X-Codes

(Invited Paper)

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***Abstract*—We propose X-Codes for a time division duplex system with $n_t \times n_r$ multiple-input multiple-output (MIMO), using singular value decomposition (SVD) precoding at the transmitter. It is known that SVD precoding transforms the MIMO channel into parallel subchannels, resulting in a diversity order of only one. To improve the diversity order, X-Codes can be used prior to SVD precoding to pair the subchannels, i.e., each pair of information symbols is encoded by a fixed 2×2 real rotation matrix. X-Codes can be decoded using n_r low complexity two-dimensional real sphere decoders. Error probability analysis for X-Codes enables us to choose the optimal pairing and the optimal rotation angle for each pair. Finally, we show that our new scheme outperforms other existing precoding schemes.**

I. Introduction

In time division duplex (TDD) MIMO systems, where channel state information (CSI) is fully available at the transmitter, precoding techniques can provide large performance improvements and therefore have been extensively studied [1], [2], [4], [5], [11], [12].

In this paper, we consider singular value decomposition (SVD) of the channel, i.e., the MIMO channel can be seen as parallel subchannels [1], [2]. Note that it results in no diversity gain. To improve it, we propose X-Codes, whose name is due to the structure of their encoding matrix. Specifically, the X-Code pairs subchannels with low diversity orders with those having high diversity orders. The pairing is achieved by jointly coding the two subchannels with a two-dimensional real orthogonal matrix (which is effectively parametrized by a single angle). These angles are chosen *a priori* and do not change with each realization of the channel, and therefore we use the term "Code" instead of "Precoder". At the receiver, low complexity sphere decoders (SDs) can be used for maximum likelihood (ML) decoding.

Another precoding scheme that pairs subchannels to improve diversity has been recently proposed in [10], called E-dmin, which is only optimized for 4-QAM symbols. Hence for higher spectral efficiencies, X-Codes have better error performance. Moreover, X-Codes can be decoded with n_r 2-dimensional real SDs, whereas E-dmin requires $\frac{n_r}{2}$ 4-dimensional real SDs.

II. System model

We consider a TDD system with $n_t \times n_r$ MIMO ($n_r \le n_t$), where the channel state information (CSI) is known perfectly at both the transmitter and receiver. Let $\mathbf{x} = (x_1, \dots, x_{n_t})^T$ be the vector of symbols transmitted by the n_t transmit antennas, where $(\cdot)^T$ denotes transposition, and let $\mathbf{H} = (h_{ij})$, $i = 1, \dots, n_r$, $j = 1, \dots, n_t$, be the $n_r \times n_t$ channel coefficient matrix, with h_{ij} as the complex channel gain between the j-th transmit antenna and the i-th receive antenna. The standard Rayleigh flat fading model is assumed with $h_{ij} \sim \mathcal{N}_c(0,1)$, i.e., i.i.d. complex Gaussian random variables with zero mean and unit variance. The received vector with n_r symbols is given by

$$\mathbf{y} = \mathbf{Hx} + \mathbf{z} \tag{1}$$

where $\mathbf{z}$ is a spatially uncorrelated Gaussian noise vector such that $\mathbb{E}[\mathbf{zz}^\dagger] = N_0 \mathbf{I}_{n_r}$, where † denotes the Hermitian transpose and $\mathbb{E}[.]$ is the expectation operator. Such a system has a maximum multiplexing gain of n_r. Let the number of information symbols transmitted be n_s ($n_s \le n_r$). Let $\mathbf{T}$ be the $n_t \times n_s$ precoding matrix which is applied to the information vector $\mathbf{u} = (u_1, \dots, u_{n_s})^T$ to yield the transmitted vector $\mathbf{x} = \mathbf{Tu}$. In general $\mathbf{T}$ is derived from the perfect knowledge of $\mathbf{H}$ at the transmitter. The transmission power constraint is given by $\mathbb{E}[\|\mathbf{x}\|^2] = P_T$ where $\|\cdot\|$ denotes the Euclidean norm. Finally, we define the signal-to-noise ratio as $\gamma \triangleq \frac{P_T}{N_0}$.

