Modulation Diversity in Fading Channels with Quantized Receiver

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***Abstract*—In this paper, we address the design of codes which achieve modulation diversity in block fading single-input single-output (SISO) channels with signal quantization at receiver and low-complexity decoding. With an unquantized receiver, coding based on algebraic rotations is known to achieve modulation coding diversity. On the other hand, with a quantized receiver, algebraic rotations may not guarantee diversity. Through analysis, we propose specific rotations which result in the codewords having equidistant component-wise projections. We show that the proposed coding scheme achieves maximum modulation diversity with a low-complexity minimum distance decoder.**

I. Introduction

In practical communication receivers, the analog received signal is quantized into a finite number of bits for further digital baseband processing. With increasing bandwidth requirements of modern communication systems, analog-to-digital converters (ADC) are required to operate at high frequencies. However, at high operating frequencies, the precision of ADC's is limited [1]. Limited precision generally leads to high quantization noise, which degrades performance. In case of fading channels, high error floors in the bit error performance have been reported, and it seems difficult to avoid this behavior [2][3]. On the other hand, channel capacity results show that even with 2-bit quantizers, the capacity of a quantized output channel is not far from that of a channel with unquantized output [4][5]. Therefore, there appears to be a gap between the theoretical limits of communication with quantized receivers, and the current state of art.

In communication systems with fading, an important performance metric is the reliability of reception. For single antenna fading scenarios, modulation diversity is a well known signal space diversity technique to improve the reliability/diversity of reception [6][7]. However, with a quantized receiver, this coding alone *does not* guarantee improvement in diversity.

In this paper, we propose 2-dimensional constellations rotated by an angle θ. With a quantized receiver, the maximum likelihood (ML) decoder is not the usual minimum distance decoder, and would be much more complex to implement. We therefore assume a minimum distance decoder operating on the quantized channel outputs. We observe that, with a quantized receiver, *i*) for a given rate of information transmission in bits per channel use, there is a minimum requirement on the number of quantization bits, without which floors[2] appear in the error probability performance, *ii*) there is only a small subset of *admissible* rotation angles which can guarantee diversity improvement, and *iii*) for a quantized receiver with perfect channel knowledge and minimum distance decoding, we analytically show that, among all *admissible* rotation angles, a good angle is one in which the transmitted vectors have *equidistant projections* along both the transmitted components. We then show that the square M^2-QAM constellation rotated by $\theta = \tan^{-1}(1/M)$ has equidistant projections.

In this paper, we assume perfect channel state information at the receiver. However, in a separate work, we relax the perfect channel knowledge assumption, and propose novel training sequences and channel estimation scheme, which are shown to achieve an error probability performance close to that achieved with perfect channel knowledge [11].

II. System model and Quantized Receiver

We consider SISO block fading channels with single transmit and single receive antenna. The channel gains are assumed to be quasi-static for the coherence interval of the channel, and change to an independent realization in the next coherence interval. We further assume that the signaling bandwidth is much smaller than the coherence bandwidth of the channel (frequency flat fading), and therefore the channel frequency response is assumed to have constant magnitude and linear phase within the signalling bandwidth. Let the radio frequency band used for transmission be $(f_c - W/2, f_c + W/2)$, where $f_c \gg W$ is the carrier frequency and W is the signaling bandwidth. The complex channel frequency response is then given by

$$H(f) = |h| e^{-j2\pi\tau f} \, , \, |f - f_c| \leq \frac{W}{2}, \tag{1}$$

and zero elsewhere. Therefore the transmitted signal is scaled by $|h|$, and is delayed by τ seconds. The transmitted signal is given by

$$x(t) = \sum_k (x_k^I \cos(2\pi f_c t) + x_k^Q \sin(2\pi f_c t)) g(t - kT), \tag{2}$$

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[2]Error probability performance is said to *floor*, if and only if it converges to a non-zero positive constant as the signal-to-noise ratio tends to infinity.

