CDMA design through asymptotic analysis: Fading channels

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Abstract — *Using asymptotic analysis, we study the effect of frequency-flat fading on CDMA systems with linear receivers and random spreading sequences. Specifically, we let the number of users grow without bound while the ratio of number of users to spreading sequence length is kept fixed to a value α. We treat separately the case of slow fading with lognormal shadowing (nonergodic channel) and of fast Rayleigh fading (ergodic channel). For the former channel we derive the outage probability, while for the latter we compute the channel capacity. We show in particular that, with fading, as $\alpha \to \infty$ the system throughput tends to the same limit of 1.44 bit/symbol as for the non-fading channel. On the contrary, the outage probability exibits a floor for all α values when a single-user matched filter is employed, while with a linear minimum-mean-square-error receiver the floor is present only for $\alpha \geq 1$. We also address the tradeoffs involved in the allocation of available bandwidth between spreading and coding.*

I. INTRODUCTION

We examine a CDMA system with error-control coding, operating on a channel affected by frequency-flat fading. The receiver consists of a linear front-end, viz., either a single-user matched filter (SUMF) or a linear minimum-mean-square error filter (MMSE) [8], followed by single-user decoding. The key performance measure here is the Signal-to-Interference plus Noise Ratio (SINR) at the output of the linear filter: users' quality of service can be expressed in terms of a target SINR [7]. We treat separately the cases of slow and fast fading, yielding non-ergodic and ergodic channels, respectively. In the non-ergodic case, the SINR cumulative distribution function (cdf) yields immediately the *outage probability*, i.e., the probability that the actual SINR is below the SINR target. In the ergodic case, performance is given in terms of user capacity, i.e., the maximum transmission rate at which arbitrarily small error probabilites are achievable over the single-user channel induced by the linear receiver. Assuming that all users transmit "Gaussian codes," the user ergodic capacity is also determined by the SINR cdf [1].

Our study is asymptotic, in the sense that the number of users grows without bound, while the ratio of number of users to spreading-sequence length is kept finite and fixed [6, 9]. The spreading sequences are random, and the system is chip-synchronous and single-cell.

After a description of the system model (Section 2), we examine the outage probability of slow-fading channels (Section 3), the capacity of fast-fading channels, and the coding-vs.-spreading tradeoff (Section 4).

II. SYSTEM MODEL

We consider the uplink of a single-cell, chip-synchronous DS-CDMA system. Our model involves K users, random spreading sequences, and a spreading-sequence length $L = T/T_c$, where $1/T$ is the symbol rate and $1/T_c$ the chip rate. As in [6, 9], we assume a large number of users ($K \to \infty$) and $K/L \to \alpha$ (a constant "channel load" as the length of the spreading sequences increases to accommodate the K users). The received signal L-chip column vector corresponding to one symbol interval is given by

$$\mathbf{y} = \sum_{k=1}^{K} \sqrt{z_k} x_k \mathbf{s}_k + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ is a complex circularly-symmetric AWGN vector $\sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0 \mathbf{I})$, x_k is the complex modulation symbol of user k, $\mathbf{s}_k$ is the spreading sequence of user k, made of binary antipodal chips $\pm 1/\sqrt{L}$ generated at random with uniform probability, and z_k is the (unit-mean) frequency-flat fading power gain, which remains constant over the time necessary to transmit a symbol. We assume that the base station receiver has perfect knowledge of all fading gains (the "channel-state information") and that the demodulation is coherent.

User k is received with an instantaneous signal-to-noise ratio (SNR) $\gamma_k = z_k \Gamma_k$, where $\Gamma_k = \mathcal{E}_k / N_0$ is the *transmit* SNR and $\mathcal{E}_k$ is the user transmit average energy per symbol. We assume that as $K \to \infty$ the empirical cdf of the received SNR's, defined by

$$F_\gamma^{(K)}(x) \triangleq \frac{1}{K} \sum_{k=1}^{K} \mathbf{1}\{\gamma_k \leq x\}$$

($\mathbf{1}\{\mathcal{A}\}$ the indicator function of the event $\mathcal{A}$) converges almost everywhere to the cumulative distribution function $F_\gamma(x)$.