III. SVD precoding and X-Codes

SVD precoding is based on the singular value decomposition of the channel matrix $\mathbf{H} = \mathbf{U\Lambda V}$ ($\mathbf{U} \in \mathbb{C}^{n_r \times n_r}$, $\mathbf{\Lambda} \in \mathbb{C}^{n_r \times n_r}$ and $\mathbf{V} \in \mathbb{C}^{n_r \times n_t}$), where $\mathbf{UU}^\dagger = \mathbf{I}_{n_r}$, $\mathbf{VV}^\dagger = \mathbf{I}_{n_r}$ and $\mathbf{I}_{n_r}$ denotes the $n_r \times n_r$ identity matrix. The diagonal matrix $\mathbf{\Lambda}$ contains the singular values λ_i ($i = 1, \dots n_r$) of $\mathbf{H}$ in decreasing order ($\lambda_1 \ge \lambda_2 \cdots \ge \lambda_{n_r} \ge 0$). Let $\tilde{\mathbf{V}} \in \mathbb{C}^{n_s \times n_t}$ be the submatrix with the first n_s rows of $\mathbf{V}$. The precoder uses $\mathbf{T} = \tilde{\mathbf{V}}^\dagger$ and the received vector is $\mathbf{y} = \mathbf{HTu} + \mathbf{z}$. Let $\tilde{\mathbf{U}} \in \mathbb{C}^{n_r \times n_s}$ be the submatrix with the first n_s columns of $\mathbf{U}$. The receiver then computes

$$\mathbf{r} = \tilde{\mathbf{U}}^\dagger \mathbf{y} = \tilde{\mathbf{\Lambda}} \mathbf{u} + \mathbf{w} \tag{2}$$

where $\mathbf{w} \in \mathbb{C}^{n_s}$ is still a uncorrelated Gaussian noise vector ($\mathbb{E}[\mathbf{ww}^\dagger] = N_0 \mathbf{I}_{n_s}$). $\tilde{\mathbf{\Lambda}} \triangleq diag(\lambda_1, \lambda_2, \cdots \lambda_{n_s})$, and $\mathbf{r} = (r_1, \dots, r_{n_s})^T$. The overall error performance is dominated by the minimum singular value λ_{n_s}. In the special case of full-rate transmission ($n_s = n_r$), the resulting diversity order is only one. This problem is alleviated by the proposed X-Codes, where pairs of subchannels are jointly coded.

We consider only the full-rate SVD precoding scheme with even n_r and $n_s = n_r$ (In general it is possible to have X-Codes with $n_s < n_r$ and odd n_s). Prior to SVD precoding,

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we now add a linear encoder $\mathbf{X} \in \mathbb{C}^{n_r \times n_r}$, which allows us to pair different subchannels in order to improve the diversity order of the system. The precoding matrix $\mathbf{T} \in \mathbb{C}^{n_t \times n_r}$ and the transmitted vector $\mathbf{x}$ are then given by

$$\mathbf{T} = \mathbf{V}^{\dagger}\mathbf{X}, \quad \mathbf{x} = \mathbf{V}^{\dagger}\mathbf{X}\mathbf{u} \tag{3}$$

The code matrix $\mathbf{X}$ is determined by the list of pairings of the subchannels and the linear code generating matrix for each pair. Let the list of pairings be $\{(i_k, j_k),\ k = 1, 2 \cdots \frac{n_r}{2}\}$, where all i_k and j_k are distinct positive integers between 1 and n_r and $i_k < j_k$. On the k-th pair of subchannels i_k and j_k, the symbols u_{i_k} and u_{j_k} are jointly coded using a 2×2 matrix $\mathbf{A}_k$. In order to reduce the ML decoding complexity, we restrict the entries of $\mathbf{A}_k$ to be real valued. In order to avoid transmitter power enhancement, we impose an orthogonality constraint on each $\mathbf{A}_k$ and parametrize it with a single angle θ_k.

$$\mathbf{A}_k = \begin{bmatrix} \cos(\theta_k) & \sin(\theta_k) \\ -\sin(\theta_k) & \cos(\theta_k) \end{bmatrix} \quad k = 1, \ldots n_r/2 \tag{4}$$

Each $\mathbf{A}_k$ is a 2×2 submatrix of the code matrix $\mathbf{X}$ as shown below.