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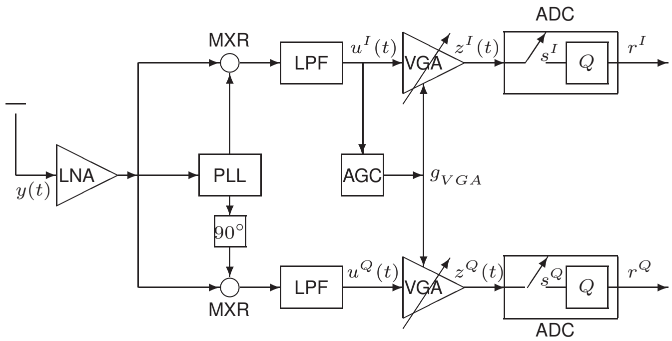

Fig. 1. Receiver analog front end (AFE) .

where $1/T$ is the rate at which information symbols are transmitted, and $x_k = x_k^I + jx_k^Q$ is the k-th transmitted information symbol. We assume pulse shaping signals which result in no inter-symbol interference (ISI) (e.g., $g(t) = \text{sinc}(Wt)$). Prior to the transmission of K information symbols, there is a training phase in which a known preamble sequence of P symbols is transmitted to enable carrier frequency synchronization in the receiver (i.e., enabling the phased locked loop (PLL) in the receiver to lock to the transmitter's local oscillator) and also for tuning the receiver gain. In this paper we assume the preamble to be a constant amplitude carrier obtained by setting $x_k^I = A$ and $x_k^Q = 0$ in (2). The received signal during the training phase of duration P symbols, is given by $y(t) = A|h|\cos(2\pi f_c t - 2\pi f_c \tau)\sum_{k=0}^{P-1} g((t-\tau)-kT)$.

Figure 1 shows the signal path of the analog front end of a typical heterodyne receiver. Let the combined gain of the Low Noise Amplifier (LNA), Mixer (MXR) and Low Pass Filter (LPF) be denoted by g_{AFE}. In the training phase, after the PLL has locked, the LPF output (I Path) is given by $u^I(t) = Ag_{AFE}|h|\sum_{k=0}^{k=P-1} g((t-\tau)-kT)+n^I(t)$, where $n^I(t)$ is the white Gaussian noise in the receiver (I path). The LPF output is digitized using a Nyquist rate sample & hold type analog-to-digital converter (ADC), as shown in Fig. 1. Let the input dynamic range of the ADC be $-c_q/2$ to $c_q/2$. We also refer to $c_q/2$ as the clip level, since any input greater than $c_q/2$ would be limited to $c_q/2$. For optimum performance, it is desirable that the range of the input signal to the ADC matches with the ADC dynamic range (ADC range matching). Due to fading, the input level at the ADC may vary, and therefore a variable gain amplifier (VGA) is generally used to ensure ADC range matching. The gain of the VGA is controlled by the automatic gain control (AGC) module [8]. During the training phase, the AGC detects the peak of the signal $u^I(t)$ using a conventional analog peak detector whose output is given by

$$V_{agc-pk} = Ag_{AFE}|h|. \tag{3}$$

Let X denote the peak absolute value of the transmitted symbols, x_k^I and x_k^Q, during normal data communication phase. During data communication phase, ADC range matching (i.e., $\frac{c_q}{2} = g_{VGA}g_{AFE}|h|X$) requires the VGA gain to be

$$g_{VGA} = \frac{c_q}{2}\frac{A}{X}\frac{1}{V_{agc-pk}}. \tag{4}$$

Since the ratio A/X and $c_q/2$ are known *a priori*, this computation is done in the AGC using simple analog circuits [9]. In the rest of the paper, we assume that this computation is perfect.

During the information transmission phase, the PLL tracking loop is turned off and the VGA gain setting is frozen to the value given by (4). Therefore, during this phase, the ADC input signal (I path) is given by $z^I(t) = g_{VGA}(g_{AFE}|h|\sum_{k=P}^{K+P-1} x_k^I g(t-\tau-kT) + n^I(t)) = \frac{c_q}{2}\sum_{k=P}^{K+P-1}\frac{x_k^I}{X}g(t-\tau-kT) + g_{VGA}n^I(t)$. The ADC input (Q-path) is similar. Subsequently, without loss of generality, we assume an ADC with a normalized clip level of $c_q/2 = 1$. Assuming perfect timing synchronization (i.e., receiver can perfectly estimate τ), the k-th output of the sample & hold circuit, at time $t = \tau + kT$ is given by

