RQ Precoding for the Cooperative Broadcast Channel

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***Abstract*—We consider a broadcast channel with multiple transmit antennas, where receiving terminals (users), with single antenna, can cooperate by exchanging information through a secondary local channel.**

Similar to zero-forcing dirty paper coding (ZF-DPC) in [1], we consider a RQ precoding, which is based on the RQ decomposition of the channel matrix and transforms the channel matrix into an upper triangular matrix. Without DPC, the RQ precoding is used in conjunction with successive interference cancelation by the users, which send to each other their respectively detected information symbols.

We conduct error probability performance analysis for the RQ precoded cooperative broadcast channel and show how errors on the local link degrade the overall performance. Finally we compare the RQ precoding to standard beamforming precoding where the users do not cooperate and observe: (i) improvements in the overall error rates and throughput, (ii) different diversity gains for the different users, (iii) performance degradation resulting in an error floor due to errors on the local channels. This last feature enables either the application of RQ precoding to a system with different quality of service users or the possibility of scheduling the users in order to improve the overall error rates and throughput.

I. Introduction

Recently, great research interest focuses on precoding techniques for the broadcast channel. In [1], for the case of two transmit antennas and two users (one receive antenna per user), information theoretic results show that optimal sum-rate is achievable by using a zero-forcing dirty paper coding (ZF-DPC) based on QR decomposition. The QR decomposition converts the channel matrix into a lower triangular form, or equivalently, a series of sub-channels, where the interference to subsequent sub-channels is known. Then dirty paper coding is used to mitigate the effect of this known interference. Extension to an arbitrary number of users can be found in [1], [2], [3]. Detailed surveys on general precoding schemes can be found in [4], [5], [6].

The above research results focus on the non-cooperative broadcast channels only. More recently research interest shifted from the non-cooperative broadcast channel to the cooperative broadcast channel (see [7], [10] and references therein). The cooperative broadcast channel refers to the scenario where 1) the transmitter has multiple transmit antennas, 2) the receiving terminals (users) have single antennas, and 3) the receiving users can cooperate by exchanging information through a secondary local channel.

A realistic scenario where cooperation could be envisaged relies on the availability of a reliable local channel between the users. Assuming the users are found in a short range from each other, a Bluetooth link could provide such a reliable local link without affecting the main broadcast channel based on different radio technology. The short range link should have a sufficient capacity to accommodate all the all information exchanges within one transmission slot.

For this cooperative broadcast channel, the RQ precoding scheme is considered in our paper. The RQ precoding is based on the RQ decomposition of the channel matrix and yields an equivalent upper triangular channel matrix. This decomposition is similar to the LQ decomposition used for ZF-DPC [1], where the equivalent channel matrix is in a lower triangular form.

We compared the RQ precoding technique to the standard ZF beamforming precoding, where the users do not cooperate. We conduct performance analysis in terms of bit error rate (BER) and then we observe: (i) improvements in the overall error rates and throughput, (ii) different diversity gains for the different users, (iii) performance degradation resulting in an error floor due to errors on the local channels. This last feature enables either the application of RQ precoding to in a system with different quality of service users or the possibility of scheduling the users in order to improve the overall error rates and throughput.

The rest of the paper is organized as follows. Section II introduces the system model. Section III presents some properties of RQ precoding. In Section IV, performance analysis in terms of BER was proposed for RQ precoded cooperative broadcast channels. Section V shows simulation results. Conclusions are drawn in Section VI.

II. System model

We consider a broadcast channel with multiple transmit antennas, where receiving terminals (users), with single antennas, can cooperate by exchanging information through a secondary local channel. For example, we may find such a scenario in the downlink of a cellular system, where a group of users can be locally connected through a Bluetooth link.