The receiver for user 1 (our reference user) is formed by a linear filter $\mathbf{h}_1$ producing the output $y_1 = \mathbf{h}_1^\dagger \mathbf{y}$ followed by a single-user decoder operating on the sequence of filter output samples. We shall consider either the single-user matched filter (SUMF) $\mathbf{h}_1 = \mathbf{s}_1$ and the linear minimum mean-square error filter (MMSE)

$$\mathbf{h}_1 = \left[\sum_{k=1}^{K} z_k \mathcal{E}_k \mathbf{s}_k \mathbf{s}_k^\dagger + N_0 \mathbf{I} \right]^{-1} \mathbf{s}_1$$

The SINR at the filter output is defined as

$$\beta_1 \triangleq \frac{\gamma_1 |\mathbf{h}_1^\dagger \mathbf{s}_1|^2}{|\mathbf{h}_1|^2 + \sum_{k=2}^{K} \gamma_k |\mathbf{h}_1^\dagger \mathbf{s}_k|^2}$$

Under the above assumptions, we have the following results [6]:

1. The SUMF output SINR converges in probability for large K to
$$\beta_1 = \frac{\gamma_1}{1+\alpha\int_0^\infty x\,dF_\gamma(x)} \tag{2}$$
2. The MMSE output SINR converges in probability for large K to the (unique) real non-negative solution of the equation
$$\beta_1 = \frac{\gamma_1}{1+\alpha\displaystyle\int_0^\infty \frac{x\gamma_1}{\gamma_1+x\beta_1}\,dF_\gamma(x)} \tag{3}$$

A. Distribution of the output SINR

From (2), it is immediately apparent that the SINR β_1 is proportional to γ_1. Similarly, since equation (3) depends only on the ratio β_1/γ_1, this turns out to be a deterministic quantity. Thus, for both SUMF and MMSE receivers the SINR has, apart from a scale factor, the same probability distribution as the fading power gain.

B. More than one class of users

We may assume, following [6], that the users are partitioned into J classes, each class i being characterized by a transmit SNR Γ_i. We can think of the Γ_is as the transmit SNRs determined by some power-control mechanism, and of the fading z_k as some channel attenuation that the power control is not able to compensate, either because it is too fast (as for example with Rayleigh multipath) or because of inaccuracies in the power control loop (as for example with residual shadowing [10]). Each class has p_iK users, where p_i is the fraction of users belonging to class i (obviously, $\sum_{i=1}^J p_i = 1$). Moreover, we assume that the fading gains z_k are i.i.d. (the fading statistics is the same for all users) and normalized so that $\int_0^\infty x\,dF_z(x) = 1$, where $F_z(x)$ is the fading-gain cdf. With these assumptions it is immediate to see that
$$F_\gamma(x) = \sum_{i=1}^J p_iF_z(x/\Gamma_i)$$
Let user 1 belong to class i. Because of the uncompensated fading, user-1 SINR is a random variable $\beta_{i,1}$. However, as shown in the previous subsection, the ratio $\xi = \beta_{i,1}/(\Gamma_iz_1)$ is a non-random constant independent of i. From (2), for the SUMF we obtain
$$\xi^{(s)} = \frac{1}{1+\alpha\displaystyle\sum_{i=1}^J p_i\Gamma_i} \tag{4}$$
From (3), for the MMSE we obtain $\xi = \xi^{(m)}$ as the unique real non-negative solution of the equation
$$\xi = \frac{1}{1+\alpha\displaystyle\sum_{i=1}^J p_i\Gamma_i\int_0^\infty \frac{x}{1+x\xi\Gamma_j}\,dF_z(x)} \tag{5}$$

We conclude that any user k belonging to class i has (asymptotically) SINR $\beta_{i,k} = z_k\Gamma_i\xi$ (where $\xi = \xi^{(s)}$ or $\xi = \xi^{(m)}$ depending on the receiver employed). Then, the SINR cdf for all users is just given by the fading cdf after a scale change. For users of class i it is given by
$$F_i(x) = \mathbb{P}(\beta_{i,k} \le x)F_z(x/(\Gamma_i\xi)) \tag{6}$$
Moreover, the SINRs of different users are (asymptotically as $K\to\infty$) statistically independent.

