Optimal Transmission Strategy in Full-duplex Relay Networks

Alessandro Nordio
CNR-IEIIT, Italy

Carla-Fabiana Chiasserini
Politecnico di Torino and CNR-IEIIT, Italy

Emanuele Viterbo
ECSE, Monash University, Australia

***Abstract*—In this work, we consider a dual-hop, decode-and-forward network where the relay can operate in FD mode. We model the residual self interference as an additive Gaussian noise with variance proportional to the relay transmit power, and we assume a Gaussian input distribution at the source. Unlike previous work, however, we assume that the source is only aware of the transmit power distribution adopted by the relay over a given time horizon, not of the symbols that the relay is currently transmitting. This scenario better reflects practical situations in which the relay node may also have to forward signaling traffic, or data originated by other sources. Under these conditions, we show that the optimal communication strategy that source and relay can adopt is a time-division scheme, and, for each slot, we determine the optimal transmit power level that source and relay should adopt depending on the channel gains. Interestingly, the distribution of the optimal transmit power turns out to be discrete with two probability masses.**

I. Introduction

Recent advances in self-interference suppression in full-duplex (FD) systems has made such a technology, and its use in relay networks, very attractive. In this work, we consider a dual-hop, decode-and-forward network where the relay can operate in FD mode, and we model the residual self interference as an additive Gaussian noise with variance proportional to the relay transmit power [1]–[4].

The capacity of such system was recently presented in [4], where it is shown that the conditional probability distribution of the source input, given the relay input, is Gaussian while the optimal distribution of the relay input is either Gaussian or symmetric discrete with finite mass points. This result implies that a capacity achieving scheme requires the source to know at each time instant what the relay is transmitting. This can be realized with the aid of a buffer at the relay, which holds the data previously transmitted by the source and correctly decoded by the relay. The relay re-encodes such data before forwarding it to the destination in the next available channel use. The source can use the same encoder as the relay, in order to know what will be transmitted by the relay and hence guarantee a capacity achieving transmission.

Different from the above scenario, in this paper we consider the case where the source does not know what symbols are transmitted by the relay and only has the transmit power distribution adopted by the relay over a given time horizon. In such a case, this knowledge is exploited by the source in order to optimally set its own transmit power and decide whether the relay should operate in half duplex or full duplex. Therefore, our scenario can accommodate the case where the relay node has to handle multiple, simultaneous traffic flows, e.g., in-band signaling as well as data traffic originated at the relay itself or previously received from other sources.

Additionally, we consider that the average transmit power at the source and at the relay are constrained to some target values. Under this scenario, we find that the optimal network communication strategy is based on time-slot division and we derive the optimal self-interference distribution at the relay. Such distribution depends on the channel gains and turns out to be discrete, composed of either one or two delta functions, depending on the target value of average transmit power at source and relay. We then obtain the power allocation policy at the relay and at the source that allows the system to achieve the maximum data rate.

II. System model

We consider a two-hop relay network including a source node s, a relay r and a destination d. All network nodes are equipped with a single antenna, and the relay is assumed to be FD enabled. No direct link exists between source and destination, thus information delivery from the source to the destination necessarily takes place through the relay. We remark, however, that source and relay do not need to be synchronized on per-symbol basis, and that the relay can handle multiple (data or control) traffic streams originated at different network nodes, according to any scheduling scheme of its choice. This implies that the source is not required to have full knowledge about the information the relay is transmitting. We assume instead that the source is aware of the transmit power distribution adopted by the relay over a given time horizon. As explained later in the paper, such knowledge is exploited by the source in order to optimally set its own transmit power. We also consider that the instantaneous transmit power at the relay is limited by a maximum value $p^{\max}$.

As far as the channel is concerned, we consider independent, memoryless block fading channels with additive Gaussian noise, between source and relay as well as between relay and destination. Additionally, when the relay transmits to the destination, a residual self interference (after analog and digital suppression) adds up to what the relay receives from the source.