Consider a transmitter with n antennas in a broadcast (or downlink) system with $K \leq n$ users with single antennas (see Fig.1). Let $\mathbf{x} = (x_1, \dots, x_n)^T$ be the vector of symbols

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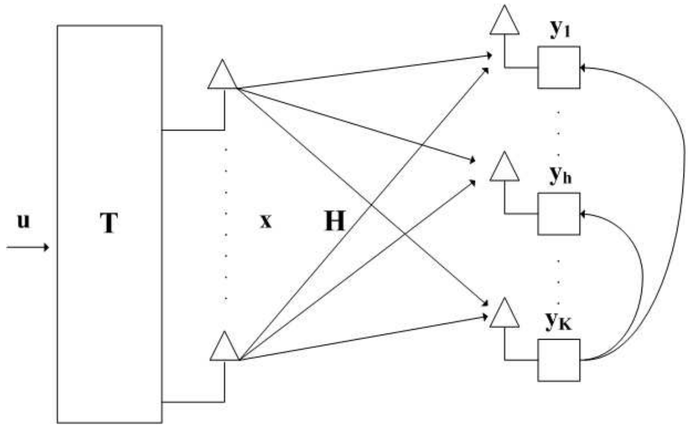

Fig. 1. Broadcast Channel

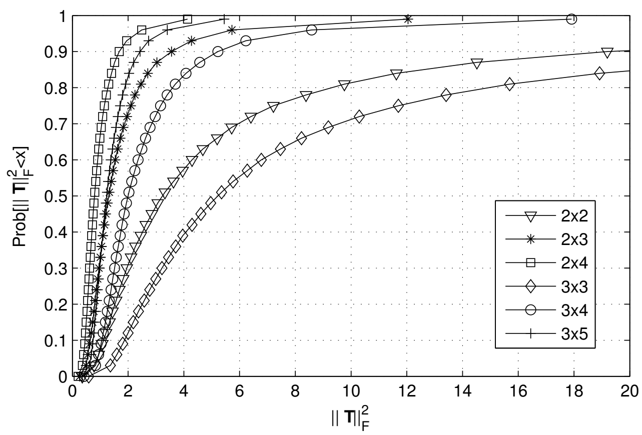

Fig. 2. C.d.f. of the power enhancement with beamforming

sent by the transmitting antennas and let $\mathbf{H} = (h_{ij})$ be the $K \times n$ channel coefficient matrix between the j-th transmit antenna and the i-th user. Using the standard Rayleigh flat fading model we assume $h_{ij} \approx \mathcal{N}_c(0, 1)$ i.e., i.i.d. complex Gaussian random variables with zero mean and unit variance. The received symbols by the K users are then given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$$

Let $\mathbf{T}$ be the $n \times K$ precoding matrix which is applied to an information vector $\mathbf{u} = (u_1, \ldots, u_K)^T$ to yield the transmitted vector

$$\mathbf{x} = \mathbf{T}\mathbf{u} \ .$$

The component u_k represents the information symbol for user k and is assumed to be taken from a QAM constellation.

A. Beamforming

The *ZF-beamforming* precoder is given by the right pseudo-inverse of $\mathbf{H}$

$$\mathbf{T} = \mathbf{H}^\dagger(\mathbf{H}\mathbf{H}^\dagger)^{-1}$$

where $\dagger$ denotes the Hermitian transpose. This precoder enables the users to operate independently. In fact, the beamforming precoder yields K independent parallel channels between the transmitter and the K users

$$\mathbf{y} = \mathbf{H}\mathbf{T}\mathbf{u} + \mathbf{z} = \mathbf{u} + \mathbf{z}$$

where $\mathbf{z}$ is the additive white Gaussian noise vector with i.i.d. elements $z_k \approx \mathcal{N}_c(0, N_0)$. The benefit of a simple detection for the users comes at the price of an increase in the average transmitted power by

$$10 \log_{10}(\mathbb{E}\|\mathbf{T}\|_F^2) \text{ dB}$$

where $\|\mathbf{T}\|_F^2$ denotes the Frobenius norm of $\mathbf{T}$. For example we show in Fig. 2 the c.d.f. of $\|\mathbf{T}\|_F^2$ for $K \times n = 2 \times 2$, $2 \times 3, 2 \times 4, 3 \times 3, 3 \times 4, 3 \times 5$. We can observe how the power enhancement increases with the number of users but decreases when the number of transmit antennas increases. We conclude that the main problems of ZF-beamforming are both the increase in the average transmitted power and the non-bounded peak transmitted power.

B. RQ precoding

Let us now consider the RQ precoding for a system where the users can exchange information on a local channel. We assume that the local channel is a short range link between the K users and is based on a different radio technology (e.g. Bluetooth) from the main broadcast channel (e.g. of a cellular system).