$$s_k^I = \frac{x_k^I}{X} + \frac{w_k^I}{|h|X} \quad , \quad s_k^Q = \frac{x_k^Q}{X} + \frac{w_k^Q}{|h|X}, \tag{5}$$

where $w_k^I \triangleq n^I(\tau+kT)/g_{AFE}$, and $w_k^Q \triangleq n^Q(\tau+kT)/g_{AFE}$. Since $n^I(t)$ and $n^Q(t)$ are Gaussian random processes, w_k^I and w_k^Q are i.i.d. Gaussian random variables with variance denoted by $\sigma^2/2$. Let the average transmit power be denoted by $P_T \triangleq \mathbb{E}[|x_k|^2]$. Then the instantaneous signal to noise ratio (SNR) at the output of the sample & hold circuit is given by $\gamma_{inst} \triangleq P_T|h|^2/\sigma^2$. Assuming a Rayleigh fading model with $h \sim \mathcal{CN}(0,1)$, the average signal to noise ratio (SNR) is given by $\gamma \triangleq \mathbb{E}_h[\gamma_{inst}] = P_T/\sigma^2$. The output of the sample & hold circuit is then quantized by a b-bit uniform quantizer Q, as shown in Fig. 1. The quantizer is modeled by the function $Q_b(t), t \in \mathbb{R}$, which is given by

$$Q_b(t) = \begin{cases} +1, & \zeta(t) \geq (2^{b-1}-1) \\ -1, & \zeta(t) \leq -(2^{b-1}-1) \\ \frac{(2\zeta(t)+1)}{2^b-1}, & \text{otherwise} \end{cases} \tag{6}$$

$$\zeta(t) \triangleq \left\lfloor \frac{t\,(2^b-1)}{2} \right\rfloor \tag{7}$$

where $\lfloor x \rfloor$ denotes the largest integer not greater than x. For a n-dimensional complex vector $\mathbf{z} = (z_1, z_2, \cdots, z_n)$, let $\mathbf{Q}_b(\mathbf{z})$ denote the n-dimensional component-wise quantized version of $\mathbf{z}$. That is, $\tilde{\mathbf{z}} = (\tilde{z}_1, \tilde{z}_2, \cdots, \tilde{z}_n) = \mathbf{Q}_b(\mathbf{z})$ implies that

$$\tilde{z}_i^I = Q_b(z_i^I)\ ,\ \tilde{z}_i^Q = Q_b(z_i^Q)\ \ i = 1, 2, \ldots, n. \tag{8}$$

The k-th quantized received symbol, $r_k = r_k^I + jr_k^Q$ is therefore given by

$$r_k^I = Q_b(s_k^I)\, ,\, r_k^Q = Q_b(s_k^Q) \tag{9}$$

where s_k^I and s_k^Q are the real and imaginary components of the k-th sample & hold output symbol given by (5).

Modulation diversity coding is illustrated in Fig. 2. Coding is performed across $n > 1$ information symbols resulting in n coded symbols/codeword. These n coded symbols are interleaved and then transmitted over n independent channel coherence intervals (realizations). At the receiver, the channel outputs during the n coherence intervals are buffered, followed

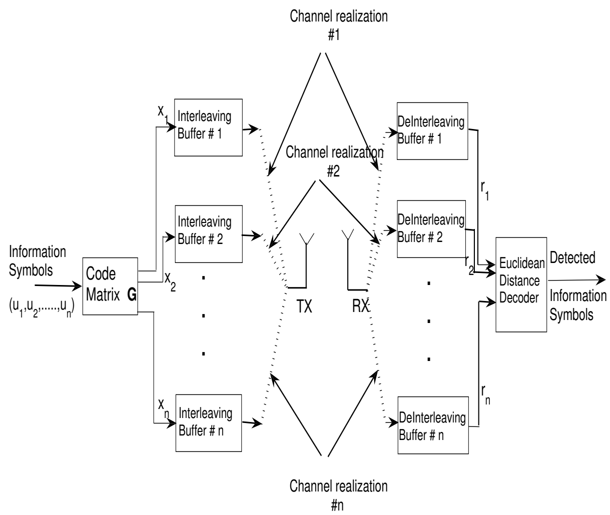

Fig. 2. Achieving modulation diversity by coding across n different channel realizations.

by de-interleaving and detection. Suitable coding across n independent channel realizations results in an n-fold increase in the diversity of reception. In fading channels, codes designed using algebraic lattices can achieve modulation diversity, and are therefore employed to improve the diversity of reception [6]. With an unquantized receiver, it is known that lattice codes based on algebraic rotations can achieve full modulation diversity [7][10]. *However, with quantized receivers, this is no longer true*.

