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Golden Space-Time Trellis Coded Modulation

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Abstract

In this paper, we present a multidimensional trellis coded modulation scheme for a high rate 2×2 multiple-input multiple-output (MIMO) system over slow fading channels. Set partitioning of the Golden code [2] is designed specifically to increase the minimum determinant. The branches of the outer trellis code are labeled with these partitions. Viterbi algorithm is applied for trellis decoding. In order to compute the branch metrics a sphere decoder is used. The general framework for code optimization is given. Performance of the proposed scheme is evaluated by simulation and it is shown that it achieves significant performance gains over uncoded Golden code.

1. INTRODUCTION

High speed wireless networks for multimedia traffic require high spectral efficiency schemes with low packet delay. MIMO systems and algebraic space-time coding offer a good set of solutions to this challenging design problem. Wireless channels are commonly modeled as block fading, where it is assumed that the channel is fixed over the duration of a frame. For such channels, concatenated coding schemes are appropriate. Space-time trellis codes (STTCs), proposed in [1], used PSK or QAM symbols and were designed according to both rank and determinant criteria. A more refined concatenated scheme enables to split these two design criteria. As an inner code, we can use a simple space-time block coding scheme, which can guarantees full diversity for any spectral efficiency (e.g. Alamouti scheme). An outer code is then used to improve the coding gain.

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In this paper, we consider a concatenated scheme, where the inner code is Golden code [2] and outer code is trellis code. As it will become clear in the following, we can view this as a multidimensional TCM, where the Golden code acts as the signal set to be partitioned. This Golden Space-Time Trellis Coded Modulation (GST-TCM) scheme is appropriate for high data rate systems thanks to the great flexibility in the choice of the modulation spectral efficiency. A first attempt to design such a scheme was made in [3]. However, the resulting *ad hoc* scheme suffered from a high trellis complexity.

We develop a systematic design approach for GST-TCM. In [4–7], lattice set partitioning combined with a trellis code is used to increase the minimum square Euclidean distance between codewords. Here, it is used to increase the minimum determinant. The Viterbi algorithm is used for trellis decoding, where the branch metrics are computed using a sphere decoder for the inner code.

We propose a design approach that is similar to Ungerboeck design rules [5, 8]. We design different GST-TCM and optimize their performance according to the design criterion. It is shown for example, that a 16 state TCM achieves significant performance gain of 4.2dB over the uncoded Golden code, at an frame error rate (FER) of 10^{-3}, over the uncoded Golden code at the spectral efficiency of 6 bits per channel use (bpcu).

2. SYSTEM MODEL

The following notations are used: T denotes transpose and † denotes Hermitian transpose. Let $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{C}$ and $\mathbb{Z}[i]$ denote the ring of rational integers, the field of rational numbers, the field of complex numbers, and the ring of Gaussian integers, where $i^2 = -1$. Let $\mathbb{Q}(\theta)$ denote an algebraic number field generated by the primitive element θ. Let $GF(2) = \{0, 1\}$ denote the Galois field of degree two. The

$m \times m$ dimensional identity matrix is denoted by $\mathbf{I}_m$. The $m \times n$ dimensional zero matrix is denoted by $\mathbf{0}_{m\times n}$.

We consider a 2×2 ($n_T = 2, n_R = 2$) MIMO system over slow fading channels. The received signal matrix $\mathbf{Y} \in \mathbb{C}^{2\times 2L}$ ($2L$ is the *frame length*), is given by

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}, \quad (1)$$

where $\mathbf{Z} \in \mathbb{C}^{2\times 2L}$ is the complex white Gaussian noise matrix with i.i.d. samples $\sim \mathcal{N}_{\mathbb{C}}(0, N_0)$, $\mathbf{H} \in \mathbb{C}^{2\times 2}$ is the channel matrix, which is constant during a frame and varies independently from one frame to another. The elements of $\mathbf{H}$ are assumed to be i.i.d. circularly symmetric Gaussian random variables $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$. The channel is assumed to be known at the receiver.