In this case, the transmitter computes the RQ decomposition of the channel matrix $\mathbf{H} = \mathbf{R}\mathbf{Q}$, where $\mathbf{R} = (r_{ij})$ is a $K \times K$ upper triangular matrix and $\mathbf{Q}$ is a $K \times n$ unitary matrix (i.e., $\mathbf{Q}\mathbf{Q}^\dagger = \mathbf{I}_K$) Then, the precoding matrix is given by

$$\mathbf{T} = \mathbf{Q}^\dagger$$

and the resulting equivalent channel becomes

$$\mathbf{y} = \mathbf{H}\mathbf{T}\mathbf{u} + \mathbf{z} = \mathbf{R}\mathbf{Q}\mathbf{Q}^\dagger\mathbf{u} + \mathbf{z} = \mathbf{R}\mathbf{u} + \mathbf{z}$$

The triangular structure of $\mathbf{R}$ suggests the following successive cancelation strategy: the K-th user estimates the information symbol $\hat{u}_K$ from its received signal

$$y_K = r_{KK}u_K + z_K$$

The estimated symbol $\hat{u}_K$ of user K is broadcast to the other $K-1$ users over the local channel. Let us first assume that $\hat{u}_K$ is correctly received by user $K-1$, then this user can cancel the interference as

$$y'_{K-1} = y_{K-1} - r_{K-1,K}u_K = r_{K-1,K-1}u_{K-1} + z_{K-1}$$

and estimates $\hat{u}_{K-1}$. User $K-1$ can then broadcast $\hat{u}_{K-1}$ to the remaining $K-2$ users. In general, user $1 \leq k < K$ will be able to cancel the previous $K-k$ interference terms as

$$y'_k = y_k - \sum_{m=k+1}^{K} r_{k,m}u_m = r_{kk}u_k + z_k \tag{1}$$

and broadcast the estimated symbol $\hat{u}_k$ to the remaining $k-1$ users.

If no error occurs both in the estimates $\hat{u}_k$ and in the local cooperative channel then the system behaves exactly like a set of parallel K fading channels with fading coefficients r_{kk}. In all other cases some error propagation occurs. In particular we observe that a wrong estimate $\hat{u}_k$ is either due to a detection error at the k-th user and/or to an error occurring on the local channel. We will show in Sec. IV that the first cause of error has a limited effect, while the second results in an error floor.

III. Properties of RQ precoding

In this section we find the analytical expression of the symbol error probability of the K users.

In order to do this, let us first observe the relation between the RQ and the QR decomposition.

Lemma 1: Let $\mathbf{H} = \mathbf{R}\mathbf{Q}$ and $\tilde{\mathbf{H}} = \mathbf{H}^T\mathbf{P} = \tilde{\mathbf{Q}}\tilde{\mathbf{R}}$ be the RQ and the QR decompositions of the $K \times n$ matrix $\mathbf{H}$ and $\tilde{\mathbf{H}}$, respectively, then

$$\mathbf{R} = (\mathbf{T}\tilde{\mathbf{R}}\mathbf{P})^T \quad \text{and} \quad \mathbf{Q} = (\tilde{\mathbf{Q}}\mathbf{T})^T$$

where

$$\mathbf{P} = \begin{pmatrix} 0 & & 1 \\ & \ddots & \\ 1 & & 0 \end{pmatrix} \qquad \mathbf{T} = \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{n-K} \end{pmatrix}$$

Proof – We first note that the matrices $\mathbf{P}$ and $\mathbf{T}$ are symmetric ($\mathbf{P} = \mathbf{P}^T$ and $\mathbf{T} = \mathbf{T}^T$) and idempotent ($\mathbf{P}^2 = \mathbf{I}$ and $\mathbf{T}^2 = \mathbf{I}$). Note that right multiplication of by $\mathbf{P}$ swaps the order of the columns of $\mathbf{H}^T$. Then we have

$$\begin{aligned} \mathbf{H} &= (\mathbf{H}^T\mathbf{P}^2)^T \\ &= (\tilde{\mathbf{H}}\mathbf{P})^T \\ &= (\tilde{\mathbf{Q}}\tilde{\mathbf{R}}\mathbf{P})^T \\ &= \mathbf{P}^T\tilde{\mathbf{R}}^T\tilde{\mathbf{Q}}^T \\ &= \mathbf{P}^T\tilde{\mathbf{R}}^T\mathbf{T}^T\mathbf{T}^T\tilde{\mathbf{Q}}^T \\ &= (\mathbf{T}\tilde{\mathbf{R}}\mathbf{P})^T(\tilde{\mathbf{Q}}\mathbf{T})^T \\ &= \mathbf{R}\mathbf{Q} \end{aligned}$$

