# Dynamic pricing in wireless networks

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*Abstract*— **In this paper, we deal with dynamic pricing strategies in wireless communication systems. Dynamic pricing policies allow the network operator to charge a cost per time unit depending on the network usage. In this way, the users behavior can be regulated and the network management is significantly improved. We model the user demand and the call duration as functions of the service price. By using standard Markovian techniques to represent the system evolution, we devise an optimal linear pricing scheme, which can be easily computed and controlled. When compared with a flat-rate policy, where a constant price for the network services is fixed, the proposed solution is able to provide a better quality of service to the users as well as a greater revenue to the network operator.**

*Keywords:* Dynamic Pricing, Markovian model.

## I. Introduction

Recently, the wireless communication market has experienced an incredible growth; market research forecasts one billion mobile phones will be active in 2002. Moreover, it is a common belief that mobile phones will be the drivers of the wireless Internet.

This enormous development of the cellular phone market has involved a great demand for radio resources. Since the availability of the radio bandwidth is limited, new solutions to increase the available capacity of radio systems need to be found. Different approaches can be used, from cell splitting and frequency reuse [1] to overlapping cell layers [2], [3] and dynamic channel allocation techniques [4]. However, these methods often imply either an increased system complexity or a significant degradation of the quality of service.

In this paper, we deal with pricing strategies for an efficient management of wireless networks. We propose a solution based on dynamic pricing techniques, i.e., on pricing schemes where the cost the network operator charges per time unit depends on the network usage and is dynamically adapted to the network status. Indeed, pricing can affect user's behavior and allow for an efficient control of the network operational conditions. For example, flat-rate pricing, i.e., a constant price for the network services, is very effective in stimulating new applications development, but it is no longer suitable for an environment with an increasing demand for network resources. On the contrary, by adjusting prices to the usage of the network, we can make a better use of the available bandwidth, and provide the desired quality of service to the users as well as a greater revenue to the network operator.

Dynamic pricing has been mainly introduced to control wired networks supporting Internet-based services [5], [6], [7]. In this case, techniques to derive the system optimal rate have been proposed, which charge users on the basis of the congestion they cause to the network. In [8], dynamic pricing has been applied to the wireless environment and simulation results have been presented for a simple pricing strategy.

We first consider connection-oriented services in third-generation wireless systems and we propose a solution to dynamic pricing based on an analytical approach. We model the call duration as a function of the service price and the user demand as a decreasing exponential function of the call price per time unit and of the call blocking probability, that is the quality of service metric we selected. Then, since customers tend to prefer transparent and easy to understand pricing policies, we impose that prices follow linear dynamics. By using standard Markovian techniques to represent the system evolution, we devise an optimal linear pricing strategy which maximizes the network revenue while providing the required quality of service to the users. Performance of the proposed pricing strategy is compared to the results obtained through a flat-rate charging policy.

Next, we give some insight in the application of dynamic pricing to wireless networks supporting data packet services. Indeed, pricing strategies in telephony and Internet based services are bound converge as the networks evolve.

## II. System model

A common econometric model assumption for the user demand function is [9]

$$D(p,Q) = \exp(-\alpha p + \beta Q) \qquad (1)$$

where $p$ is the price per time unit, $Q$ is the *quality of service* and $\alpha, \beta$ are constant parameters related to the user population behavior. We assume as quality of service index the *call success probability*; thus $Q = 1 - P_b$, where $P_b$ is the *call blocking probability*, i.e., the probability that the system can not accept any new call. This demand

function represents the users active time expressed in seconds per time unit. This model takes into account a common user behavior: user's demand drops as the price per time unit increases and the quality of service decreases. The parameters $\alpha$ and $\beta$ must be identified by adequate market research.

We use the above model to derive the traffic intensity by assuming that the demand in the time span $\Delta t$ is equal to the actual traffic that is generated in $\Delta t$, i.e., the traffic intensity $\gamma$ weighted by the call success probability $1 - P_b$

$$\Delta t \exp[-\alpha p + \beta(1 - P_b)] = \Delta t \gamma (1 - P_b) \ . \quad (2)$$

Let $N$ be the maximum number of available communication channels. The system may be described by a birth-death Markov chain where each state $i = 0, \ldots N$ represents the number of active calls in the network.