In this paper we consider the case of $n = 2$. Let the information symbol vector be denoted by $\mathbf{u} = (u_1, u_2)^T$, where the information symbols u_1 and u_2 are restricted to square M^2-QAM signal set, though a generalization to non-square QAM is trivial. Let the set $\mathcal{S}_M = \{-(M-1), \ldots, -1, 1, \cdots, (M-1)\}$ denote the M-PAM signal set. Then, M^2-QAM is denoted by the set $\mathcal{S}_M^2 \stackrel{\Delta}{=} \{w + jv \mid w, v \in \mathcal{S}_M\}$. The information symbols are coded using a 2×2 rotation matrix $\mathbf{G}$, resulting in the transmit vector $\mathbf{x} = (x_1, x_2)^T = \mathbf{G}\mathbf{u}$, where

$$\mathbf{G} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}. \tag{10}$$

Due to QAM symmetry, one can restrict the rotation angle in (10) to $[0, \pi/4)$. The set of transmitted vectors $\mathcal{X}$ and the peak component value X are given by

$$\begin{aligned} \mathcal{X} &= \left\{ \mathbf{x} \mid \mathbf{x} = \mathbf{G}\mathbf{u}, u_1, u_2 \in \mathcal{S}_M^2 \right\}, \\ X &= \max_{\mathbf{x} \in \mathcal{X}} \left\{ \max_{i=1,2} \left[\max(|x_i^I|, |x_i^Q|) \right] \right\} \end{aligned} \tag{11}$$

Also, let the channel gain during the transmission of x_1 and x_2 be denoted by $|h_1|$ and $|h_2|$, respectively. We assume h_1 and h_2 to be i.i.d. $\mathcal{CN}(0, 1)$. Let $\mathbf{r} = (r_1, r_2)^T$ denote the quantized received vector, where $r_1 = r_1^I + jr_1^Q$ and $r_2 = r_2^I + jr_2^Q$ are the ADC outputs during the transmission of x_1 and x_2, respectively. From (5) and (9) it follows that

$$r_i^I = Q_b\Big(\frac{x_i^I}{X} + \frac{w_i^I}{|h_i|X}\Big), \; r_i^Q = Q_b\Big(\frac{x_i^Q}{X} + \frac{w_i^Q}{|h_i|X}\Big). \tag{12}$$

With the above quantized receiver model, maximum likelihood decoding is no more given by the minimum distance decoder, and is rather complex. Nevertheless, due to its lower decoding complexity, we shall assume a minimum distance decoder taking $\mathbf{r}$ as its input, and the output (detected information symbols) given by

$$\widehat{\mathbf{u}} = \arg \min_{\mathbf{u} \in \mathcal{S}_M^2 \times \mathcal{S}_M^2} \left\| \mathrm{diag}(|h_1|, |h_2|) \Big(\mathbf{r} - \frac{\mathbf{G}\mathbf{u}}{X} \Big) \right\|^2 \tag{13}$$

where $\dagger$ and $\|.\|$ denote Hermitian transpose and Euclidean norm respectively.

III. Rotation Coding in Quantized Receiver

In an unquantized receiver, at high SNR, the word error probability is minimized by choosing the transmit vectors such that the minimum product distance between any two vectors is maximized [7]. There also exists algebraic rotations which guarantee a non-vanishing minimum product distance with increasing QAM size [6]. In this paper, we study the error performance of these rotated constellations with a *quantized* receiver and minimum distance decoding, and derive the conditions under which full modulation diversity can be achieved.