$$\begin{aligned} X_{i_k,i_k} &= \cos(\theta_k)\,, X_{i_k,j_k} = \sin(\theta_k) \\ X_{j_k,i_k} &= -\sin(\theta_k)\,, X_{j_k,j_k} = \cos(\theta_k) \end{aligned} \tag{5}$$

where $X_{i,j}$ is the entry of $\mathbf{X}$ in the ith row and jth column. The orthogonality constraint on each $\mathbf{A}_k$ therefore implies that $\mathbf{X}$ is also orthogonal. We shall see later, that an optimal pairing in terms of achieving the best diversity order is one in which the k-th subchannel is paired with the $(n_r - k + 1)$th subchannel. The code matrix $\mathbf{X}$ for this pairing has a cross-form structure and thus the name "X-Codes". Each symbol in $\mathbf{u}$ takes values from a regular M^2-QAM constellation which consists of the M-PAM constellation $\mathcal{S} \triangleq \{\beta(2i - (M - 1))\ |i = 0, 1, \cdots (M-1)\}$ used in quadrature on the real and the imaginary components of the channel. $\beta \triangleq \sqrt{\frac{3E_s}{2(M^2-1)}}$ and $E_s = \frac{P_T}{n_r}$ is the average symbol energy for each information symbol in the vector $\mathbf{u}$. Gray mapping is used to map the bits separately to the real and imaginary component of the symbols in $\mathbf{u}$.

IV. Decoding of X-Codes

Given the received vector $\mathbf{y}$, the receiver computes $\mathbf{r} = \mathbf{U}^{\dagger}\mathbf{y}$. Using (1) and (3), we have $\mathbf{r} = \mathbf{\Lambda}\mathbf{X}\mathbf{u} + \mathbf{w} = \mathbf{M}\mathbf{u} + \mathbf{w}$, where $\mathbf{M} \triangleq \mathbf{\Lambda}\mathbf{X}$ is the equivalent channel gain matrix and $\mathbf{w} \triangleq \mathbf{U}^{\dagger}\mathbf{z}$ is a noise vector with the same statistics as $\mathbf{z}$.

Further let $\mathbf{r}_k \triangleq [r_{i_k}, r_{j_k}]^T$, $\mathbf{u}_k \triangleq [u_{i_k}, u_{j_k}]^T$, $\mathbf{w}_k \triangleq [w_{i_k}, w_{j_k}]^T$, for $k = 1, 2, \cdots n_r/2$. For each $k \in \{1, 2, \cdots \frac{n_r}{2}\}$, let $\mathbf{M}_k \in \mathbb{R}^{2\times 2}$ denote the 2×2 submatrix of $\mathbf{M}$ consisting of entries in the i_k and j_k rows and columns. Using (5) and the definition of $\mathbf{M}$ we have

$$\mathbf{M}_k = \begin{bmatrix} \lambda_{i_k}\cos(\theta_k) & \lambda_{i_k}\sin(\theta_k) \\ -\lambda_{j_k}\sin(\theta_k) & \lambda_{j_k}\cos(\theta_k) \end{bmatrix} \tag{6}$$

With these new definitions, $\mathbf{r}$ can be equivalently written as

$$\mathbf{r}_k = \mathbf{M}_k\mathbf{u}_k + \mathbf{w}_k,\ k = 1, 2, \cdots \frac{n_r}{2}. \tag{7}$$

Since $\mathbf{M}$ has real entries ML decoding for the k-th pair can be separated into independent ML decoding of the real and imaginary components of $\mathbf{u}_k$.

V. Performance evaluation and design of X-Codes

In this section, we analyze the word (block) error probability of X-Codes. Towards this end, we shall find the following Lemma useful ([13]).

Lemma 1: Given a real scalar channel modeled by $y = \sqrt{\alpha}x + n$, where $x = \pm\sqrt{E_s}$, $n \sim \mathcal{N}(0, \sigma^2)$, and the square fading coefficient α has $\mathbb{E}[\alpha] = 1$ and a cdf (Cumulative Density Function) $F(\alpha) = C\alpha^k + o(\alpha^k)$, for $\alpha \to 0^+$, where C is a constant and k is a positive integer, then the asymptotic error probability for $\gamma = E_s/\sigma^2 \to \infty$ is given by

$$P = \frac{C((2k-1).(2k-3) \ \cdots \ 5.3.1)}{2}\gamma^{-k} + o(\gamma^{-k})$$

■

Let P_k denote the ML word error probability for the k-th pair of subchannels. The overall word error probability for the transmitted information symbol vector is given by

$$P = 1 - \Pi_{k=1}^{\frac{n_r}{2}}(1 - P_k). \tag{8}$$

It is also clear that the word error probability for the real and the imaginary components of the k-th pair are the same. Therefore without loss of generality we can compute the word error probability only for the real component (denoted by $P_k^{'}$) and then $P_k = 1 - (1 - P_k^{'})^2$. Let us further denote by $P_k^{'}(\Re(\mathbf{u}_k))$ the probability of the real part of the ML decoder decoding not in favor of $\Re(\mathbf{u}_k)$ when $\mathbf{u}_k$ is transmitted on the k-th pair.