Let $\mathbf{p} = (p_0, p_1, \ldots p_{N-1})$ be the *price vector* representing the cost per time unit of a call started when the system is in state $i$, with $0 \leq i \leq N-1$. Our main objective is to determine a convenient price vector which optimizes some network performance parameters as we describe in the following.

Let $\lambda_i$ be the call arrival rate when the system is in state $i$ ($0 \leq i \leq N-1$) and let $\mu_i$ be the average call termination rate when the system is in state $i$ ($1 \leq i \leq N$). According to the above econometric model the call arrival rate in state $i$ is modeled as

$$\lambda_i = \frac{m_i}{1 - P_b} \exp[-\alpha p_i + \beta(1 - P_b)], \ 0 \leq i \leq N-1 \quad (3)$$

where we have set $\gamma = \lambda_i / m_i$, being $m_i$ the average call duration of a call starting in state $i$.

Since we expect that users will react to the increase of price per time unit by reducing their call duration, we define $T_{min}$ and $T_{max}$ as the minimum and the maximum call duration times, respectively. We model the average call duration of a call starting in state $i$ as a decreasing function of the call price per time unit in the range $[T_{min}, T_{max}]$ as

$$\frac{1}{m_i} = T_{max} \exp[-K_m (p_i - p_0)] \quad 0 \leq i \leq N-1 \quad (4)$$

where $K_m = \ln(T_{max}/T_{min})/(p_{N-1} - p_0)$. For example, in the case of voice traffic, a reasonable choice could be $T_{max} = 180$s and $T_{min} = 60$s.

Assuming that call durations are exponentially distributed with rates $m_i$ ($0 \leq i \leq N-1$), the average call termination rate in state $i$ is given by

$$\mu_i = m_0 + m_1 + \cdots + m_{i-1} \quad 1 \leq i \leq N \ . \quad (5)$$

The steady state solution of the Markov chain shown in Figure 1 is the probability vector

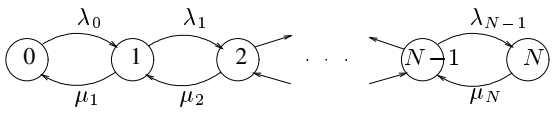


Fig. 1. Markov model.

$(\pi_0, \pi_1, \ldots \pi_N)$ where

$$\pi_0 = \left( \sum_{k=0}^{N} \prod_{j=1}^{k} \frac{\lambda_{j-1}}{\mu_j} \right)^{-1} \quad (6)$$

$$\pi_i = \pi_0 \prod_{j=1}^{i} \frac{\lambda_{j-1}}{\mu_j} \ .$$

The system blocking probability is $P_b = \pi_N$. Since $\lambda_i$ and $\mu_i$ are also functions of $P_b$, the steady state solution must be computed iteratively until convergence on $P_b$ is reached.

Let $\mathbf{g} = (g_1, \ldots g_N)$ be the revenue vector representing the total revenue per time unit of the network operator when the system is in state $1 \leq i \leq N$. We have

$$g_i = \sum_{j=0}^{i-1} p_j \quad (7)$$

since when the system is in state $i$ each user has started a call in one of the previous states $j < i$.

The total *network revenue per time unit* is

$$G = \sum_{i=1}^{N} \pi_i g_i \quad (8)$$

and is of interest to the network operator.

A useful performance parameter of interest to the users is the call *average cost per time unit*, which is given by

$$\overline{c} = \sum_{i=0}^{N-1} \pi_i p_i \ . \quad (9)$$

The aim of this paper is to evaluate a pricing policy (i.e., a price vector) maximizing the total network revenue $G$, while reducing the call blocking probability $P_b$. This optimization problem in its general form may be hard to solve numerically. We consider a suboptimal approach in the sense that we assume a linearly increasing price vector which gradually discourages users from starting new calls as the network load increases. A further motivation for the linear-rate choice is the simplicity of this pricing policy, which can be better understood by the users. The optimal slope of the linearly increasing price vector is determined in order to maximize the total network revenue $G$. We call this price vector the optimal *linear-rate policy*. As outcome of this procedure we also obtain the corresponding $P_b$, which is a metric of interest to the users.