In case of a quantized receiver, the sample & hold outputs (5), are quantized to the appropriate *quantization box* containing it. As an example, Fig. 3 illustrates the rotated 4-QAM constellation with $\theta = 20°$. The dark filled squares represent the 4 possible values taken by the real component of the normalized transmit vector $\mathbf{x}^I/X = (x_1^I/X\,,\, x_2^I/X)^T$. A $b = 2$-bit quantizer is used along both codeword components. The dashed horizontal and vertical lines represent the quantization boundaries along the 2 components. The *projections* of the 4 possible vectors onto the first component (horizontal) are marked with a cross. As an example, in Fig.3 the real component of the sample & hold output vector $\mathbf{s}^I = (s_1^I, s_2^I)^T$ (marked with a star), is therefore quantized to $\mathbf{r}^I = (r_1^I, r_2^I)^T$ (There are totally 16 different quantized outputs marked with empty circles). The quantization box corresponding to the output $\mathbf{r}^I$ is shown in the figure as a square with solid edges.

As the noise variance $\sigma^2 \to 0$, the sample & hold output $\mathbf{s}$ is almost the same as the normalized transmitted vector $\mathbf{x}/X$. Therefore at sufficiently high SNR, if there exists two different transmit vectors $\mathbf{x}$ and $\mathbf{y}$, such that $\mathbf{Q}_b(\mathbf{x}/X)$ and $\mathbf{Q}_b(\mathbf{y}/X)$ are identical, then it is obvious that the error probability performance would floor as SNR $\to \infty$. This is because, at high SNR the quantizer output would be the same irrespective of whether $\mathbf{x}$ or $\mathbf{y}$ was transmitted, which makes it impossible for the the receiver to distinguish between the two transmit vectors leading to erroneous detection. More formally, two transmit vectors $\mathbf{x}$ and $\mathbf{y}$ are said to be *distinguishable* if and only if $\mathbf{Q}_b(\mathbf{x}/X) \neq \mathbf{Q}_b(\mathbf{y}/X)$. Therefore, in order to avoid floors in the error probability performance, we propose the first code design criterion.

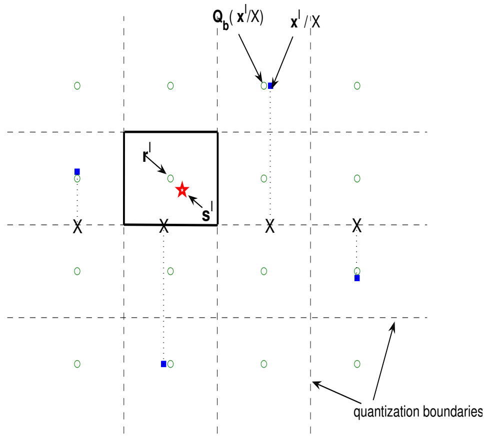

Fig. 3. Signal space at the quantizer input with $b = 2$ (real component). Rotated 4-QAM ($\theta = 20°$) depicted with dark filled squares.

Criterion I *: A necessary and sufficient condition to avoid error floors with a quantized receiver, is that any two transmit vectors must be distinguishable.*

To achieve full modulation diversity, even under deep fading conditions in one component, any two transmit vectors $\mathbf{x}$ and $\mathbf{y}$ must still be *distinguishable* in the other component. This implies that the projections of all the transmit vectors onto any one component must be *distinguishable* by the quantizer in that component. Therefore, we have the second criterion.

Criterion II *: Given a b-bit quantized receiver, in order to achieve full modulation diversity, a necessary condition on the rotation angle θ is that, any two distinct transmit vectors $\mathbf{x}$ and $\mathbf{y}$ satisfy*

$$\mathbf{Q}_b(x_i/X) \neq \mathbf{Q}_b(y_i/X), \;\; i = 1, 2. \tag{14}$$

With a rotated M^2-QAM there are totally M^2 distinct projections onto any component, and therefore the minimum number of quantization bits required for the transmit vectors to be *distinguishable* along any component is at least $\lceil 2\log_2(M) \rceil$. Hence, in order to achieve full modulation diversity a straight forward lower bound on b is[3]

$$b \geq \lceil 2\log_2(M) \rceil. \tag{15}$$

Subsequently, we assume that for a given M, b is fixed to the lower bound value in (15). We further note that, with a $b = \lceil 2\log_2(M) \rceil$-bit quantizer, Criterion II is not satisfied by all rotation angles. For example, even though $\theta = 1/2\tan^{-1}(2)$ guarantees a rotation code having non-vanishing minimum product distance, with a $b = 4$-bit uniform quantizer and $M^2 = 16$-QAM it *does not* satisfy Criterion II.