In (1), $\mathbf{X} = [X_1,...,X_t,...,X_L] \in \mathbb{C}^{2\times 2L}$ is the transmitted signal matrix, where $X_t \in \mathbb{C}^{2\times 2}$. There are three different options for selecting inner codewords $X_t, t = 1, \ldots, L$:

1. X_t are independently selected from the Golden code $\mathcal{G}$, i.e.,

$$X_t = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha\left(a_t + b_t\theta\right) & \alpha\left(c_t + d_t\theta\right) \\ i\bar{\alpha}\left(c_t + d_t\bar{\theta}\right) & \bar{\alpha}\left(a_t + b_t\bar{\theta}\right) \end{bmatrix} \quad (2)$$

   where $a_t, b_t, c_t, d_t \in \mathbb{Z}[i]$ are the information symbols, $\theta = 1 - \bar{\theta} = \frac{1+\sqrt{5}}{2}$, $\alpha = 1 + i - i\theta$, $\bar{\alpha} = 1 + i(1 - \bar{\theta})$, and the factor $\frac{1}{\sqrt{5}}$ is used to normalize energy [2].

2. X_t are independently selected from a linear subcode of the Golden code;

3. A trellis code is used as the outer code encoding across the symbols X_t selected from partitions of $\mathcal{G}$.

We denote Case 1 as the *uncoded system*, Case 2 as the *Partitioned Golden code system*, and Case 3 as the *Golden Space-Time Trellis Coded Modulation system*.

In this paper, we use Q-QAM constellations, where $Q = 2^\eta$ as information symbols in (2). We assume the constellation is scaled to match $\mathbb{Z}[i] + (1 + i)/2$, i.e., the minimum Euclidean distance is set to 1 and it is centered at the origin. The average energy E_s is 0.5, 1.5 and 2.5 for $Q =$ 4,8,16. Signal to noise ratio is defined as SNR $= E_b/N_0$, where $E_b = E_s/q$ is the energy per bit and q denotes the number of information bits per symbol. We have $N_0 = 2\sigma^2$, where σ^2 is the noise variance per real dimension, which can be adjusted as $\sigma^2 = (n_T E_b/2)10^{(\text{-SNR}/10)}$.

Assuming that a codeword $\mathbf{X}$ is transmitted, the maximum-likelihood receiver might decide erroneously in favor of another codeword $\hat{\mathbf{X}}$. Let r denote the rank of the *codeword difference matrix* $\mathbf{X} - \hat{\mathbf{X}}$. Since the Golden code is full rank, $r = n_T = 2$. Let $\lambda_j, j = 1, \ldots, r$, be the eigenvalues of the *codeword distance matrix* $\mathbf{A} = (\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^\dagger$. Let $\Delta = \prod \lambda_j$ be the determinant of the codeword distance matrix $\mathbf{A}$ and $\Delta_{\min}$ be the corresponding *minimum determinant*, which is given by $\Delta_{\min} = \min_{\mathbf{X}\neq\hat{\mathbf{X}}} \det(\mathbf{A})$. We call $n_T n_R$ the *diversity gain* and $(\Delta_{min})^{1/n_T}$ the *coding gain* [1]. In the case of linear codes we can simply consider the all-zero codeword matrix and we have $\Delta_{\min} \geq \min_{\mathbf{X}\neq\mathbf{0}_{2\times 2L}} \det\left(\mathbf{X}\mathbf{X}^\dagger\right)$, where equality holds for infinite codes [2].

In order to compare two coding schemes supporting the same information bit rate, but with different minimum determinants ($\Delta_{\min,1}$ and $\Delta_{\min,2}$) and different constellation energies ($E_{s,1}$ and $E_{s,2}$), we define the asymptotic coding gain as

$$\gamma_{as} = \frac{\sqrt{\Delta_{\min,1}}/E_{s,1}}{\sqrt{\Delta_{\min,2}}/E_{s,2}}. \quad (3)$$

In the case of $L = 1$, the codeword matrix $\mathbf{X} = X_1 \in \mathcal{G}$ is a square matrix. The Golden code $\mathcal{G}$ has full rate, full rank $r = 2$, and the minimum determinant is $\delta_{\min} = \frac{1}{5}$ [2]; thus $\Delta_{\min} = \delta_{\min}$ for the uncoded Golden code system. In all cases, we have

$$\det\left(\mathbf{X}\mathbf{X}^\dagger\right) = \det\left(\sum_{t=1}^{L}\left(X_t X_t^\dagger\right)\right). \quad (4)$$