C. Methodological preamble

The intricacies of dealing with fading channels are thoroughly described in [1], so we shall not delve into them here. Rather, we list a number of points that describe the rationale behind the calculations that follow. In particular, we motivate the different methods that are used to analyze the two situations of flat, slow fading and of flat, fast fading.

Flat, slow fading. Consider first flat, slow fading—not slow enough to be compensated by the power-control system, yet slow enough to make the channel nonergodic because a codeword cannot experience all the channel states. We may define an outage probability
$$P_{\text{out}} \triangleq \mathbb{P}(\beta < \beta_0) \tag{7}$$
which generally will not be a step function as in the nonfaded channel [3], even in our limiting case of K and L both growing to infinity. The value of β_0 may be chosen as in [3]. For example, if we use a nonideal code with rate R bit/symbol achieving the target performance at a certain E_b/N_0, we set $\beta_0 = RE_b/N_0$. If we consider instead an optimum code which operates at the Shannon limit for a Gaussian channel and we want a rate R, then we set
$$\beta_0 = 2^R - 1 \tag{8}$$
as we did in [3].

Flat, fast fading. This is the case of a fading whose variations within a code word are so fast that the channel can be considered *information stable* [1]. Here we focus on the case of a single class of users. All users generate their code book according to a complex circularly-symmetric Gaussian probability densityfunction (pdf); hence, the single-user channel seen at the output of any user's receiving filter is an additive Gaussian noise channel, whose capacity is
$$C = \mathbb{E}[\log_2(1+\beta)] = \int_0^\infty \log_2(1+x\xi\Gamma)\,dF_z(x) \tag{9}$$

System throughput. We may define the *system throughput* η as the total number of bit/symbol in the system. For large systems, it is possible to transmit close to one complex symbol per second per Hz, and hence to express η in bit/s/Hz. If all K users transmit at rate C, the system throughput is
$$\eta = \frac{KC}{L} = \alpha C \tag{10}$$

If we want all users to transmit reliably at rate R, the maximum admissible channel load is then

$$\alpha_{\max} \triangleq \max\{\alpha : R \leq C(\alpha)\} \tag{11}$$

III. Slow-fading channel

Outage probability for users of class i is defined as the probability that the SINR is below some threshold value $\bar{\beta}_i$ that depends on the coding scheme of class i. We obtain

$$P_{\text{out},i} = \mathbb{P}(\beta_{i,k} \leq \bar{\beta}_i) = F_z\left(\frac{\bar{\beta}_i}{\xi \Gamma_i}\right) \tag{12}$$

Assuming Gaussian codes and minimum distance decoding at the output of the receiving filter, each user can transmit with arbitrarily small error probability at rate $R_{i,k} = \log_2(1 + \beta_{i,k})$ [4], for sufficiently large code block lengths. Then, in the absence of further specification of class i user codes, it make sense to define the coding rate R_i bit/symbol for users in class i and choose the SINR threshold as $\bar{\beta}_i = 2^{R_i} - 1$.

A. System outage capacity

Let $\bar{\mathbf{\Gamma}} = (\bar{\Gamma}_1, \ldots, \bar{\Gamma}_J)$ be a vector of input SNR constraints, $\boldsymbol{\epsilon} = (\epsilon_1, \ldots, \epsilon_J)$ a vector of target outage probabilities, and $\mathbf{R} = (R_1, \ldots, R_J)$ be a vector of coding rates. We would like to find the set $\mathcal{R} \subseteq \mathbb{R}_+^J$ of rate vectors $\mathbf{R}$ that can be assigned to the J classes such that, for all $i = 1, \ldots, J$

$$P_{\text{out},i} \leq \epsilon_i \qquad \text{and} \qquad \Gamma_i \leq \bar{\Gamma}_i \tag{13}$$

We refer to the region $\mathcal{R}$ as the *system outage capacity* region for the input and outage constraints specified by $\bar{\mathbf{\Gamma}}$ and $\boldsymbol{\epsilon}$.