This work was supported by the FLAG projects and by ARC under Grant Discovery Project No. DP160101077

978-1-5090-3097-2/17/$31.00 ©2017 IEEE

Then the signal received at the relay and destination can be written as:

$$\begin{aligned} y_r &= \sqrt{P} h_1 x_s + \nu + n_r \\ y_d &= \sqrt{p} h_2 x_r + n_d \end{aligned}$$

where

- h_1 and h_2 are the channel gains associated with, respectively, the source-relay and relay-destination links;
- P and p are the source and relay transmit powers, respectively;
- x_s and x_r are the input symbols transmitted by, respectively, the source and the relay. We assume the input at both source and relay to be zero-mean complex Gaussian distributed, with $\mathbb{E}[|x_s|^2] = \mathbb{E}[|x_r|^2] = 1$ where $\mathbb{E}[\cdot]$ is the expectation operator;
- n_r and n_d represent zero-mean complex Gaussian noise over, respectively, the source-relay and the relay-destination links, with $\mathbb{E}[|n_r|^2] = N_0$ and $\mathbb{E}[|n_d|^2] = N_0$;
- ν represents the instantaneous residual self interference at the relay. As typically done in previous studies [1]–[4], we model ν as a Gaussian noise whose power is proportional to the transmission power at the relay, i.e., $\mathbb{E}[|\nu|^2] = N_\nu$. Note that: $N_\nu = \beta p \leq \beta p^{\max}$, where β denotes the self-interference attenuation factor at the relay. Also, we remark that, as shown in [4], assuming ν as a zero-mean i.i.d. Gaussian random variable represents the worst-case linear residual self-interference model.

Since the relay transmit power may vary over time, the relay power p can be modeled as a time-varying continuous random variable. We define $f(p)$ as the probability density function of p, whose support is $[0, p^{\max}]$.

Finally, as often required in, e.g., energy harvesting systems, we consider that the long-run average power at the source and at the relay is constrained to given target values, denoted by $\bar{p}$ and $\bar{P}$, respectively. The average power at the relay is therefore given by:

$$\bar{p} = \int_0^{p^{\max}} p f(p) \, \mathrm{d}p \tag{1}$$

III. Problem formulation

In our study, we aim at determining the power allocation at the source and relay that maximizes the achievable rate of the dual-hop network described above. To this end, we start by recalling some fundamental concepts:

(a) the network rate will be determined by the minimum between the rate achieved over the source-relay link and that achieved over the relay-destination link, hereinafter referred to as R_1 and R_2, respectively;

(b) R_1 depends on the source transmit power as well as on the residual self interference at the relay, which, on its turn, depends on the relay transmit power;

(c) R_2 depends on the relay transmit power;

(d) the transmit power at source and relay may vary over time. Whenever $P > 0$, $p > 0$ corresponds to the relay working in FD mode, while $p = 0$ corresponds to the relay being in receiving mode; instead, when $P = 0$ and $p > 0$, the relay is transmitting while the source is silent.

Due to the dependency that the residual self interference creates between the performance of the first and second hop, it is clear that, in order to maximize the network rate, source and relay should coordinate their power allocation strategies.

In our study, we take the distribution of the power at the relay ($f(p)$) as the driving factor, based on which the power allocation, hence the network rate, can be optimized. As a first step, we write the source power as a function of p, $P(p)$, which is subject to the following constraint:

$$\int_0^{p^{\max}} f(p) P(p) \, \mathrm{d}p = \bar{P} \, . \tag{2}$$

As a second step, we fix $f(p)$ and derive the expressions of the rates R_1 and R_2 as detailed below.

Source-relay rate. The average rate on the source-relay channel is given by:

$$R_1 = \int_0^{p^{\max}} f(p) \log\left(1 + \frac{P(p)|h_1|^2}{N_0 + \beta p}\right) \mathrm{d}p \, . \tag{3}$$

Under the assumption that $f(p)$ is given, the above rate can be maximized with respect to $P(p)$. It can be shown (see Appendix A in [5]) that, given $f(p)$, the source transmit power maximizing R_1 is given by:

$$P(p) = \frac{\beta}{|h_1|^2} [\omega - p]^+ \tag{4}$$

where ω is a parameter which satisfies the average transmit power constraint, i.e.,

$$\int_0^{p^{\max}} f(p) [\omega - p]^+ \, \mathrm{d}p = \bar{\mathcal{P}} \tag{5}$$