A code design criterion attempting to maximize $\Delta_{\min}$ is hard to exploit, due to the non-additive nature of the determinant metric in (4). Since $X_t X_t^\dagger$ are positive definite matrices, we use the following determinant inequality [10]

$$\Delta_{\min} \geq \min_{\mathbf{X}\neq\mathbf{0}_{2\times 2L}} \sum_{t=1}^{L} \det\left(X_t X_t^\dagger\right) = \Delta'_{\min}. \quad (5)$$

The lower bound $\Delta'_{\min}$ will be adopted as the guideline of our concatenated scheme design. In particular we will design trellis codes that attempt to maximize $\Delta'_{\min}$, by using set partitioning to increase the minimum number of non zero terms $\det\left(X_t X_t^\dagger\right)$ in (5).

3. TRELLIS CODED MODULATION

The uncoded system (Case 1) and partitioned Golden code system (Case 2) are discussed in [3, 11]. Here, we propose a systematic design approach for Case 3. We analyze the systematic design problem of this concatenated scheme by using Ungerboeck style set partitioning rules for coset codes [5–7]. The design criterion for the trellis code is developed in order to maximize $\Delta'_{\min}$, since this results in the maximum lower bound on the asymptotic coding gain of the GST-TCM over the uncoded Golden code

$$\gamma_{as} \geq \frac{\sqrt{\Delta'_{\min}}/E_{s,1}}{\sqrt{\delta_{\min}}/E_{s,2}}. \quad (6)$$

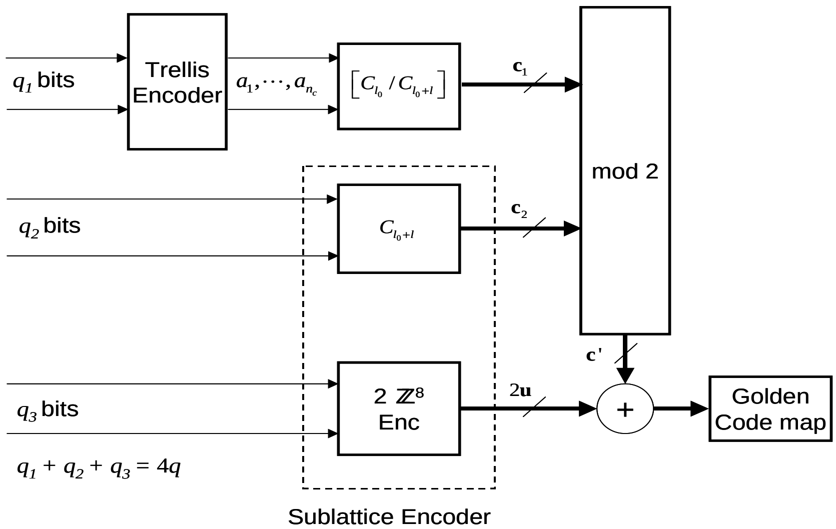

Figure 1: General encoder structure of the concatenated scheme.

Before we design the coding scheme, we briefly recall the set partition chain in [3].

Partitioning the Golden code – Let us consider a subcode $\mathcal{G}_k \subseteq \mathcal{G}$ for $k = 1, \ldots, 4$, obtained by

$$\mathcal{G}_k = \{XB^k, X \in \mathcal{G}\}, \tag{7}$$

where

$$B = \begin{bmatrix} i(1-\theta) & 1-\theta \\ i\theta & i\theta \end{bmatrix}. \tag{8}$$

This provides the minimum square determinant $2^k\delta_{\min}$ (see Table 1). It is shown that the codewords of $\mathcal{G}_k$, when vectorized, correspond to different sublattices of $\mathbb{Z}^8$. It can be verified that these lattices form the lattice partition chain

$$\mathbb{Z}^8 \supset D_4^2 \supset E_8 \supset L_8 \supset 2\mathbb{Z}^8 \tag{9}$$

where D_4^2 is the direct sum of two four-dimensional Shäfli lattices, E_8 is the Gosset lattice and L_8 is a lattice of index 64 in $\mathbb{Z}^8$. Any two consecutive lattices $\Lambda \supset \Lambda'$ in this chain form a four way partition, i.e., the quotient group Λ/Λ' has order 4. Let $[\Lambda/\Lambda']$ denote the set of coset leaders of the quotient group Λ/Λ'. The lattices in the partition chain can be obtained by Construction A [9], using the nested sequence of linear binary codes $C_k = (8, 8-2k, d_{\min})$, where $d_{\min}$ is the minimum Hamming distance and $k = 0, \ldots, 4$,

$$\begin{aligned} C_0 = (8,8,1) \supset C_1 = (8,6,2) \supset C_2 = (8,4,4) \\ \supset C_3 = (8,2,4) \supset C_4 = (8,0,\infty) \end{aligned} \tag{10}$$

