Turbo code at 0.03 dB from capacity limit

J. Boutros, G. Caire, E. Viterbo, H. Sawaya & S. Vialle
ENST University, 46 Rue Barrault
FR-75013 Paris, France
e-mail: boutros@enst.fr

Abstract — **We propose a rate 1/3 infinite-length irregular turbo code capable of achieving an error rate $P_{eb} = 10^{-5}$ at signal-to-noise ratio $E_b/N_0 = -0.46dB$, which is at distance 0.03 dB from Shannon capacity limit. Our turbo code is derived by a simple modification of the original Berrou-Glavieux code.**

I. Introduction

Channel coding theory and related information theory themes made an impressive progress these last two years toward accomplishing the major objectives established since Shannon's original work. One of the goals is to approach the signal-to-noise ratio capacity limit with a vanishing error probability, i.e., find an error-correcting code and an error-correcting decoder that perform as close as possible to channel capacity. Among the recently discovered solutions, we cite irregular binary LDPC codes under iterative binary APP (a posteriori probability) decoding [4, 5, 6] and regular binary turbo codes under iterative quaternary APP decoding [2]. In this paper, we show that binary irregular turbo codes [1, 3] also exhibit near-capacity performance under classical iterative binary APP decoding. It is assumed that the interleaver size is infinite ($N = +\infty$) and that the graph representation of the turbo code is free from cycles.

II. Code structure and parameters

The general structure of an irregular turbo encoder is the cascade of a non-uniform repetition, a pseudo-random interleaver and a recursive systematic convolutional (RSC) code. The non-uniform repetition divides the information bits into classes indexed by $i = 2 \ldots d_{max}$, where d_{max} is the maximum bitnode degree. The number of bits in a class i is a fraction f_i of the total number of information bits at the turbo encoder input. The bits belonging to class i are repeated i times by the non-uniform repetition. The following simple relations are satisfied by the irregular turbo code parameters:

$$\sum_{i=2}^{d_{max}} f_i = 1 \quad and \quad \sum_{i=2}^{d_{max}} i \times f_i = \bar{d} \tag{1}$$

where $\bar{d}$ is the average degree of bitnodes. The rate R of the turbo code and the rate ρ of its RSC constituent satisfy

$$R = \frac{1}{1 + (\frac{1}{\rho} - 1)\bar{d}} \quad and \quad \rho = \frac{1}{2 - f_p} = \frac{\bar{d}}{\bar{d} + 2} \tag{2}$$

We limited our search to $R = 1/3$. A small fraction f_p of parity bits has been punctured to yield $\rho \geq \rho_0 = 1/2$. The RSC code is the classical 16-state $(37, 21)_8$.

III. Numerical Results

We assume an ideal coherent additive white Gaussian noise (AWGN) channel. Coded digits are transmitted via a binary phase shift keying (BPSK) modulation. As shown in the table below, we checked different configurations of 2-class irregularity turbo codes and we computed the density evolution threshold and distance to capacity limit. The best irregular turbo codes are those obtained by non-zero fractions (f_2, f_8) and (f_2, f_9). Then, we mixed both configurations and we considered the irregular turbo code defined by the 3 fractions (f_2, f_8, f_9). The number of decoding iterations does not exceed 200. The total bit error probability $P_{eb}(m)$ at iteration m is evaluated from $P_{eb}(m) = \sum_{i=2}^{d_{max}} f_i \times P_{eb}(i, m)$ where $P_{eb}(i, m)$ is the bit error probability of class i. Let $p_0(x)$ be the density of a real Gaussian variable $\mathcal{N}(-2/N_0, 4/N_0)$. Let $p_{\Lambda_m}(x)$ be the density function of the extrinsic log ratio. The partial BER $P_{eb}(i, m)$ is equal to the area under the tail of $p_{i,m}(x)$, where the latter is expressed via Fourier transform as

$$p_{i,m}(x) = \mathcal{F}^{-1}\left[\mathcal{F}[p_0(x)] \times \mathcal{F}^i[p_{\Lambda_m}(x)]\right] \tag{3}$$

Non Zero Fractions	Distance to Capacity
$f_2 = 0.812,\ f_6 = 0.188$	0.30 dB
$f_2 = 0.850,\ f_7 = 0.150$	0.19 dB
$f_2 = 0.875,\ f_8 = 0.125$	0.09 dB
$f_2 = 0.892,\ f_9 = 0.108$	0.08 dB
$f_2 = 0.906,\ f_{10} = 0.094$	0.13 dB
$f_2 = 0.917,\ f_{11} = 0.083$	0.14 dB
$f_2 = 0.925,\ f_{12} = 0.075$	0.19 dB
$f_2 = 0.888,\ f_8 = 0.060,\ f_9 = 0.052$	0.03 dB

IV. Conclusions

The proposed turbo code has a limited number of irregularity classes, namely the initial class of degree-2 bits and only two classes of high degree bits. Due to its low complexity encoding and its faster convergence speed, such a family of irregular turbo codes is a serious alternative to irregular low-density parity-check codes.

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