With a $b = \lceil 2\log_2(M) \rceil$-bit quantizer, the set of angles (between 0 and $\pi/4$) which result in *distinguishable* projections along both the codeword components will be referred to as the *admissible* angles (i.e., angles which satisfy Criterion II). For example, with 4- and 16-QAM, the admissible angles lie in the range $(\tan^{-1}(1/5)\,, \pi/4)$ and $(11.3°\,, 16.9°)$, respectively. With increasing QAM size, the interval of admissible rotation angles reduces. For example, with 256-QAM, the range of admissible angles is only $(3.47°\,, 3.68°)$. Another interesting fact is that, for M^2-QAM, $\theta = \tan^{-1}(1/M)$ is always in the set of admissible angles. Further, as M grows larger, $\tan^{-1}(1/M) \pm \epsilon$ are observed to be the only admissible angles.

[3] $\lceil x \rceil$ denotes the smallest integer not smaller than x.

Apart from the fact that the chosen angle must have distinguishable projections, it can be analytically shown that for M^2-QAM, any rotation angle for which the rotated constellation satisfies

$$\mathbf{Q}_b(\mathbf{x}/X) = \mathbf{x}/X \;,\;\; \mathbf{x} \in \mathcal{X} \tag{16}$$

does indeed achieve a diversity order of 2 (i.e., full modulation diversity since $n = 2$), with a $b = \lceil 2\log_2(M) \rceil$-bit quantized receiver and minimum distance decoding given by (13) (See Theorem A.1, Appendix A in [11]). Subsequently, a rotated constellation which satisfies (16) shall be referred as being *matched* to the quantizer. It is easy to see that a rotated M^2-QAM constellation is matched to a $b = 2\lceil \log_2(M) \rceil$-bit uniform quantizer, if and only if, the projections of the transmit vectors are component-wise *equidistant* and distinguishable.

Even with a mismatched rotated constellation having distinguishable projections (i.e., when the projections are not equidistant), full modulation diversity may be achieved, but then the error probability would be higher, since some transmit vectors would be closer to the edge of their quantization boxes (making it easier for noise to move the transmitted vector to another quantization box when received) (illustrated through Fig. 10 in Appendix A of [11]). Following along the same lines as the proof of Theorem A.1 in [11], it can be shown that *mismatched* constellations result in a higher error probability when compared to matched constellations. This therefore leads us to the third code construction criterion.

Criterion III *: In order to minimize the error probability of a rotated M^2-QAM constellation with a $b = \lceil 2\log_2(M) \rceil$-bit quantized receiver, the rotation angle must be such that the rotated M^2-QAM constellation is matched to the quantizer.*

We next construct rotated M^2-QAM constellations which satisfy Criterion III. We had earlier observed that, for M^2-QAM, a rotation by $\theta = \tan^{-1}(1/M)$ appeared to be always in the set of admissible angles. In fact, it can be shown analytically that a rotation by $\theta = \tan^{-1}(1/M)$, actually guarantees equidistant projections along both codeword components.

For M^2-QAM with $\theta = \tan^{-1}(1/M)$, it can be shown that the minimum product distance of the code is $4M/(M^2+1)$ ($\approx 4/M$ for $M \gg 1$). On the other hand, a rotation angle of $\theta = 1/2\tan^{-1}(2)$ is known to have a minimum product distance of at least $4/\sqrt{5}$ irrespective of the QAM size. Therefore, with increasing M, the error performance of a quantized receiver with $\theta = \tan^{-1}(1/M)$ is expected to be increasingly less power efficient than that of a unquantized receiver with $\theta = 1/2\tan^{-1}(2)$. With increasing M, the set of admissible angles appeared to be only $\tan^{-1}(1/M) \pm \epsilon$ and therefore, it can be argued that, the best possible error performance with a $b = \lceil 2\log_2(M) \rceil$-bit quantized receiver would have a loss in power efficiency when compared to an unquantized receiver. However, this appears to be the cost to achieve full modulation