Let G_k denote the generator matrix of the code C_k for $k = 1, 2, 3$. We have

$$G_1 = \begin{bmatrix} 1&0&0&1&0&0&0&0 \\ 0&1&0&1&0&0&0&0 \\ 0&0&1&1&0&0&0&0 \\ 0&0&0&0&1&0&0&1 \\ 0&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&1&1 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0&1&0&1&0&1&0&1 \\ 0&0&1&1&0&0&1&1 \\ 0&0&0&0&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0&0&0&0&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \end{bmatrix}$$

Following the track of [5–7], we consider the lattice partition chain $\Lambda \supset \Lambda' \supset \Lambda_\ell$, where $\Lambda, \Lambda', \Lambda_\ell$ are any three consecutive lattices in the partition chain. We can write

$$\Lambda = \Lambda_\ell + [\Lambda/\Lambda_\ell] = \Lambda_\ell + [\Lambda/\Lambda'] + [\Lambda'/\Lambda_\ell].$$

Let C, C' and C'' be the corresponding codes in (10). Then we can write[1]

$$\Lambda = \Lambda_\ell + [C/C''] = \Lambda_\ell + [C/C'] + [C'/C''] \,. \tag{11}$$

The coset leaders in $[C/C']$ form a group of order 4 ($\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$), which is generated by two binary generating vectors $\mathbf{h}_1$ and $\mathbf{h}_2$

$$[C/C'] = \{b_1\mathbf{h}_1 + b_2\mathbf{h}_2 \mid b_1, b_2 \in GF(2)\}$$

If we consider all the lattices in (9), and the corresponding nested sequence of linear binary codes C_k in (10), we have:

$$\begin{aligned} [C_0/C_1] &: \begin{cases} \mathbf{h}_1^{(1)} = (0,0,0,0,0,0,0,1) \\ \mathbf{h}_2^{(1)} = (0,0,0,1,0,0,0,0) \end{cases} \\ [C_1/C_2] &: \begin{cases} \mathbf{h}_1^{(2)} = (0,0,0,0,0,0,1,1) \\ \mathbf{h}_2^{(2)} = (0,0,0,0,0,1,0,1) \end{cases} \\ [C_2/C_3] &: \begin{cases} \mathbf{h}_1^{(3)} = (0,1,0,1,0,1,0,1) \\ \mathbf{h}_2^{(3)} = (0,0,1,1,0,0,1,1) \end{cases} \\ [C_3/C_4] &: \begin{cases} \mathbf{h}_1^{(4)} = (0,0,0,0,1,1,1,1) \\ \mathbf{h}_2^{(4)} = (1,1,1,1,1,1,1,1) \end{cases} \end{aligned} \tag{12}$$

[1]Note that the binary components in $GF(2)$ of the coset leaders are lifted to the ring of integers with an abuse of notation.

Level	Subcode	Lattice	Binary code	$\Delta_{\min}$
0	$\mathcal{G}$	$\mathbb{Z}^8$	$C_0 = (8,8,1)$	$\delta_{\min}$
1	$\mathcal{G}_1$	D_4^2	$C_1 = (8,6,2)$	$2\delta_{\min}$
2	$\mathcal{G}_2$	E_8	$C_2 = (8,4,4)$	$4\delta_{\min}$
3	$\mathcal{G}_3$	L_8	$C_3 = (8,2,4)$	$8\delta_{\min}$
4	$\mathcal{G}_4 = 2\mathcal{G}$	$2\mathbb{Z}^8$	$C_4 = (8,0,\infty)$	$16\delta_{\min}$

Table 1: The Golden code partition chain with corresponding lattices, binary codes, and minimum squared determinants.