SUMF receiver. Define

$$\mu_i \triangleq \frac{2^{R_i} - 1}{\sup\{x \in \mathbb{R}_+ : F_z(x) = \epsilon_i\}} \tag{14}$$

(which reduces to $\mu_i \triangleq (2^{R_i} - 1)/F_z^{-1}(\epsilon_i)$ when $F_z(\cdot)$ is strictly monotonic and continuous) and, by using (12) and (4), rewrite the outage constraint as

$$\frac{\Gamma_i}{1 + \alpha \sum_{j=1}^J p_j \Gamma_j} \geq \mu_i$$

The maximum R_i is achieved when the above inequalities are satisfied with equality. The solution of the resulting system of equations has the form $\Gamma_i/\mu_i = \kappa$, where κ does not depend on i. Solving for κ we obtain

$$\kappa = \frac{1}{1 - \alpha \sum_{j=1}^{J} p_j \mu_j}$$

provided that

$$\alpha \sum_{j=1}^{J} p_j \mu_j \leq 1 \tag{15}$$

The resulting transmit SNR assignment (power control) is obtained from $\Gamma_i = \kappa \mu_i$ in the form

$$\Gamma_i = \frac{\mu_i}{1 - \alpha \sum_{j=1}^{J} p_j \mu_j}$$

By imposing the input constraint $\Gamma_i \leq \bar{\Gamma}_i$ for all $i = 1, \ldots, J$, we obtain the desired final result

$$\alpha \sum_{j=1}^{J} p_j \mu_j \leq \min_{1 \leq i \leq J} \left\{1 - \frac{\mu_i}{\bar{\Gamma}_i}\right\} \tag{16}$$

MMSE receiver. By using (12) and (14), rewrite the outage constraint as

$$\Gamma_i \xi^{(m)} \geq \mu_i$$

Since $\xi^{(m)}$ does not depend on i, the solution for the above system (taken with equality) has the form $\Gamma_i/\mu_i = \kappa'$, where $\kappa' = 1/\xi^{(m)}$. By substituting in (5) and solving for κ' we obtain

$$\kappa' = \frac{1}{1 - \alpha \sum_{j=1}^{J} p_j \mu_i'},$$

where

$$\mu_i' \triangleq \int_0^\infty \frac{x\mu_j}{1 + x\mu_j}\, dF_z(x),$$

provided that

$$\alpha \sum_{j=1}^{J} p_j \mu_j' \leq 1 \tag{17}$$

The resulting transmit SNR assignment (power control) is obtained from $\Gamma_i = \kappa' \mu_i$ in the form

$$\Gamma_i = \frac{\mu_i}{1 - \alpha \sum_{j=1}^{J} p_j \mu_j'}$$

By imposing the input constraint $\Gamma_i \leq \bar{\Gamma}_i$ for all $i = 1, \ldots, J$, we obtain the desired final result

$$\alpha \sum_{j=1}^{J} p_j \mu_j' \leq \min_{1 \leq i \leq J} \left\{1 - \frac{\mu_i}{\bar{\Gamma}_i}\right\} \tag{18}$$

Example 1. Fig. 1 shows the outage capacity region with SUMF and MMSE receiver in a system with two user groups: the first one has 90% of the users transmitting with an outage probability 0.1 and a signal-to-noise ratio $\bar{\Gamma} = 10$ dB; the second one has 10% of the users transmitting with an outage probability 0.01 and a signal-to-noise ratio $\bar{\Gamma} = 13$ dB. The channel statistics are log-normal with mean value 1 and log-standard deviation $\sigma = 2$ dB, and $\alpha = 0.2, 0.5, 1,$ and 2.

B. Outage probability floor and near-far resistance

Consider the case of a single class, and neglect for simplicity the class index. The outage probability is given by $P_{\rm out} = F_z(\bar{\beta}/(\xi\Gamma))$. As $\Gamma \to \infty$, for the SUMF receiver we have

$$\lim_{\Gamma\to\infty} \xi^{(s)}\Gamma = 1/\alpha$$

Therefore, the outage probability has a floor at $F_z(\alpha\bar{\beta})$ for all $\alpha > 0$. This is a consequence of the fact that CDMA with SUMF reception is interference-limited, and because of fading, there is always a non-zero probability that some interferer is so strong that drives the SINR below the target threshold.