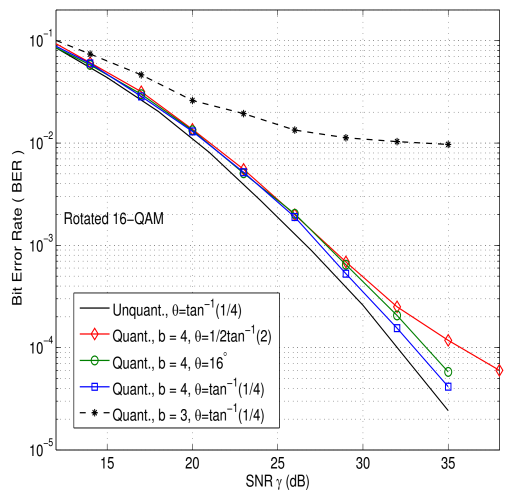

Fig. 4. BER vs. SNR for a quantized receiver. $b = 4$, 16-QAM and perfect channel state information at receiver.

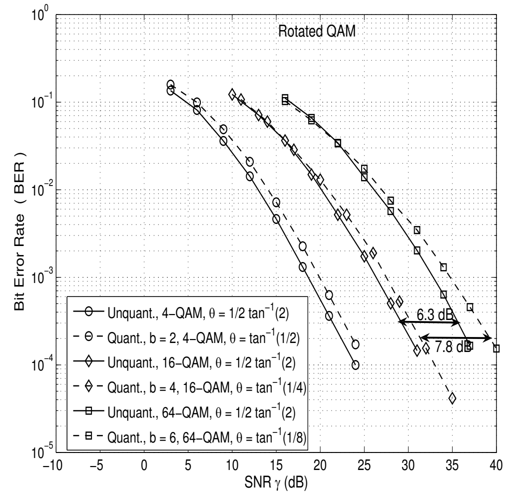

Fig. 5. BER comparison between quantized and unquantized receivers. 4-,16-,64-QAM. Perfect channel state information at receiver.

diversity in quantized receivers with limited precision.

IV. Simulation Results

All error probabilities reported in this Section have been averaged over the Rayleigh flat fading statistics of the channel. Also, the receiver is assumed to have perfect channel state information. In Fig. 4, we plot the average bit error rate/probability (BER), for rotated 16-QAM constellation (M=4) and a b=4-bit quantized receiver. The following four important observations can be made in Fig. 4: *i*) with $\theta = 1/2\tan^{-1}(2)$ (which is known to achieve full modulation diversity in an unquantized receiver), the BER performance with a quantized receiver fails to achieve full diversity (note the difference in slope at high SNR), which validates Criterion II, *ii*) with $\theta = \tan^{-1}(1/4)$, which results in equidistant projections, the quantized receiver achieves full modulation diversity with b=4. Further, the quantized receiver performs only 1 dB away from an ideal *unquantized* receiver at a BER of 10^{-4}, *iii*) with a quantized receiver a rotation angle of $\theta = 16^\circ$ also appears to achieve full modulation diversity, but perform poor when compared to a matched rotated constellation with $\theta = \tan^{-1}(1/4)$. This supports Criterion III, and *iv*) In Fig. 4 it is also observed that with 16-QAM rotated constellation (θ = $\tan^{-1}(1/4)$), the error performance floors with b=3 $<$ 4 quantization bits, which validates code design Criterion I.

It was discussed in Section III, that with increasing QAM size, a quantized receiver would be increasingly less power efficient when compared to an unquantized receiver. This fact is illustrated in Fig. 5, where the BER performance of both unquantized receiver with θ=$1/2\tan^{-1}(2)$ and quantized receiver with θ=$\tan^{-1}(1/M)$ are plotted for M^2=4-,16- and 64-QAM. The number of quantization bits with M^2-QAM is b=$\lceil 2\log_2(M)\rceil$. It is observed that for a fixed BER of 2×10^{-4}, an unquantized receiver requires 6.3 dB more transmit power when the QAM size is increased from 16 to 64. For the same increase in QAM size, a quantized receiver would require 7.8 dB more transmit power. However, when the QAM size is increased from 4 to 16, the extra transmit power required for a fixed target BER of 2×10^{-4} is roughly the same (about 7.7 dB) for both quantized and unquantized receiver.

In Fig. 11, Appendix A of [11], we report the BER performance of a rotated 16-QAM constellation for varying θ and fixed SNR. It is observed that, the rotation angle which results in a matched constellation, achieves the minimum BER. This then supports code design Criterion III.

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