QED

Lemma 2.1 in [9] gives the distribution of the random elements of the triangular matrix $\mathbf{R}$ in the QR decomposition of a standard complex Gaussian matrix and, in particular, it states that the random variables $|r_{kk}|^2$ are distributed as a chi-square distribution with $2(K-k+1)$ degrees of freedom.

Hence, combining the Lemma 2.1 in [9] with Lemma 1 we obtain the following

Lemma 2: Let $\mathbf{H}$ be a Gaussian complex random matrix of size $K \times n$ ($K \leq n$) with i.i.d. zero mean and unit variance elements. Consider its RQ decomposition $\mathbf{H} = \mathbf{R}\mathbf{Q}$, then the elements of $\mathbf{R}$ are independent and in particular the ones on the diagonal r_{kk} are such that $\alpha = |r_{kk}|^2$ are distributed as a chi-square distribution with $2k$ degrees of freedom

$$f_{\chi^2_{2k}}(\alpha) = \frac{1}{(k-1)!}\alpha^{k-1}e^{-\alpha} \qquad \alpha \geq 0$$

Proof – Given an complex standard Gaussian random matrix $\mathbf{H}$, the matrix $\tilde{\mathbf{H}}$ in Lemma 1 has the same statistical properties, since it is simply a permutation of the matrix elements. After RQ decomposition of $\mathbf{H}$, the elements on the diagonal of $\mathbf{R}$ are in reverse order of the ones on the diagonal of $\tilde{\mathbf{R}}$ obtained from the QR decomposition of $\tilde{\mathbf{H}}$ and this concludes the proof. QED

Remark – The above Lemma is similar to the one in [1], where it is applied to the lower triangular matrix $\mathbf{G}$ in the Gram-Schmidt orthogonal decomposition of $\mathbf{H} = \mathbf{G}\mathbf{Q}$. In that case the order of successive cancelation is reversed.

IV. Performance Analysis

In this section we provide the analytical symbol error performance $P(e_k)$ of each user. We note that in a point-to-point system where all users are co-located to form a multiple receive antenna array we would be interested in the total error probability which is given by

$$P(e) = \sum_{k=1}^{n} P(e_k)$$

In the RQ precoded system, when the interference from other users is canceled, the k-th receiver gets:

$$y_k = r_{kk}u_k + z_k \qquad k = 1, \ldots, n$$

The k-th user will observe an instantaneous signal-to-noise ratio SNR_k :

$$\text{SNR}_k = |r_{kk}|^2\frac{E_s}{N_0}$$

where E_s is the average information symbol energy. Let us consider for simplicity the case of a QPSK modulation. The exact symbol error probability conditioned on $\alpha = |r_{kk}|^2$ is given by

$$P(e|\alpha) = 2Q\left(\sqrt{\alpha\frac{E_s}{N_0}}\right) - Q\left(\sqrt{\alpha\frac{E_s}{N_0}}\right)^2 \qquad (2)$$

where $Q(x)$ is the Gaussian tail probability function defined as:

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}dt$$

Finally, the average symbol error probability for user k is:

$$\begin{aligned} P(e_k) &= E_\alpha[P(e|\alpha)] \\ &= \int_0^\infty f_{\chi^2_{2k}}(\alpha)\left[2Q\left(\sqrt{\alpha\frac{E_s}{N_0}}\right) - Q\left(\sqrt{\alpha\frac{E_s}{N_0}}\right)^2\right]d\alpha \end{aligned}$$

Unfortunately, the above expression cannot be evaluated in closed form. The $P(e|\alpha)$ in (2) can be tightly upper bounded as

$$P(e|\alpha) < 2Q\left(\sqrt{\alpha\frac{E_s}{N_0}}\right) \qquad (3)$$

Instead, by using the upper bound in (3) we obtain:

$$P(e_k) < \int_0^\infty f_{\chi^2_{2k}}(\alpha)2Q\left(\sqrt{\alpha\frac{E_s}{N_0}}\right)d\alpha$$