With an MMSE receiver, if we let $\Gamma \to \infty$ we obtain from (5)

$$\frac{\xi^{(m)}}{\xi^{(m)}+\alpha} = \xi^{(m)}$$

This has a positive solution for $\xi^{(m)}$ if and only if $\alpha < 1$ (recall that $0 \le \xi^{(m)} \le 1$), otherwise $\xi^{(m)} = 0$ is the only solution. We conclude that, if $\alpha < 1$ then $\lim_{\Gamma\to\infty} P_{\rm out} = 0$ (no outage probability floor), otherwise there is an error floor. This is a consequence of the fact that the MMSE receiver is near-far resistant (i.e., not interference-limited) if the user spreading sequences are linearly independent, and that with sufficiently long random sequences linear independence is achieved with arbitrarily large probability if $\alpha < 1$ and with arbitrarily small probability if $\alpha > 1$ [8].

Example 2. Assume for the fading gains a log-normal distribution with log-standard deviation σ dB and mean value $\mathbb{E}[z] = 1$. Letting $\kappa_s(\alpha, \Gamma) \triangleq 1/(\Gamma^{-1}+\alpha)$, for the SUMF we get

$$P^{\rm s}_{\rm out} = 1 - Q\left[\sigma^{-1}\left(\mu_z + 10\,\log_{10}[(2^R-1)/\kappa_s(\alpha,\Gamma)]\right)\right]$$

where $Q(x) \triangleq \mathbb{P}(\mathcal{N}(0,1) > x)$. For the MMSE we obtain the equation

$$z_1/\beta = \Gamma^{-1} + \alpha\mathbb{E}_z\left[\frac{z}{z_1/\beta+z}\right](z_1/\beta) \qquad (19)$$

which can be solved iteratively. Its solution can be written in the form

$$\beta = \kappa_m(\alpha,\Gamma)z_1 \qquad (20)$$

which shows that, as already observed in a general context, β is proportional to z_1 by a deterministic factor depending only on the parameters α and Γ. The outage probability is

$$P^{\rm s}_{\rm out} = 1 - Q\left[\sigma^{-1}\left(\mu_z + 10\,\log_{10}[(2^R-1)/\kappa_{\rm m}(\alpha,\Gamma)\right)\right]$$

With the SUMF receiver, since $\lim_{\Gamma\to\infty}\kappa_s(\alpha,\eta) = \alpha^{-1}$, the outage probability floor is

$$1 - Q\left[\sigma^{-1}\left(\mu_z + 10\,\log_{10}[(2^R-1)\alpha]\right)\right] \qquad (21)$$

for any value of α.

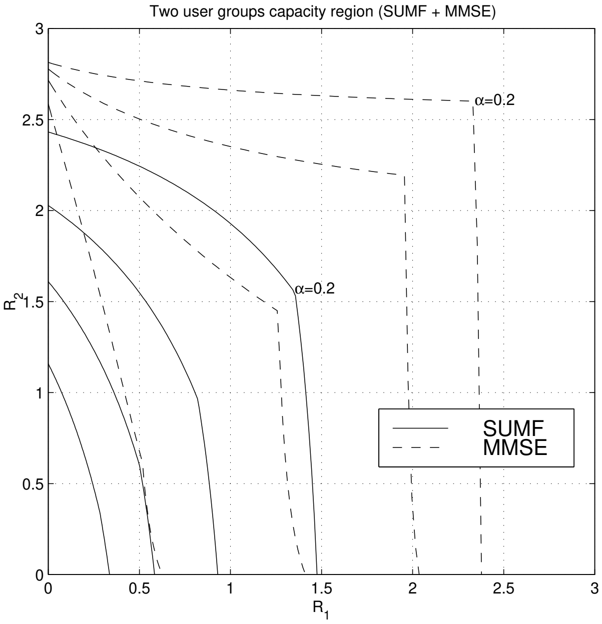

Fig. 1: *System outage capacity region with two user groups. The first one has 90% of the users transmitting with an outage probability* $P_{\rm out,1} = 0.1$ *and a signal-to-noise ratio* $\bar{\Gamma} = 10$ *dB; the second one has 10% of the users transmitting with an outage probability* $P_{\rm out,2} = 0.01$ *and a signal-to-noise ratio* $\bar{\Gamma} = 13$ *dB. The channel statistics are log-normal with log-standard deviation* $\sigma = 2$ *dB, and* $\alpha = 0.2$, 0.5, 1, *and* 2.

