,

$$\theta_i = \theta \times e^{4j\pi(i-1)/n}, i = 1...n/2$$

and the minimal polynomial must be written as

$$\mu_\theta(x) = x^n + \epsilon x^{n/2} + 1$$

where the constant ϵ takes the values 0 or -1 (see Table 1). The real matrix form of size $n \times n$ is obtained by splitting each complex entry into a 2×2 real matrix. As an example, for $n = 24$ and $N = 72$, we build a 24-dimensional rotation with diversity order equal to 12. The diversity is sufficiently high to convert the Rayleigh channel into a Gaussian channel [8]. Figure 4 shows the performance of the lattice $\mathbf{Z}_{24,12}$ on the Rayleigh channel for a spectral efficiency of 2 bits/dimension and the performance of the 16-QAM on the Gaussian channel.

The bit error rates of all rotations listed in Table 1 with 1 bit per dimension are shown in Figure 3. This supports the theoretical analysis shown in Figure 1. All the BER curves in Fig. 3 lay between the Gaussian curve (at the most left) and the Rayleigh curve with no diversity $L = 1$ (at the most right). The comparison is done with a 4-PSK modulation and we see that a 32-dimensional rotation with $L = 16$ is only 1 dB away from AWGN.

n	N	$\mu_\theta(x)$
4	8	$x^4 + 1$
	12	$x^4 - x^2 + 1$
8	16	$x^8 + 1$
	24	$x^8 - x^4 + 1$
12	36	$x^{12} - x^6 + 1$
16	32	$x^{16} + 1$
	48	$x^{16} - x^8 + 1$
24	72	$x^{24} - x^{12} + 1$
32	64	$x^{32} + 1$
	96	$x^{32} - x^{16} + 1$

Table 1: Minimal polynomial of $Z_{n,n/2}$ lattices from cyclotomic number fields $\mathbf{Q}(e^{2j\pi/N})$.

3 Lattices versus TCMs

Let us compare the rotated constellations to trellis coded modulations. It is obvious that rotated QAMs have no gain on the Gaussian channel. A trellis coded modulation may have a gain up to 6 dB over the Gaussian channel.

The diversity order of a TCM is equal to the minimum Hamming distance between all the possible signal sequences in the trellis. If we consider an n-dimensional TCM, its diversity L can be bounded by $n \leq L \leq n \times (\nu/k + 1)$ when the trellis has 2^ν states and the convolutional encoder is of rate $k/(k+1)$. The lower bound is reached if the trellis contains parallel transitions. The upper bound is reached if we find a special trellis encoder where the minimum length of a diverging path is $(\nu/k) + 1$ and all the components of the n-dimensional points are distinct.

For one dimensional TCMs, the diversity is practically limited to 6 or 7 (2048 states for 2 bits/dimension). For higher dimensions, the trellis diversity is also limited by its size, and in all dimensions higher than 4, parallel transitions are needed to reduce the number of states. In these cases the diversity is still limited by the dimension of the coded signal set ($L \leq 8$ if we rotate the Wei 8-dimensional TCM [4]).

Nevertheless, for comparison reasons on the Rayleigh

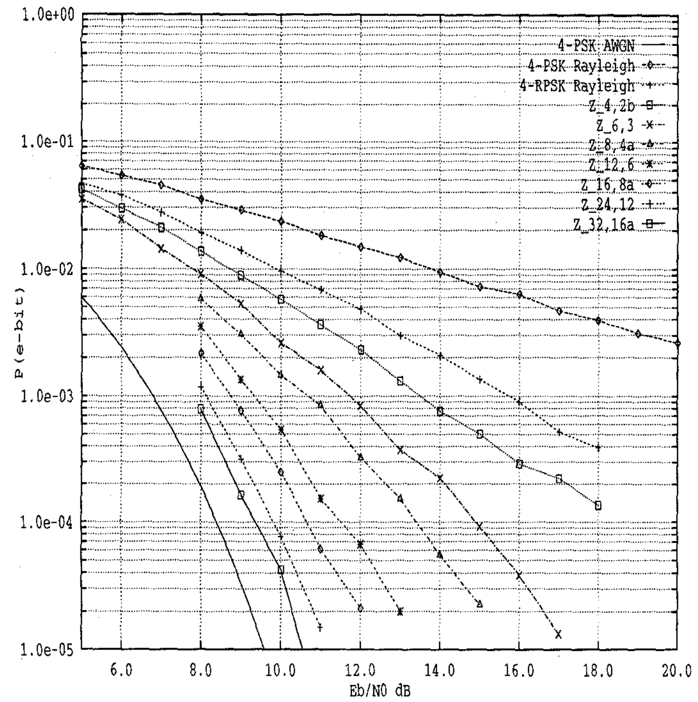

Figure 3: Bit error rates for the family of $Z_{n,n/2}$ constellations, 2 bits per symbol.