Getting an exact analytic expression is difficult, and therefore we try to get tight upper bounds. Towards this end let $\{\Re(\mathbf{u}_k) \to \Re(\mathbf{v}_k)\}$ denote the pairwise error event, whose probability is denoted by $P_k^{'}(\Re(\mathbf{u}_k) \to \Re(\mathbf{v}_k))$ (PEP) ($\Re(\cdot)$ denotes the real parts of a complex argument). Using the union bounding technique, $P_k^{'}(\Re(\mathbf{u}_k))$ is then upper bounded by the sum of all the possible PEPs. It is clear that this upper bound on $P_k^{'}(\Re(\mathbf{u}_k))$ induces an upper bound on $P_k^{'}$. The difference vector $\mathbf{z}_k = \Re(\mathbf{u}_k) - \Re(\mathbf{v}_k)$ can be written as $\sqrt{\frac{6E_s}{(M^2-1)}}(p\ ,\ q)^T$, where $(p, q) \in \mathbb{S}_M$ and $\mathbb{S}_M \triangleq \{(p, q)|0 \le p \le (M-1), 0 \le q \le (M-1), (p, q) \ne (0, 0)\}$. Then, the PEP $P_k^{'}(\Re(\mathbf{u}_k) \to \Re(\mathbf{v}_k))$ is given by

$$P_k^{'}(\Re(\mathbf{u}_k) \to \Re(\mathbf{v}_k)) = \mathbb{E}_{(\lambda_{i_k},\lambda_{j_k})}\left[Q\left(\sqrt{\frac{3\gamma d_k^2(p, q, \theta_k)}{n_r(M^2-1)}}\right)\right] \tag{9}$$

where

$$\begin{aligned} d_k^2(p, q, \theta_k) \triangleq\ & \lambda_{i_k}^2(p\cos(\theta_k) + q\sin(\theta_k))^2 \\ & + \lambda_{j_k}^2(q\cos(\theta_k) - p\sin(\theta_k))^2 \end{aligned}$$

and $Q(x)$ is the Gaussian tail function. Since $\lambda_{i_k} \geq \lambda_{j_k} \geq 0$, we have the inequality

$$\lambda_{i_k}^2 (p\cos(\theta_k) + q\sin(\theta_k))^2 < d_k^2(p,q,\theta_k) < \lambda_{i_k}^2(p^2+q^2). \tag{10}$$

Since $Q(x)$ is a monotonically decreasing function with increasing argument, the PEP in (9) can be bounded as

$$P_k^{'}(\Re(\mathbf{u}_k) \to \Re(\mathbf{v}_k)) < \mathbb{E}_{\lambda_{i_k}}\left[Q\left(\sqrt{\frac{3\gamma\, \tilde{d}_k(p,q,\theta_k)\, \lambda_{i_k}^2}{n_r(M^2-1)}} \right) \right] \tag{11}$$

where $\tilde{d}_k(p,q,\theta_k) \triangleq (p^2+q^2)\cos^2(\theta_k - \tan^{-1}(\frac{q}{p}))$. Using Lemma 1 and the marginal pdf of the s-th eigenvalue λ_s^2 (for $\lambda_s^2 \to 0$) as given in [9], the bound in (11) can be further written as

$$P_k^{'}(\Re(\mathbf{u}_k) \to \Re(\mathbf{v}_k)) < b_k \left(\frac{3\gamma \tilde{d}_k(p,q,\theta_k)}{n_r(M^2-1)} \right)^{-\delta_k} + o(\gamma^{-\delta_k}) \tag{12}$$

where $\delta_k \triangleq (n_t - i_k + 1)(n_r - i_k + 1)$ and $b_k \triangleq \frac{C(i_k)((2\,\delta_k - 1)\,.\ \cdots\ 5\,.\,3\,.\,1)}{2\,\delta_k}$, where C is defined in [9]. Using the upper bound in (12), the union bound is given by

$$P_k^{'} \leq \frac{b_k}{M^2} \Big[\sum_{(p,q)\in \mathbb{S}_M} \left(\frac{3\gamma \tilde{d}_k(p,q,\theta_k)}{n_r(M^2-1)} \right)^{-\delta_k} \Big] + o(\gamma^{-\delta_k}) \tag{13}$$