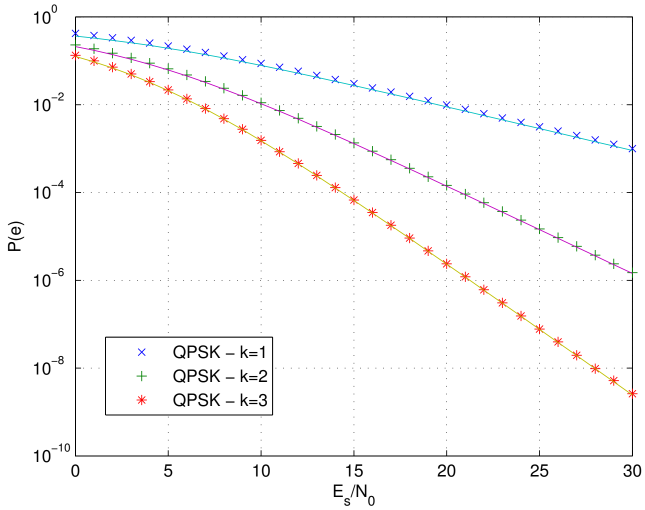

Fig. 3. Error probability of each user with QPSK: comparison of exact expressions (continuous lines) and upper bounds (markers)

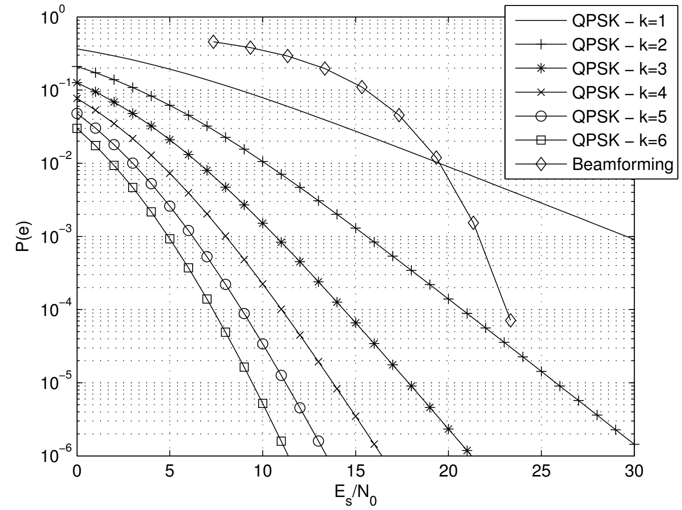

Fig. 4. Error probability of each user with QPSK compared to ZF beamforming

which can be evaluated in closed form for any k and yields:

$$
\begin{aligned}
P(e_1) &< 1-\frac{1}{\sqrt{1+\frac{2N_0}{E_s}}} \approx \frac{1}{\frac{E_s}{N_0}} \qquad (4)\\
P(e_2) &< 1-\frac{3+\frac{E_s}{N_0}}{(2+\frac{E_s}{N_0})\sqrt{1+\frac{2N_0}{E_s}}} \approx \frac{3}{2(\frac{E_s}{N_0})^2}\\
P(e_3) &< 1-\frac{15+5\frac{E_s}{N_0}+(\frac{E_s}{N_0})^2}{2(2+\frac{E_s}{N_0})^2\sqrt{1+\frac{2N_0}{E_s}}} \approx \frac{5}{2(\frac{E_s}{N_0})^3}\\
&\cdots
\end{aligned}
$$

These provide a tight upper bound on the symbol error probability of the k-th user in the proposed system. The approximation for $\frac{E_s}{N_0} \to \infty$ shows that the diversity order for user k is k (i.e., the power of E_s/N_0 is $-k$). In the Figure 3 we show the symbol error rate for three users and compare the exact error probability expression computed by numerical integration of (2), to the upper bound expression using (4), which can be computed in a closed form. We note a small gap only for user 1 at low $\frac{E_s}{N_0}$.