As a baseline for comparisons we consider the *flat-rate* policy where the price vector is constant and the average call duration is fixed to $T_{max}$. In particular, we select a flat price which lays between the minimum and the maximum value of the linear-rate prices. The fact that the initial values of the linear-rate price vector are lower than the flat price is the main incentive for the users to place calls during off-peak time.

## III. Results

In this section, we show some results obtained using our analytical model in the case of voice traffic. We consider different user populations by fixing $\alpha = 0.2$ and choosing $\beta/\alpha = 1$ and 4, in networks with $N = 8$ and $N = 16$ radio channels. The flat-rate policy is compared to the linear-rate policy on the basis of the network revenue per time unit and of the blocking probability.

In the following, all curves comparing the different strategies are given in terms of the traffic load $\rho$, that we have when the flat-rate policy is applied. Denoting by $\lambda_f$ the call arrival rate relative to the flat-rate policy, we write

$$\rho = \lambda_f T_{max} \, . \tag{10}$$

Figure 2 shows how the network revenue can be increased by using the linear-rate strategy. For any value of $\rho$ there is a significant improvement in network revenue when using the linear policy. For example for $\rho = 0.8$ we observe a $50\%$ improvement in $G$ of the linear-rate over the flat-rate policy. We also note that results are mildly affected by the parameter $\alpha/\beta$.

Figure 3 shows how the blocking probability can be reduced by using the linear-rate policy. This may be a secondary desired goal of the network operator but it is of great interest to the users. By applying the linear-rate pricing scheme, $P_b$ can be reduced by a factor ranging between 2 and 10 as $\rho$ decreases. Here the dependence on $\alpha$ and $\beta$ is more critical for high $\rho$s.

Figure 4 compares on a single plot $G$ and $P_b$ as functions of $\rho$ in the range $[0.1, 1]$ in steps of $0.1$, for $N = 8$ and $N = 16$. Globally better performance can be noticed when the curves move toward the upper left corner of the plot. Only the case $\alpha/\beta = 1$ is shown since we observed only minor differences when $\alpha/\beta = 4$.

Figure 5 shows the average cost per time unit as a function of $\rho$. Note that in the flat-rate case the average cost per time unit decreases when traffic load is high; this is explained by looking at (9) and considering that for high traffic load the blocking probability becomes relevant. The plot suggests that the linear dynamic policy is always more advantageous for the operator.

We observe that the results given here are a simple example of the possibilities offered by our dy-

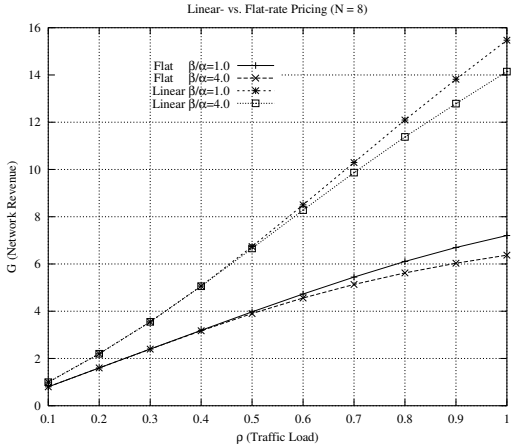


Fig. 2. Linear- vs. Flat-rate: network revenue per time unit vs. traffic load for different users' behavior ($N = 8$).

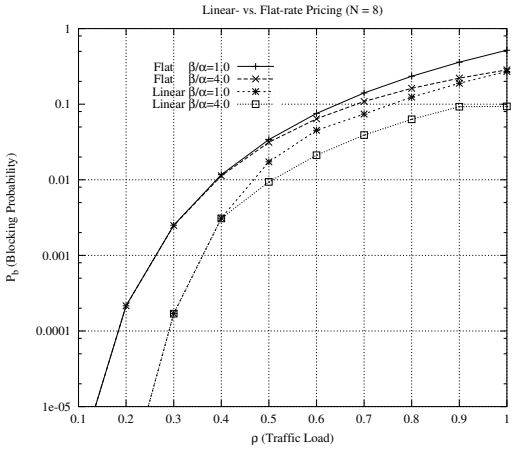


Fig. 3. Linear- vs. Flat-rate: call blocking probability vs. traffic load for different users' behavior ($N = 8$).