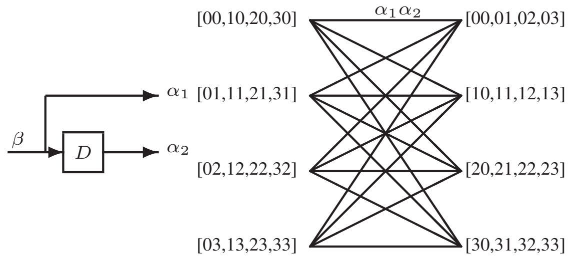

Figure 2: The 4-state encoder with $g_1(D) = 1$ and $g_2(D) = D$ and corresponding trellis diagram. Labels on the left are outgoing from each state clockwise, labels on the right are incoming counterclockwise.

Encoder structure – Fig. 1 shows the encoder structure of the proposed concatenated scheme. The input bits feed two encoders, an upper trellis encoder and a lower lattice encoder.

Generalizing (11), we consider two lattices Λ and Λ_ℓ from the lattice partition chain in Table 1, such that Λ_ℓ is a proper sublattice of the lattice Λ, where ℓ denotes the *relative partition level* of Λ_ℓ with respect to Λ. Let ℓ_0 denote the *absolute partition level* of the lattice Λ. For example, with $\ell_0 = 0, \ell = 2$, we have $\Lambda = \mathbb{Z}^8$ and $\Lambda_\ell = 2\mathbb{Z}^8$.

For two lattices Λ and Λ_ℓ, we have the quotient group Λ/Λ_ℓ with order $N_c = |\Lambda/\Lambda_\ell| = 4^\ell$, which corresponds to the total number of cosets of the sublattice Λ_ℓ in the lattice Λ. We assume that we have $4q$ input bits. The upper encoder is a trellis encoder that operates on q_1 information bits. Given the relative partition depth ℓ, we select a trellis code rate $R_c = 1/\ell$. The trellis encoder outputs $n_c = q_1/R_c$ bits, which are used by the coset mapper to label the coset leader $\mathbf{c}_1 \in [\Lambda/\Lambda_\ell]$. The mapping is obtained by the product of the n_c bit vector with a binary coset leader generator matrix H_1 with rows $\mathbf{h}_1^{(\ell_0+1)}, \mathbf{h}_2^{(\ell_0+1)}, \cdots, \mathbf{h}_1^{(\ell_0+\ell)}, \mathbf{h}_2^{(\ell_0+\ell)}$, where the rows are taken from (12). This will limit $q_1 = 2$.

The lower encoder is a sublattice encoder for Λ_ℓ and operates on q_2+q_3 information bits, where $q_2 = 2\times(4-\ell-\ell_0)$ and $q_3 = 4q - q_1 - q_2$. The q_2 bits label the cosets of $2\mathbb{Z}^8$ in Λ_ℓ by multiplying the matrix H_2 with rows $\mathbf{h}_1^{(\ell_0+\ell+1)}, \mathbf{h}_2^{(\ell_0+\ell+1)}, \cdots, \mathbf{h}_1^{(4)}, \mathbf{h}_2^{(4)}$, which generates the coset leaders $\mathbf{c}_2 \in [\Lambda_\ell/2\mathbb{Z}^8]$. We finally add both coset leaders of $\mathbf{c}_1$ and $\mathbf{c}_2$ modulo 2 to get $\mathbf{c}'$. The q_3 bits go through $2\mathbb{Z}^8$ encoder and generate vector $2\mathbf{u}$, $\mathbf{u} \in \mathbb{Z}^8$, which is added to $\mathbf{c}'$ (lifted to have integer components) and mapped to the Golden codeword X_t.

We now focus on the structure of the trellis code to be used. We consider linear convolutional encoders over the quaternary alphabet $\mathbb{Z}_4 = \{0,1,2,3\}$ with mod 4 operations, in order to match the four way partitions. We assume the natural mapping between pairs of bits and $\mathbb{Z}_4$ symbols, i.e., $0 \to 00, 1 \to 01, 2 \to 10, 3 \to 11$. Let $\beta \in \mathbb{Z}_4$ denote the input symbol and $\alpha_1, \ldots, \alpha_\ell \in \mathbb{Z}_4$ denote the ℓ output symbols generated by the generator polynomials $g_1(D), \ldots g_\ell(D)$ over $\mathbb{Z}_4$. For example, Figure 2 shows a 4 state encoder and the trellis labels for outgoing and incoming branches are listed from top to bottom. Figure 3 shows how the N_c cosets can be addressed through a partition tree of depth 2.