IV. Ergodic fading

Assume Rayleigh fading; from (9) we have $F_z(x) = (1-\exp(-x))u(x)$, which yields

$$\begin{aligned} C &= \int_0^\infty \log_2(1+x\xi\Gamma)e^{-x}dx \\ &= e^{1/(\xi\Gamma)}\,\mathrm{Ei}(1,1/(\xi\Gamma))\log_2 e \qquad (22)\end{aligned}$$

where $\xi = \xi^{(s)}$ or $\xi = \xi^{(m)}$, depending on the receiver used.

A. System throughput

Fig. 2 shows both C and η vs. α at $\Gamma = 20$ dB, for Rayleigh fading and a channel with no fading. This chart illustrates the behavior of the system throughput as the channel load increases. The following effects (which are generally present irrespectively of the fading dsitribution) may also be observed.

- As $\alpha \to \infty$, with both receivers η approaches the same limit, viz., $\log_2 e = 1.44$ bit/symbol, regardless of Γ. In fact, as $\alpha \to \infty$ with the SUMF receiver we have

$$\xi^{(s)} = \frac{1}{1+\alpha\Gamma} = (\Gamma\alpha)^{-1} + o(1/\alpha)$$

and with the MMSE receiver, after rearranging terms

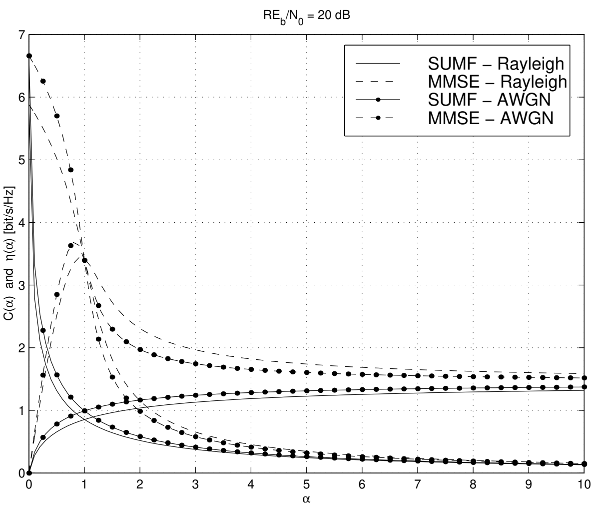

Fig. 2: *Plot of $C(\alpha)$ and $\alpha\,C(\alpha)$ vs. α for the MMSE (dashed) and SUMF (solid) receiver for $\Gamma = 20$ dB.*

in (5) we have

$$\alpha = \frac{1-\xi^{(m)}}{\displaystyle\int_0^\infty \frac{x\Gamma\xi^{(m)}}{1+x\Gamma\xi^{(m)}}\,dF_z(x)} \tag{23}$$

which, for $\alpha \to \infty$, yields $\xi^{(m)} = (\alpha\Gamma)^{-1} + O(1/\alpha)$. Then, for any fading distribution, $\eta \to \log_2 e$ as $\alpha \to \infty$. This is the same limit achieved over a non-fading (Gaussian) channel: thus, fast fading does not decrease the system throughput in the limit for large channel load.