We further define $g(\theta_k, M)$ as follows,

$$g(\theta_k, M) = \min_{(p,q)\in \mathbb{S}_M} \tilde{d}_k(p,q,\theta_k) \tag{14}$$

Using (14) in (13), we can further upper bound $P_k^{'}$ as follows.

$$P_k^{'} \leq \frac{4(M-1)b_k}{M} \left(\frac{3\gamma g(\theta_k, M)}{n_r(M^2-1)} \right)^{-\delta_k} + o(\gamma^{-\delta_k}) \tag{15}$$

From (15) it is clear that the diversity order achievable by the k-th pair is at least δ_k. The diversity order achievable for the overall system (combined effect of all the pairs) is determined by the pair with the lowest diversity order. Let δ_{ord} denote the overall diversity order. Based on the above discussion δ_{ord} can be lower bounded as follows.

$$\delta_{ord} \geq \min_k \delta_k. \tag{16}$$

For a given MIMO configuration (n_t, n_r), the design of optimal X-Codes depends upon the optimal pairing of subchannels and the optimal angle for each pair. From the lower bound on δ_{ord} (16) it is clear that the following pairing of subchannels achieves the best lower bound

$$i_k = k \quad\quad j_k = (n_r - k + 1),\ k = 1, 2 \cdots \frac{n_r}{2}. \tag{17}$$

Note that this corresponds to a cross-form generator matrix $\mathbf{X}$. The lower bound on the overall diversity order is then given by $\delta_{ord} \geq (\frac{n_r}{2} + 1)(n_t - \frac{n_r}{2} + 1)$. Finding the optimal angle for the k-th pair is a difficult problem, hence we choose the angle which maximizes $g(\theta_k, M)$. Maximization of $g(\theta_k, M)$ can be computed offline as the angles for X-Codes are fixed *a priori*.

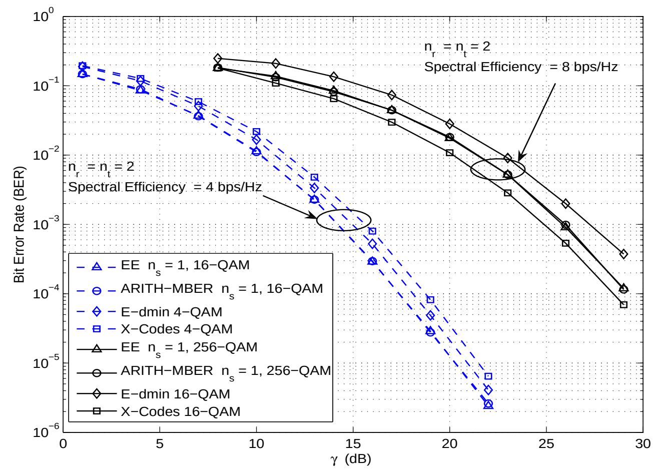

Fig. 1. Comparison between various precoders for $n_r = n_t = 2$ and target spectral efficiency = 4,8 bps/Hz.

VI. Simulation results

For all the simulations we assume $n_r = n_t$. The subchannel pairing for the X-Code is given by (17). The angle used for the subchannels is derived as discussed in section V (by optimizing upper bounds on the error probability expression).

Comparisons are made with *i*) the E-dmin (equal dmin precoder proposed in [10]), *ii*) the Arithmetic mean BER precoder (ARITH-MBER) proposed in [11], *iii*) the Equal Energy linear precoder (EE) based upon optimizing the minimum eigenvalue for a given transmit power constraint [12]), *iv*) the THP precoder based upon the idea of Tomlinson-Harashima precoding applied in the MIMO context [6]) and *v*) the channel inversion (CI) known as Zero Forcing precoder [3].

Among all the considered precoding schemes (except CI), E-dmin and X-Codes have the best diversity order. Though CI achieves infinite diversity, it suffers from power enhancement at the transmitter. We also observed that THP exhibit poor performance, when compared to the other precoders.

In Fig. 1, we plot the bit error rate (BER) for $n_r = n_t = 2$, and a target spectral efficiency of 4,8 bps/Hz. It is observed that for a target spectral efficiency of 4 bps/Hz, the best performance is achieved by ARITH-MBER and EE using only n_s=1 subchannel with 16-QAM modulation. X-Codes with 4-QAM modulation performs the worst. X-codes perform about 1.2 dB worse (at BER = 10^{-3}) compared to ARITH-MBER and EE. For a target spectral efficiency of 8 bps/Hz the results are totally different. X-Codes with 16-QAM modulation performs the best, and E-dmin performs the worst. Also the performance of X-codes is better than that of ARITH-MBER/EE by about 0.8 dB (at BER = 10^{-3}).