Trellis labeling– In order to increase the potential coding gain, the lower bound $\Delta'_{\min}$ in (5) should be maximized. Let

$$\Delta_{\text{par}} = 2^{\ell_0+\ell}\delta_{\min} \tag{13}$$

denote the minimum determinant of the trellis parallel transitions corresponding to the Golden code partition Λ_ℓ of absolute level $\ell_0 + \ell$. Let

$$\Delta_{\text{sim}} = \min_{\mathbf{X}\neq \mathbf{0}_{2\times 2L}} \sum_{t=t_o}^{t_o+L'-1} \det(X_t X_t^\dagger) \tag{14}$$

denote the minimum determinant on the shortest simple error event, where L' is the length of the shortest simple error event diverging from the zero state at t_o and merging to the zero state at $t_i = t_o + L' - 1$.

The lower bound $\Delta'_{\min}$ in (5) is determined either by the parallel transition error events or by the shortest simple error events in the trellis, i.e.,

$$\begin{aligned} \Delta'_{\min} &= \min\{\Delta_{\text{par}}, \Delta_{\text{sim}}\} \\ &\geq \min\left\{\Delta_{\text{par}}, \min_{X_{t_o}} \det(X_{t_o}X_{t_o}^\dagger) + \min_{X_{t_i}} \det(X_{t_i}X_{t_i}^\dagger)\right\}. \end{aligned}$$

The corresponding coding gain will be

$$\gamma'_{as} = \min\{\gamma'_{as}(\Delta_{\text{par}}), \gamma'_{as}(\Delta_{\text{sim}})\}. \tag{15}$$

Therefore, we have the following:

Design Criterion – *We focus on $\Delta'_{\min}$. The incoming and outgoing branches for each state should belong to different cosets that have the common father node as deep as possible in the partition tree. This guarantees that simple error events in the trellis give the largest contribution to $\Delta'_{\min}$.*

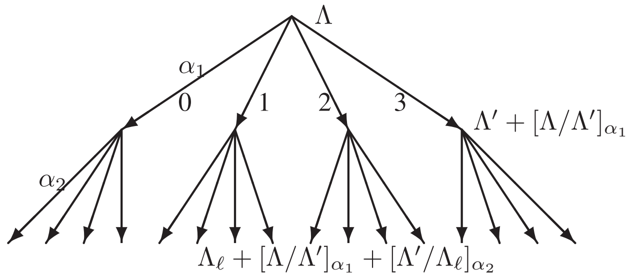

Figure 3: Two level set partitioning of Λ, output label α_1 selects the first level and α_2 selects the second level in the partition tree.

In order to fully satisfy the above criterion for a given relative partition level ℓ, the minimum number of trellis states should be $N_c = 4^\ell$. In order to reduce complexity we will also consider trellis codes with fewer states. We will see in the following that the performance loss of these suboptimal codes (in terms of the above design rule) is marginal since $\Delta_{\rm par}$ is dominating the code performance. Nevertheless, the optimization of $\Delta_{\rm sim}$ yields a performance enhancement. In fact, maximizing $\Delta_{\rm sim}$ has the effect of minimizing another relevant PWEP term.

Decoding – Let us analyze the decoding complexity. The decoder is structured as a typical TCM decoder, i.e. a Viterbi algorithm using a branch metric computer. The branch metric computer should output the distance of the received symbol from all the cosets of Λ_ℓ in Λ. The decoding complexity depends on two parameters: N_c and 2^{q_1}.

4. CODE DESIGN EXAMPLES

In this section, we give two examples of GST-TCM with different numbers of trellis states. We assume each frame contains $L = 130$ symbols (2×2). We first describe the uncoded Golden code schemes with the same frame length, which are used as a reference system for performance comparison. The subscript t in (2) will be omitted for brevity.

Uncoded Golden code 7bpcu – A total of 14 bits must be sent in a Golden codeword (2): the symbols a and c are in a 8-QAM (3bits), while the symbols b and d are in a 16-QAM (4bits). In this case we have $E_{s,2} = (1.5{+}2.5)/2 = 2$ and $q = 3.5$ bits.