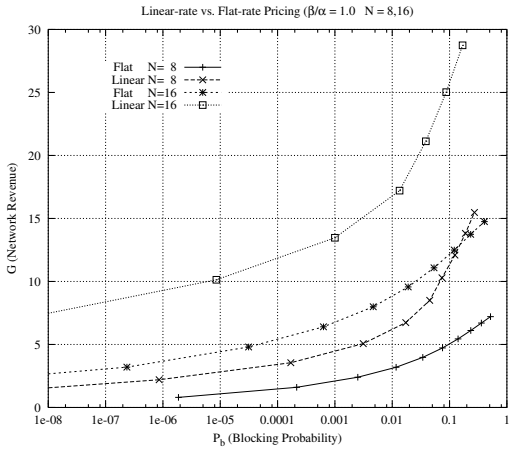


Fig. 4. Linear- vs. Flat-rate: network revenue per time unit vs. call blocking probability for $N = 8, 16$.

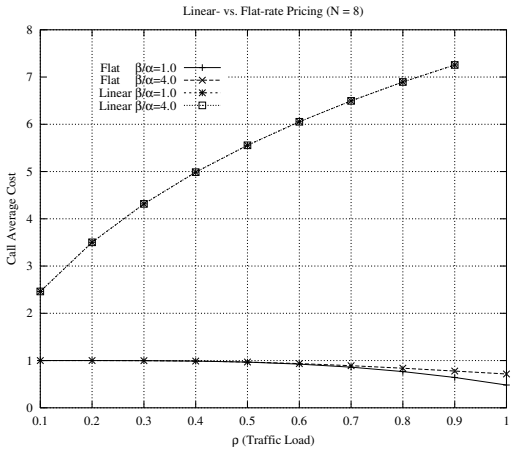


Fig. 5. Linear- vs. Flat-rate: user average cost per time unit vs. traffic load for $N = 8$ and $\beta/\alpha = 1, 4$.

namic pricing scheme. More realistic parameters should be evaluated by the network operator according to their user population behavior analysis. The proposed method is an extremely flexible and simple tool to estimate the performance of dynamic pricing policies. As already noted, the linear pricing strategy is not necessarily the optimal one. Further investigation is needed to solve the pricing problem to optimum.

## IV. Future work on pricing for data packet services

In this section, we consider the possibility to apply pricing strategies to connection-less services. This problem was already addressed in [5], [6], [7], [10], [11].

We consider different classes of users according to their desired information bit rate and actual bandwidth occupancy. Consider, for example, users that require an information bit rate $R_i$; depending on the channel conditions and on their mobility profile the actual used bandwidth $B_i$ may be greater than $R_i$ due to the overhead of error correction functions. Hence, a user class is identified by the pair $(R_i, B_i)$.

The network operator supplies a total shared bandwidth $B$. The pricing policy can be designed in order to maximize the operator revenue and the total users information bit rate.

The baseline strategy may consist in setting the price for each user class proportionally to $B_i$ independently of the network traffic load. Alternatively, we propose to apply a different pricing policy for each users class in such a way that user demand is suitably shaped to reduce congestion.

In order to select the optimal pricing policy, we model the system as follows. We assume that the total bandwidth, $B$, is virtually divided into basic rate channels of width equal to $b$, and we express $B_i$ as multiples of $b$. We model users' demand in each class similarly to (1) and use a multidimensional Markovian model to represent the system evolution.

From the operator viewpoint, performance is measured in terms of network revenue $G$ and average blocking probability over all user classes. Whereas, from the user viewpoint, the average cost per information bit and the class blocking probability must be considered.

By solving the analytical model we can optimize the pricing strategies for the different user classes. It will then be possible to answer questions such as: (i) Is it better to charge high rate users more than low rate users? (ii) How much can we shape the users demand for network resources as the network traffic load varies over time?

## V. Conclusions

In this paper, a dynamic pricing scheme for wireless communication systems was presented. We modeled the user demand and call duration as functions of the service price. Then, by using standard Markovian techniques, an optimal linear pricing policy was derived in order to obtain a transparent and easily controllable pricing strategy. The performance of the proposed solution was derived in the case of voice services and compared to the results obtained when a flat-rate policy is applied. It was shown that by adjusting prices to the usage of the network, we can make a better use of the available bandwidth, and provide a greater revenue to the network operator as well as an improved quality of service to the users. We also gave directions for future research for data packet services on the lines of the proposed methodology.

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