In Fig. 2, we plot the BER for $n_r = n_t = 4$, and a target spectral efficiency of 8,16 bps/Hz. It is observed that for a target spectral efficiency of 8 bps/Hz, the best performance is achieved by E-dmin with 4-QAM modulation. ARITH-MBER with N=3 subchannels (16-QAM modulation on one channel and 4-QAM on the other two) has the worst performance. X-codes perform worse than the E-dmin precoder by about 1 dB

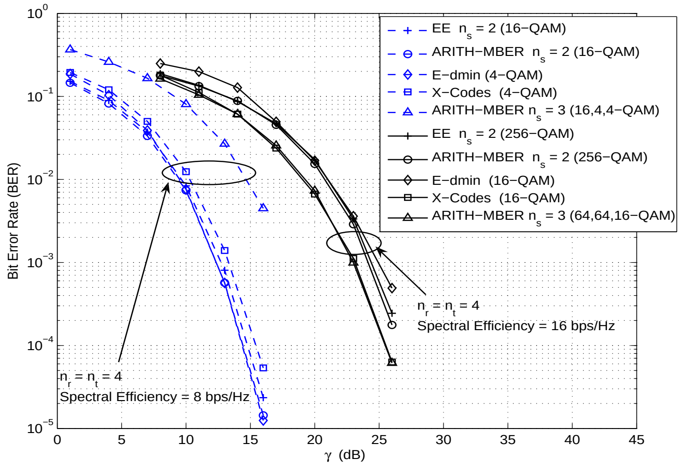

Fig. 2. Comparison between various precoders for $n_r = n_t = 4$ and target spectral efficiency = 8,16 bps/Hz.

(at BER = 10^{-3}).

For a target spectral efficiency of 16 bps/Hz X-codes with 16-QAM modulation performs the best. E-dmin performs the worst and is 2 dB away from X-Codes (at BER = 10^{-3}). E-dmin has poor performance since the precoder proposed in [10] has been optimized only for 4-QAM modulation, and therefore it does not perform that well for higher spectral efficiencies. E-dmin optimization for higher order QAM modulation is prohibitively too complex. It can be observed from Figs. 1 and 2 that for higher spectral efficiencies X-Codes perform the best when compared to other precoders.

VII. Complexity

All the considered precoders need to compute either SVD, QR or the pseudo-inverse of $\mathbf{H}$, whose complexity is $O(n_r^3)$. Generally, TDD is employed in a slowly fading channel, and therefore these computations can be performed at a very low rate compared to the rate of transmission. We, therefore, do not account for the complexity of these decompositions in the discussion below.

The encoding complexity of all the schemes have the same order. The complexity of the transmit pre-processing filter is $O(n_r n_t)$. If the number of operations were to be computed, CI and X-Codes would have the lowest complexity, since the linear and the THP precoders need extra pre-processing. E-dmin and X-Codes need to only compute SVD, which automatically gives the pre-processing matrices. X-Codes have lower encoding complexity compared to E-dmin, since the coding matrices $\mathbf{A}_k$ are fixed *a priori*. CI has an even lower complexity since there is no spatial coding.

The decoding complexity of all the schemes have a square dependence on n_r. This is due to the post-processing matrix filter at the receiver. The linear precoders, CI and THP employ post processing at the receiver, which enables independent ML decoding for each subchannel. E-dmin and X-Codes on the other hand use sphere decoding to jointly decode pairs of subchannels. ML decoding for X-Codes is accomplished by using n_r two-dimensional real sphere decoders.

However E-dmin requires $\frac{n_r}{2}$ 4-dimensional real sphere decoders. The average complexity of sphere decoding is cubic in the number of dimensions (and is invariant w.r.t modulation alphabet size M) [7], and therefore X-Codes have a much lower decoding complexity when compared to E-dmin.

VIII. Conclusion and Future Work

The proposed X-Codes are able to achieve full-rate and high diversity at a low complexity by pairing the subchannels before SVD precoding. Future work will focus on a generalization of X-Codes, which jointly codes more than two subchannels. Additional work will also address the reduction in decoder complexity and the generation of soft outputs.

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