Uncoded Golden code 6bpcu – A total of 12 bits must be sent in a Golden codeword (2): the symbols a, b, c, d are in a 8-QAM (3bits). This guarantees that the same average energy is transmitted from both antennas. In this case we have $E_{s,2} = 1.5$ and $q = 3$ bits.

Example 1 – *The 4 and 16 state trellis codes using 16-QAM constellation gain 3dB and 3.3dB, respectively, over uncoded transmission at the rate of 7bpcu. We use $\Lambda = \mathbb{Z}^8$ and $\Lambda_\ell = E_8$, where $\ell_0 = 0, \ell = 2$. We have $E_{s,1} = 2.5$ and $q = 3.5$ bits. We have $L' = 2, L' = 3$ for 4 and 16 state trellis codes.*

We consider a two level partition with the quotient group $\Lambda/\Lambda_\ell = \mathbb{Z}^8/E_8$ of order $N_c = 16$. The quaternary trellis encoders for 4 and 16 states with rate $R_c = 1/2$, have $q_1 = 2$ input information bits and $n_c = 4$ output bits, which label the coset leaders. The sublattice encoder has $q_2 = 4$ and $q_3 = 8$ input bits, giving a total number of input bits per information symbol $q = (q_1{+}q_2{+}q_3)/4 = 14/4 = 3.5$bits.

The 4 state trellis structure is shown in Fig. 2. In such a case, α_1 chooses the cosets from L_8 and α_2 chooses the cosets from $\Lambda_\ell = 2\mathbb{Z}^8$. The four trellis branches merging in each state belong to four different cosets of $2\mathbb{Z}^8$ in L_8, since α_1 is fixed and α_2 varies. This guarantees an increased Δ'_{min}. On the other hand, the four trellis branches departing from each state are in the cosets of L_8 in E_8, this does not give the highest possible increase to Δ'_{min}, since α_1 varies.

The above problem suggests the use of a 16 state encoder. In fact we note that the first output label α_1 is fixed for all outgoing and incoming states. This satisfies the proposed design criteria. Compared to 4 state GST-TCM, 16 state GST-TCM has a higher decoding complexity.

Performance of both the proposed TCM and uncoded transmission (7 bpcu) schemes is compared in Fig. 4. It is shown that the proposed 4 and 16 state TCMs outperform the uncoded case by 3.0dB and 3.3dB at the FER of 10^{-3}.

Example 2– *The 16 and 64 state trellis codes using 16-QAM gain 4.2 and 4.3 dB, respectively, over an uncoded transmission scheme at the rate of 6 bpcu and $\Lambda = \mathbb{Z}^8, \Lambda_\ell = L_8$, where $\ell_0 = 0, \ell = 3$. We have $E_{s,1} = 2.5$ and $q = 3$ bits.*

We consider a three level partition with quotient group $\Lambda/\Lambda_\ell = \mathbb{Z}^8/L_8$ of order $N_c = 64$. The quaternary trellis encoders for 16 and 64 states with rate $R_c = 1/3$ have $q_1 = 2$ input information bits and $n_c = 6$ output bits, which label the coset leaders. The sublattice encoder has $q_2 = 2$ and $q_3 = 8$ input bits, giving a total number of input bits per information symbol $q = (q_1 + q_2 + q_3)/4 = 12/4 = 3$bits.

The 16 state GST-TCM has the following generator polynomials: $g_1(D) = D, g_2(D) = D^2, g_3(D) = 1 + D^2$, where D is a delay operator mod 4. For the 16 state GST-TCM, at each trellis state, four outgoing branches are labeled with $\alpha_1, \alpha_2, \alpha_3$, corresponding to input $\beta \in \mathbb{Z}_4$. In this case, since α_1 and α_2 are fixed, α_3 varies. This guarantees an increased $\Delta'_{\rm min}$. The four trellis branches arriving in each state are in cosets of E_8. This does not give the highest possible increase to $\Delta'_{\rm min}$ since α_2 varies. This results in a suboptimal design, which yields

$$\Delta'_{\rm min} \geq \min(8\delta_{\rm min}, 4\delta_{\rm min} + \delta_{\rm min} + 2\delta_{\rm min}) = 7\delta_{\rm min}.$$