For the sake of notation simplicity, in the above expression we defined $\bar{\mathcal{P}} = \bar{P} \frac{|h_1|^2}{\beta}$. By substituting (4) in (3) and by defining $\beta_0 = \frac{\beta}{N_0}$, we get

$$\begin{aligned} R_1 &= \int_0^{p^{\max}} f(p) \log\left(1 + \beta \frac{[\omega - p]^+}{N_0 + \beta p}\right) \mathrm{d}p \\ &= \int_0^{p^{\max}} f(p) \log\left(1 + \frac{\beta_0}{1 + \beta_0 p} [\omega - p]^+\right) \mathrm{d}p \, . \end{aligned}$$

Relay-destination rate. The average rate achieved on the relay-destination channel is given by:

$$R_2 = \int_0^{p^{\max}} f(p) \log\left(1 + v_2 p\right) \mathrm{d}p \tag{6}$$

where $v_2 = \frac{|h_2|^2}{N_0}$. Next, having expressed the source power as a function of p, we need to find the optimal distribution $f(p)$ that maximizes the network data rate. We therefore formulate the following optimization problem, subject to the system constraints:

978-1-5090-3097-2/17/$31.00 ©2017 IEEE

$$\textbf{P1:}\quad R = \max_{f(p)} \min\{R_1, R_2\} \qquad \text{s.t.}$$

$$(a)\quad R_1 = \int_0^{p^{\max}} f(p)\log\left(1+\frac{\beta_0[\omega-p]^+}{1+\beta_0 p}\right)\,\mathrm{d}p$$

$$(b)\quad R_2 = \int_0^{p^{\max}} f(p)\log\left(1+v_2 p\right)\,\mathrm{d}p$$

$$(c)\quad \int_0^{p^{\max}} f(p)[\omega-p]^+\,\mathrm{d}p = \bar{\mathcal{P}}$$

$$(d)\quad \int_0^{p^{\max}} p f(p)\,\mathrm{d}p = \bar{p}\,; \qquad (e)\quad \int_0^{p^{\max}} f(p)\,\mathrm{d}p = 1\,;$$

$$(f)\quad 0 \le p \le p^{\max}$$

In the above formulation,

- constraints (a) and (b) represent the average rates achieved on the source-relay and relay-destination links, respectively;
- (c) is the average power constraint at the source;
- (d) is the average power constraint at the relay;
- (e) grants that $f(p)$, being a distribution, integrates to 1;
- (g) limits the relay transmit power to $p^{\max}$.

In the following, due to the lack of room, we limit our analysis to the case $\omega \ge p^{\max}$.

IV. Optimal power allocation

Under the above assumptions, by using constraints *(c)*, *(d)* and *(e)* in **P1**, we obtain: $\omega = \bar{\mathcal{P}} + \bar{p}$. Since $\omega > p^{\max}$, this implies that a solution to problem **P1** exists if $\bar{\mathcal{P}} \ge \mathcal{P}_0 = p^{\max} - \bar{p}$. Moreover, by using the above expression for ω in *(a)* and in (4), we obtain

$$R_1 = \log\left(1+\beta_0(\bar{\mathcal{P}}+\bar{p})\right) - \int_0^{p^{\max}} f(p)\log(1+p\beta_0)\,\mathrm{d}p \quad (7)$$

and $P(p) = \frac{\beta}{|h_1|^2}[\bar{\mathcal{P}}+\bar{p}-p]$. Since (i) $\log(1+cp)$, $c>0$, is a concave function of p and (ii) $f(p)$ has average $\bar{p}$, we can apply Lemma B.1 reported in [5] and write:

$$R_1 \le r_1^{\max} = \log\left(1+\beta_0(\bar{\mathcal{P}}+\bar{p})\right) - \frac{\bar{p}}{p^{\max}}\log(1+p^{\max}\beta_0) \quad (8)$$

$$R_2 \ge r_2^{\min} = \frac{\bar{p}}{p^{\max}}\log(1+p^{\max}v_2) \quad (9)$$

with the equality holding when $f(p) = \left(1-\frac{\bar{p}}{p^{\max}}\right)\delta(p) + \frac{\bar{p}}{p^{\max}}\delta(p-p^{\max})$ where $\delta(\cdot)$ is the Dirac delta function. Similarly, by applying again Lemma B.1 in [5], we get:

$$\begin{aligned} R_1 &\ge r_1^{\min} = \log\left(1+\beta_0(\bar{\mathcal{P}}+\bar{p})\right) - \log(1+\bar{p}\beta_0) \\ R_2 &\le r_2^{\max} = \log(1+\bar{p}v_2) \end{aligned} \quad (10)$$

with the equality holding when $f(p) = \delta(p-\bar{p})$. We now consider the following three cases.