V. Simulation results

In this section, the performance of the proposed RQ decomposition precoding technique is investigated with simulations. We specify a system in which the number of transmit antennas at the BS is n and the number of users is $n = K$. We first compare the system with $K = 6$ users to the ZF beamforming in Fig. 4. We observe how the power enhancement shifts the error curve to the right and that the error probabilities of each user in the cooperative system are significantly better in a wide range of $\frac{E_s}{N_0} < 20$dB. This figure also suggests that a system with $n = 6$ transmit antenna and only $K = 4$ users could outperform beamforming by over 10dB at 10^{-2}.

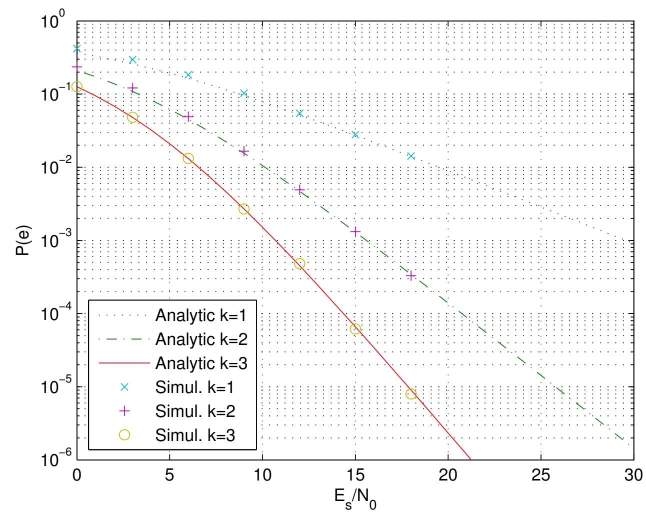

Fig. 5. Effects of error propagation on the error probability of each user with QPSK

Let us now focus on the effect error propagation in the successive interference cancelation. A wrong estimate $\hat{u}_k$ is either due to (i) a detection error at the k-th user or to (ii) an error occurring on the local channel.

In Fig. 5 we show the effect of error propagation from one user to the next due to a detection error, hence assuming the local channel does not introduce any errors. We can observe how the degradation is marginal. This can be explained by the fact that the first decisions are the most reliable thanks to the higher diversity.

Finally, in Fig. 6 we show the effect of the errors introduced on the local channel where we assume a bit error probability of 10^{-2} and 10^{-3}. We note that in this case an error floor appears for all the user except for the K-th one (the first one

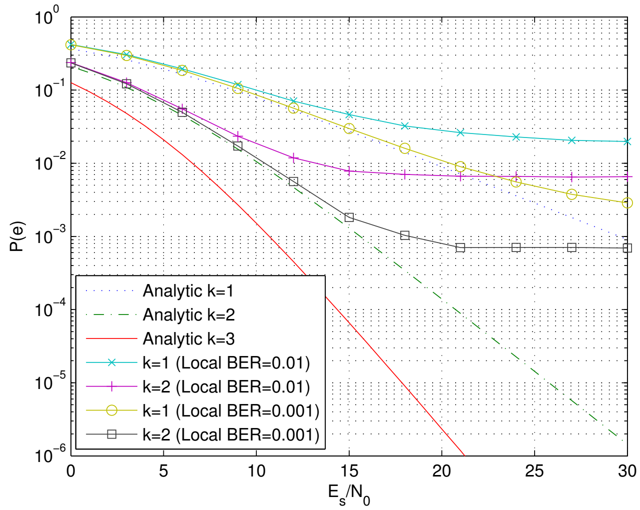

Fig. 6. Effects of local channel errors on the error probability of each user with QPSK

to decode).

VI. Conclusion

Cooperative broadcast channels are considered in this paper. Similarly to ZF-DPC in [1], we use a RQ precoding, which is based on the RQ decomposition of the channel matrix and transfer the channel matrix into an upper triangular matrix. Then successive interference cancelation is used to remove the interference between each other. We compare the RQ precoding to standard beamforming precoding where the users do not cooperate in terms of BER analysis. We observe: (i) improvements in the overall error rates and throughput, (ii) different diversity gains for the different users, (iii) performance degradation resulting in an error floor due to errors on the local channels. The different diversity enables either the application of RQ precoding to in a system with different quality of service users or the possibility of scheduling the users in order to improve the overall error rates and throughput.

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