The above problem suggests the use of a 64 state encoder with the generator polynomials: $g_1(D) = D, g_2(D) = D^2, g_3(D) = 1 + D^3$. In such a case, the output labels

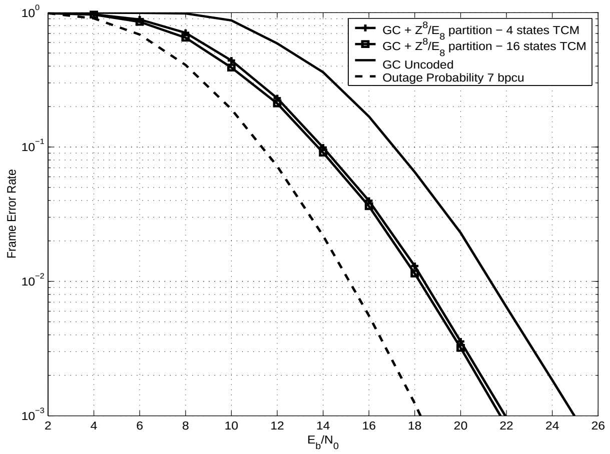

Figure 4: Performance comparison of 4- and 16-state trellis codes using 16-QAM constellation and an uncoded transmission at the rate 7bpcu, $\Lambda = \mathbb{Z}^8$, $\Lambda_\ell = E_8$, $\ell = 2$.

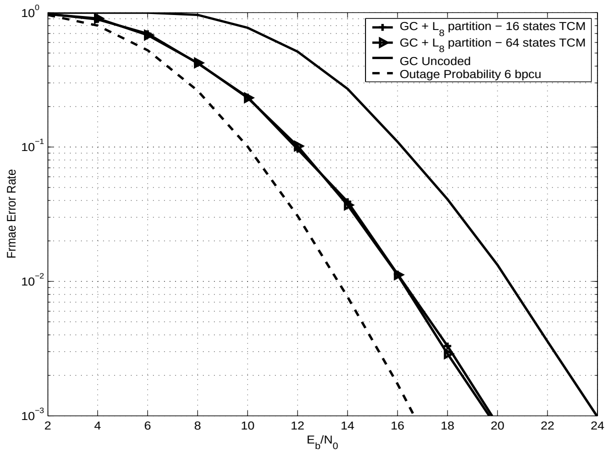

Figure 5: Performance comparison of 16- and 64-state trellis codes using 16-QAM constellation and an uncoded transmission at the rate 6bpcu, $\Lambda = \mathbb{Z}^8$, $\Lambda_\ell = L_8$, $\ell = 3$.

$\alpha_1(D_4^2), \alpha_2(E_8)$ are fixed for all outgoing and incoming states. Only $\alpha_3(L_8)$ varies to choose different subgroups from the deepest partition level in this example. This fully satisfies our design rule and yields

$$\Delta'_{\min} \geq \min(8\delta_{\min}, 4\delta_{\min}+\delta_{\min}+2\delta_{\min}+4\delta_{\min}) = 8\delta_{\min}.$$

Compared to 16 state GST-TCM, the 64 state GST-TCM has a higher decoding complexity. It requires $N_c = 256$ lattice decoding operations in each trellis section, while the 16 state TCM only requires $N_c = 64$. Performance of the proposed codes and the uncoded scheme with 6 bpcu is compared in Fig. 5. We can observe that a 16 state GST-TCM outperforms the uncoded scheme by 4.2 dB and 3.1 dB away from outage probability at the FER of 10^{-3}. The 64 state GST-TCM outperforms the uncoded case by 4.3 dB and 3 dB away from outage probability at FER of 10^{-3}.

5. CONCLUSIONS

In this paper, we presented GST-TCM, a coding scheme suitable for slow fading 2×2 MIMO systems. The inner modulation is the Golden code, which provides the full diversity and non-vanishing determinant property. Lattice set partitioning is designed specifically to increase the minimum determinant of the Golden codewords, which label the branches of the trellis code. Viterbi algorithm is applied in trellis decoding, where branch metrics are computed by using a lattice decoder. The general framework for GST-TCM design and optimization is based on Ungerboeck TCM design rules.

Future work will explore further code optimization, by an extensive search based on the determinant distance spectrum and extension to MIMO systems with more than two tx/rx antennas.

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