1) If $r_2^{\min} \ge r_1^{\max}$, then $R = r_1^{\max}$ and the optimal relay power distribution is $f^\star(p) = \left(1-\frac{\bar{p}}{p^{\max}}\right)\delta(p) + \frac{\bar{p}}{p^{\max}}\delta(p-p^{\max})$. Solving for $\bar{\mathcal{P}}$ the inequality $r_2^{\min} \ge r_1^{\max}$, we obtain $\bar{\mathcal{P}} \le \mathcal{P}_1 = \frac{1}{\beta_0}\left[(1+p^{\max}\beta_0)(1+p^{\max}v_2)\right]^{\frac{\bar{p}}{p^{\max}}} - \frac{1+\bar{p}\beta_0}{\beta_0}$ and $R = \log\left(1+\beta_0(\bar{\mathcal{P}}+\bar{p})\right) - \frac{\bar{p}}{p^{\max}}\log\left(1+p^{\max}\beta_0\right)$.
2) If $r_1^{\min} \ge r_2^{\max}$, then $R = r_2^{\max}$ and the optimal relay power distribution is $f^\star(p) = \delta(p-\bar{p})$. Solving for $\bar{\mathcal{P}}$ the inequality $r_1^{\min} \ge r_2^{\max}$, we get $\bar{\mathcal{P}} \ge \mathcal{P}_2 = \frac{\bar{p}v_2}{\beta_0}(1+\bar{p}\beta_0)$ and $R = \log\left(1+\bar{p}v_2\right)$.
3) Otherwise, we find solutions for $f(p)$ such that $R = R_1 = R_2$. Indeed, for $\mathcal{P}_1 \le \bar{\mathcal{P}} \le \mathcal{P}_2$, problem **P1** becomes:

$$\begin{aligned} \textbf{P2:}\quad R &= \log\left(1+\beta_0(\bar{\mathcal{P}}+\bar{p})\right) \\ &\quad - \min_{f(p)} \int_0^{p^{\max}} f(p)\log(1+p\beta_0)\,\mathrm{d}p \qquad \text{s.t.} \end{aligned}$$

$$(a)\quad \int_0^{p^{\max}} f(p)\log\left[(1+p\beta_0)(1+pv_2)\right]\mathrm{d}p = \log\left(1+\beta_0(\bar{\mathcal{P}}+\bar{p})\right)$$

$$(b)\quad \int_0^{p^{\max}} p f(p)\,\mathrm{d}p = \bar{p}$$

$$(c)\quad \int_0^{p^{\max}} f(p)\,\mathrm{d}p = 1\,.$$

In this case the minimizer of the functional can be found by applying the following theorem.

Theorem 4.1: Consider the following constrained minimization problem

$$\min_{f(p)} \int_a^b f(p)\phi(p)\,\mathrm{d}p \qquad \text{s.t.} \quad (11)$$

$$(a)\quad \int_a^b f(p)\psi(p)\,\mathrm{d}p = c$$

$$(b)\quad \int_a^b p f(p)\,\mathrm{d}p = m$$

$$(c)\quad \int_a^b f(p)\,\mathrm{d}p = 1$$

$$(d)\quad f(p) \ge 0, \forall p \in [a,b]$$

where $\phi(p) = \log(1+\gamma_1 p)$, $\eta(p) = \log(1+\gamma_2 p)$, $\psi(p) = \phi(p)+\eta(p)$, and $f(p)$ is a probability distribution with support in $p \in [a,b]$, $a>0$. Moreover, $\gamma_1 > 0$, $\gamma_2 > 0$ and c are constant parameters. Then the minimizer has the following expression

$$f^\star(p) = \begin{cases} \frac{p_2-m}{p_2-a}\delta(p-a) + \frac{m-a}{p_2-a}\delta(p-p_2) & \text{if } \gamma_1 \ge \gamma_2 \\ \frac{b-m}{b-p_1}\delta(p-p_1) + \frac{m-p_1}{b-p_1}\delta(p-b) & \text{if } \gamma_1 < \gamma_2 \end{cases} \quad (12)$$

where the constants $p_1 \in [a,m]$ and $p_2 \in [m,b]$ are obtained by replacing the first and second case, respectively, of (12) in the constraint *(a)* in (11).