- The maximum system throughput with SUMF is achieved for $\alpha \to \infty$. On the contrary, with MMSE receivers an optimal $\alpha = \alpha_{\rm opt}$ exists which maximizes η. Fig. 3 shows the optimal system throughput and the optimal load versus the SNR Γ for the AWGN and the Rayleigh fading channel. With both channel models, for high SNR the system throughput increases linearly. For low SNR, the Rayleigh fading channel has a marginally higher throughput.
  Notice that $\alpha_{\rm opt}$ exhibits a minimum (for Rayleigh fading, this occurs at $\Gamma = 30.9$ dB). This rather unexpected behavior can be explained by noting that for low SNR, the MMSE receiver approaches the SUMF (noise dominates multiple-access interference in this case), and the system capacity is maximum when α is large. For high SNR, the MMSE receiver approaches the decorrelating detector [8] (this is the optimal linear receiver in the absence of noise), and system capacity is maximum for $\alpha \to 1$. Moreover, α must tend to 1 from below, since for large Γ we get $\alpha \simeq 1 - \xi^{(m)}$ with $\xi^{(m)} \le 1$ (see (23)). Then, by continuity, α must have a minimum for some $0 < \Gamma < \infty$.
- With SUMF both capacity and throughput decrease when Rayleigh fading affects transmission in addition to Gaussian noise. However, with MMSE and high values of the channel load Rayleigh fading causes a modest increase of both capacity and throughput (a similar effect was observed in [2, Fig. 1]).

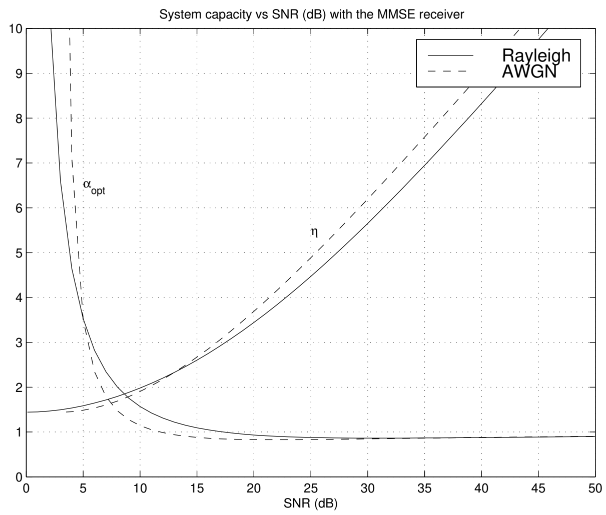

Fig. 3: *System capacity $C_{\rm sys}$ and optimal α versus the signal-to-noise ratio Γ for the MMSE receiver: AWGN and Rayleigh fading channel.*

B. Spreading-coding trade-off

In a non-asymptotic CDMA system with total bandwidth W and user information bit-rate R_b, the total bandwidth expansion of W/R_b dimensions/bit should be apportioned between spreading and coding, so that $L/R = W/R_b$, where R is the user coding rate, expressed in information bits per symbol, and L is the spreading factor, expressed in dimensions per symbol. Our asymptotic analysis offers some insight into the optimal trade-off between spreading and coding. For a system based on SUMF, the system throughput is maximized for $\alpha \to \infty$. This implies a very large number of users, each of which transmitting at a low coding rate. In this case, as it is well-known, the whole bandwidth expansion should be devoted to (low-rate) coding, while devoting a minimum amount of spreading to acquisition and synchronization. On the contrary, for a system equipped with an MMSE receiver, we observe that there is an optimal finite α. Then, the optimal coding rate should be close to C calculated for $\alpha = \alpha_{\rm opt}$ and, consequently, the optimal spreading gain is given by $L_{\rm opt} = (W/R_b)C(\alpha_{\rm opt})$. This should provide guidelines for the dimensioning of practical (large) CDMA systems.

V. Conclusions

We have examined a CDMA system operating on a channel affected by frequency-flat fading. The receiver consists of either a SUMF or an MMSE filter. We have treated separately the cases of slow and fast fading, yielding non-ergodic and ergodic channels, respectively. In the non-ergodic case, we have studied the outage probability, i.e., the probability that the actual SINR is below a required value. In the ergodic case, performance was expressed in terms of channel capacity and user throughput. Among our findings, we have proved that as the signal-to-noise ratio gets large the SUMF receiver exhibits an error floor for all channel loads, while the MMSE receiver does not whenever $\alpha < 1$. On the contrary, with fading, as $\alpha \to \infty$ the system capacity with SUMF and MMSE tends to the same limit of 1.44 bit/symbol as for the non-fading channel. We also address the tradeoffs involved in the allocation of available bandwidth between spreading and coding, and compare the decrease in capacity and the increase in optimum loads due to the presence of fading.

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