ν	Generators in octal systematic recursive	Diversity
3	11, 02, 16	2
4	23, 12, 16	3
5	41, 52, 34	3
6	117, 52, 164	4

Table 2: Optimal trellis coded 8-PAM modulation for the Rayleigh fading channel.

channel, we computed the best trellis coded 8-PAM modulations shown in Table 2. The 64 states 8-PAM ($L = 4$) exhibits a good performance on the Rayleigh channel. If we compare the slopes of the 8-PAM and the $\mathbf{Z}_{24,12}$ curves, it is possible to compute the signal-to-noise ratio for which the maximum diversity is reached (the curve becomes a straight line). The diversity 4 is reached by the 64 states 8-PAM at 9.4 dB and the diversity 12 by the 24-dimensional rotated QAM at 14.5 dB. Rotating the 8-dimensional Wei TCM gives a gain of 7 dB at 10^{-3} on the Rayleigh channel compared to the same TCM without rotation. The performance are identical to that of a 16-dimensional uncoded rotation and still 3 dB away from that of the Gaussian channel.

4 Beyond the Gaussian Channel

We have seen that a high diversity rotation converts the Rayleigh channel into a Gaussian channel. Practically, with $\mathbf{Z}_{24,12}$ the performance are still 1.5 dB away (Figure 4). An interesting question arises : On the Rayleigh channel, how can we achieve a performance better than the Gaussian channel?

We propose 3 different schemes. The first technique is to rotate a lattice with a positive fundamental gain, such as the Leech lattice [2]. It is known that the Leech lattice Λ_{24} has an asymptotic gain of 6 dB on the Gaussian channel. Thus theoretically, the performance should go 6 dB beyond the Gaussian channel if we rotate Λ_{24} into $\Lambda_{24,12}$. Practically, the gain should be 3 or 4 dB instead of 6 because of the relatively high kissing number and the finite diversity order.

The second scheme is the rotation of a trellis coded modulation. In this scheme, the lattice decoder is combined with the Viterbi decoder and hence a soft output lattice decoder must be available. The soft output decoding of convolutional codes [5][6] has been used to decode powerful concatenated schemes (Turbo codes). The soft output lattice decoding is still too complex and has not been completely studied, but it would enable us to further approach the channel capacity.

The third scheme is the simplest one. It is possible to push the rotated lattice performance beyond the Gaussian channel by adding a high rate error control code. As an example, we concatenate the 24-dimensional rotation with a (252,220) Reed-Solomon code. As shown in Figure 4 the gain is 3 dB at 10^{-5}. The price to pay is a bandwidth expansion factor of 1.14 .

5 Conclusions

It is possible to improve the performance on the Rayleigh fading channel with a multidimensional uncoded rotation. The fading effect becomes negligible when very high diversity rotations are applied. It is even possible to go beyond the Gaussian channel with some price to pay (complexity or bandwidth expansion).

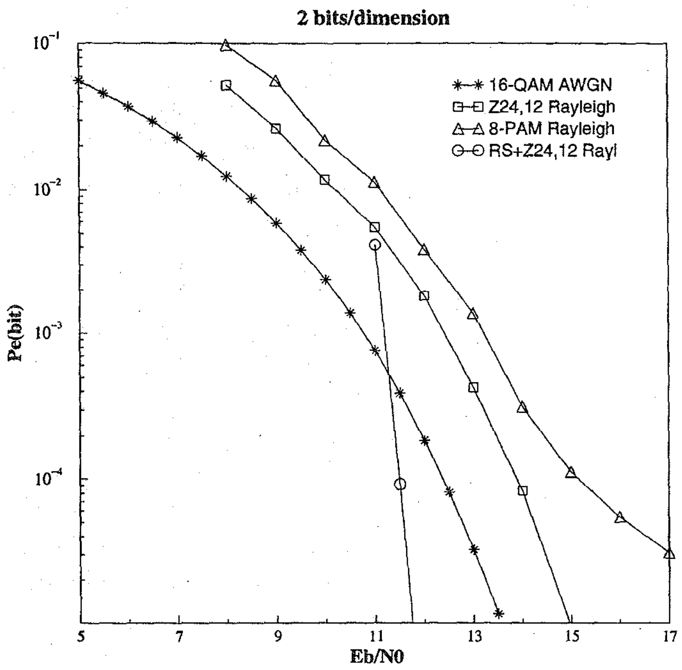

Figure 4: Performance over the Rayleigh channel.

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