Proof: The proof is given in [5, Appendix C]. ■

Through the above theorem and considering $v_2 \ge \beta_0$, the maximizer of the rate in **P2** is given by

$$f^\star(p) = \frac{p^{\max}-\bar{p}}{p^{\max}-p_1}\delta(p-p_1) + \frac{\bar{p}-p_1}{p^{\max}-p_1}\delta(p-p^{\max})$$

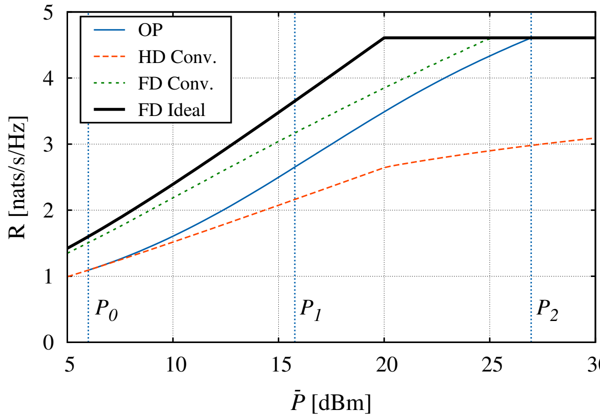

Fig. 1. Achieved rate vs. $\bar{\mathcal{P}}$ for $\bar{p} = 20\,\text{dBm}$, $p^{\max} = 23\,\text{dBm}$, $\beta = -135\,\text{dB}$.

where p_1 is obtained by replacing $f(p)$ with $f^\star(p)$ in constraint *(a)* in **P2**, i.e., by solving

$$\left[\frac{(1{+}p_1\beta_0)(1{+}p_1 v_2)}{k}\right]^{\frac{p^{\max}-\bar{p}}{p^{\max}-p_1}} = \frac{1{+}\beta_0(\bar{\mathcal{P}}{+}\bar{p})}{k}$$

with $k = (1{+}p^{\max}\beta_0)(1{+}p^{\max}v_2)$. When instead $v_2 < \beta_0$, the maximizer of the rate in **P2** is given by

$$f^\star(p) = \frac{p_2 - \bar{p}}{p_2}\delta(p) + \frac{\bar{p}}{p_2}\delta(p - p_2)$$

where p_2 is obtained again using $f^\star(p)$ in constraint *(a)*, i.e., by solving

$$[(1 + p_2\beta_0)(1 + p_2 v_2)]^{\frac{\bar{p}}{p_2}} = 1 + \beta_0(\bar{\mathcal{P}} + \bar{p})\,. \quad (13)$$

From the above results, we observe that the power allocation that leads to the maximum rate corresponds to source and relay operating according to a time division strategy consisting, in general, of two time slots. The time fraction associated to each slot, and the transmission power to be used at the source and relay during each slot, are given by the coefficient and the argument of the δ functions composing $f^\star(p)$.

To summarize, we report the solution of problem **P1**, for $\bar{\mathcal{P}} \geq \mathcal{P}_0$, in Table I. Looking at the top table, we remark that:

- for $\bar{\mathcal{P}} \leq \mathcal{P}_1$, during the first slot only the source transmits (i.e., the relay only receives) while in the second slot the relay operates in FD and transmits at maximum power;
- for $\mathcal{P}_1 \leq \bar{\mathcal{P}} \leq \mathcal{P}_2$, two cases are possible. When the relay-destination channel gain v_2 is greater than the self-interference attenuation β_0, the relay always operates in FD but both source and relay use different power levels in the two slots. Otherwise, the relay uses the same scheme as for $\bar{\mathcal{P}} \leq \mathcal{P}_1$ but its transmit power in the second slot is set to p_2;
- for $\bar{\mathcal{P}} \geq \mathcal{P}_2$, source and relay always transmit at their average power.

V. Results

We compare the performance of our proposed scheme against the ideal full duplex communication scheme (in the following referred to as ”FD Ideal”) where the relay does not suffer from self interference and whose expression is

$$R_{\text{FDIdeal}} = \min\left\{\log\left(1 + \frac{\bar{\mathcal{P}}|h_1|^2}{N_0}\right), \log\left(1 + \frac{\bar{p}|h_2|^2}{N_0}\right)\right\}$$

which is also reported in [4, eq.(38)]. We then consider the conventional full duplex scheme (referred to as ”FD Conv.”) where the relay always works in FD mode, the source always transmits with average power $\bar{\mathcal{P}}$ and has perfect knowledge of the instantaneous relay transmit power. The expression of the ”FD Conv.” scheme is [4, eq. (39)]:

$$\begin{aligned} R_{\text{FDConv}} \;=\; & \max_{p\leq\bar{p}}\min\left\{\int_{-\infty}^{+\infty}\log\left(1 + \frac{\bar{\mathcal{P}}|h_1|^2}{N_0 + \beta x^2}\right)\right., \\ & \left.\frac{\mathrm{e}^{-x^2/(2p)}}{\sqrt{2\pi p}}\,\mathrm{d}x, \log\left(1 + \frac{p|h_2|^2}{N_0}\right)\right\}. \quad (14) \end{aligned}$$

Furthermore, we compare to the conventional half duplex scheme (named ”HD Conv.”) whose expression is given by

$$\begin{aligned} R_{\text{HDConv}} \;=\; & \max_{\bar{p}/p^{\max}\leq t\leq 1}\min\left\{(1{-}t)\log\left(1{+}\frac{|h_1|^2\bar{\mathcal{P}}}{(1{-}t)N_0}\right)\right., \\ & \left. t\log\left(1 + \frac{\bar{p}|h_2|^2}{tN_0}\right)\right\} \quad (15) \end{aligned}$$

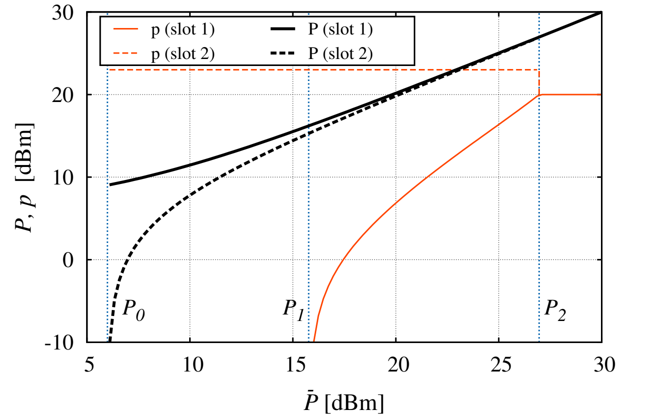

Fig. 2. Optimal source and relay transmit power for slot 1 (solid lines) and slot 2 (dashed lines) for the same scenario as in Fig. 1.

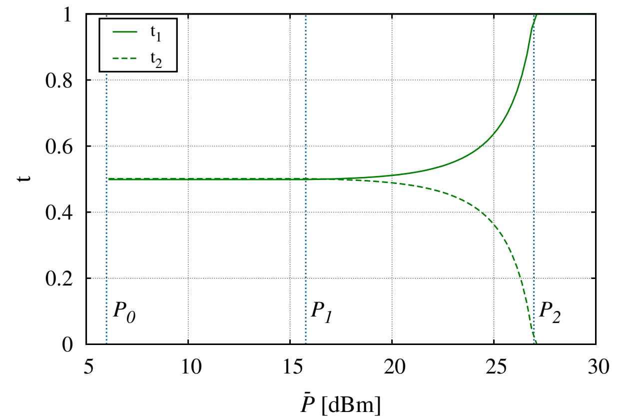

Fig. 3. Time fractions assigned to slot 1 (solid line) and slot 2 (dashed line) for the same scenario as in Fig. 1.

TABLE I
OPTIMAL POWER ALLOCATION AND RATE FOR $\mathcal{P} \geq \mathcal{P}_0$ WHERE p_1 IS THE SOLUTIONS OF (13). t_1 AND $t_2 = 1 - t_1$ ARE THE TIME FRACTIONS ASSOCIATED TO THE FIRST AND SECOND SLOTS, RESPECTIVELY

$\mathcal{P}$	Slot 1			Slot 2		
	t_1	P	p	t_2	P	p
$\bar{\mathcal{P}} \leq \mathcal{P}_1$	$1 - \frac{\bar{p}}{p^{\max}}$	$\frac{\beta}{\lvert h_1\rvert^2}(\bar{\mathcal{P}} + \bar{p})$	0	$\frac{\bar{p}}{p^{\max}}$	$\frac{\beta}{\lvert h_1\rvert^2}(\bar{\mathcal{P}} + \bar{p} - p^{\max})$	$p^{\max}$
$\bar{\mathcal{P}} \in (\mathcal{P}_1, \mathcal{P}_2)$; $v_2 \geq \beta_0$	$\frac{p^{\max} - \bar{p}}{p^{\max} - p_1}$	$\frac{\beta}{\lvert h_1\rvert^2}(\bar{\mathcal{P}} + \bar{p} - p_1)$	p_1	$\frac{\bar{p} - p_1}{p^{\max} - p_1}$	$\frac{\beta}{\lvert h_1\rvert^2}(\bar{\mathcal{P}} + \bar{p} - p^{\max})$	$p^{\max}$
$\bar{\mathcal{P}} \in (\mathcal{P}_1, \mathcal{P}_2)$; $v_2 < \beta_0$	$\frac{p_2 - \bar{p}}{p_2}$	$\frac{\beta}{\lvert h_1\rvert^2}(\bar{\mathcal{P}} + \bar{p})$	0	$\frac{\bar{p}}{p_2}$	$\frac{\beta}{\lvert h_1\rvert^2}(\bar{\mathcal{P}} + \bar{p} - p_2)$	p_2
$\bar{\mathcal{P}} \geq \mathcal{P}_2$	1	$\bar{P}$	$\bar{p}$	–	–	–

$\mathcal{P}$	R
$\bar{\mathcal{P}} \leq \mathcal{P}_1$	$\log\left(1 + \beta_0(\bar{\mathcal{P}} + \bar{p})\right) - \frac{\bar{p}}{p^{\max}} \log(1 + p^{\max}\beta_0)$
$\bar{\mathcal{P}} \in (\mathcal{P}_1, \mathcal{P}_2)$; $v_2 \geq \beta_0$	$\log\left(1 + \beta_0(\bar{\mathcal{P}} + \bar{p})\right) - \frac{p^{\max} - \bar{p}}{p^{\max} - p_1} \log\left(1 + p_1\beta_0\right) - \frac{\bar{p} - p_1}{p^{\max} - p_1} \log\left(1 + p^{\max}\beta_0\right)$
$\bar{\mathcal{P}} \in (\mathcal{P}_1, \mathcal{P}_2)$; $v_2 < \beta_0$	$\log\left(1 + \beta_0(\bar{\mathcal{P}} + \bar{p})\right) - \frac{\bar{p}}{p_2} \log\left(1 + p_2\beta_0\right)$
$\bar{\mathcal{P}} \geq \mathcal{P}_2$	$\log(1 + \bar{p}v_2)$

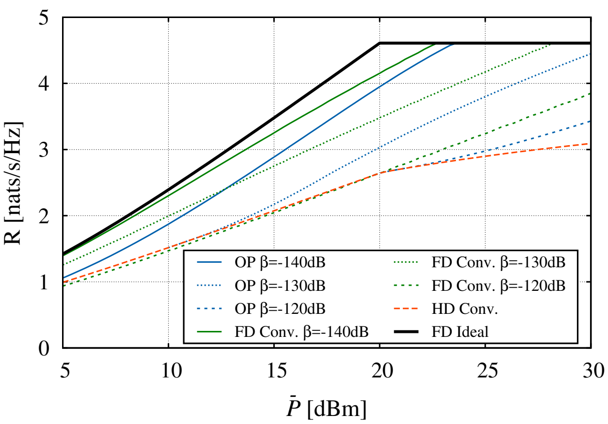

Fig. 4. Achieved rate vs. $\bar{P}$, for $\bar{p} = 20\,$dBm, $p^{\max} = 23\,$dBm, and different values of β.

where the relay always operates in half duplex and its transmit power is limited to $p^{\max}$.

We consider a scenario similar to that employed in [4] where the distance between source and relay and between relay and destination are both set to $d = 500\,$m, the signal carrier frequency is $f_c = 2.4\,$GHz, the signal bandwidth is 200 KHz, and the path loss is given by $|h_1|^2 = |h_2|^2 = \left(\frac{c}{4\pi f_c}\right)^2 d^{-3}$.

Fig. 1 compares the rate of our optimal power allocation scheme, labeled by "OP", against the performance of "FD Ideal", "FD Conv." and "HD Conv.", for $\bar{p} = 20\,$dBm, $p^{\max} = 23\,$dBm and $\beta = -135\,$dB. The results are shown as functions of the average transmit power at the source, $\bar{P}$. The plot highlights three operational regions corresponding to $\mathcal{P}_0 \frac{\beta}{|h_1|^2} \leq \bar{P} \leq \mathcal{P}_1 \frac{\beta}{|h_1|^2}$, $\mathcal{P}_1 \frac{\beta}{|h_1|^2} < \bar{P} \leq \mathcal{P}_2 \frac{\beta}{|h_1|^2}$, and $\bar{P} > \mathcal{P}_2 \frac{\beta}{|h_1|^2}$, respectively. "OP" always outperforms the "HD Conv" and gets very close to the "FD Conv.", which assumes perfect knowledge of the instantaneous power at the relay. Such performance of the "OP" scheme is achieved for the source and relay transmit power levels and for the slot durations depicted in Figs. 2 and 3, respectively. Interestingly, in the first operational region the time fractions assigned to the slots remain constant. With regard to the transmit power, the source always transmits (even if at different power levels), while the relay only receives in slot 1 and transmits at its maximum power in slot 2. In the second region, both source and relay transmit but the time fractions of the two slots vary, with $t_2 \to 0$ as $\bar{P} \to \mathcal{P}_2 \frac{\beta}{|h_1|^2}$. In the third region, both source and relay transmit at their average power level.

Fig. 4 shows the rate versus $\bar{P}$, achieved by "OP" and "FD Conv.", as β varies. For $\beta = -120\,$dB (i.e., $\beta > |h_2|^2 = -121\,$dB), "FD Conv." gives worse performance than "HD Conv.", due to the large impact of self interference and the fact that the relay is constrained to work in FD. As expected, as β decreases, the "OP" performance becomes closer to that of "FD Conv." and "FD Ideal"; in particular, for $\beta = -140\,$dB, the gap between "OP" and "FD Conv." is about 1 dB.

VI. CONCLUSIONS

We investigated the maximum achievable rate in dual-hop networks where the relay can operate in FD mode. Unlike existing work, in our scenario the source must be aware only of the distribution of the transmit power at the relay. We then derived the optimal self-interference distribution at the relay, which results to be discrete and composed of either one or two delta functions. Given such a distribution, we obtained the optimal communication strategy and power allocation to be used at the source and the relay. Future work will extend the analysis to the case where the parameter ω (on which the source transmit power depends) is smaller than the value of the maximum transmit power at the relay.

REFERENCES

[1] C. Y. A. Shang, P. J. Smith, G. K. Woodward, and H. A. Suraweera, "Linear transceivers for full duplex mimo relays," in *2014 Australian Communications Theory Workshop (AusCTW)*, Feb 2014, pp. 11–16.

[2] Q. Shi, M. Hong, X. Gao, E. Song, Y. Cai, and W. Xu, "Joint source-relay design for full-duplex mimo af relay systems," *IEEE Transactions on Signal Processing*, vol. 64, no. 23, pp. 6118–6131, Dec 2016.

[3] A. Behboodi, A. Chaaban, R. Mathar, and M. S. Alouini, "On full duplex gaussian relay channels with self-interference," in *2016 IEEE International Symposium on Information Theory (ISIT)*, July 2016, pp. 1864–1868.

[4] N. Zlatanov, E. Sippel, V. Jamali, and R. Schober, "Capacity of the gaussian two-hop full-duplex relay channel with residual self-interference," *IEEE Transactions on Communications*, vol. PP, no. 99, pp. 1–1, 2017.

[5] A. Nordio, C.-F. Chiasserini, and E. Viterbo, "Optimal power allocation strategies in full-duplex relay networks," Available on ArXiv, 2017.