On Fast–Decodable Space–Time Block Codes

Ezio Biglieri
Universitat Pompeu Fabra
Barcelona, Spain
e.biglieri@ieee.org

Yi Hong
Institute of Advanced Telecommunications
University of Wales, Swansea, UK
y.hong@swansea.ac.uk

Emanuele Viterbo
DEIS - Università della Calabria
via P. Bucci 42C, Rende (CS), Italy
viterbo@deis.unical.it

***Abstract*— In this paper, we focus on full–rate, fast–decodable space–time block codes (STBCs) for 2×2 and 4×2 multiple-input multiple-output (MIMO) transmission. We first derive conditions for reduced-complexity maximum-likelihood decoding, and apply them to a unified analysis of two families of 2×2 STBCs that were recently proposed. In particular, we describe a reduced-complexity sphere decoding algorithm suitable for QAM signal constellations. Next, we derive a novel reduced-complexity 4×2 STBC, and show that it outperforms all previously known codes with certain constellations.**

I. Introduction

Recently, Paredes *et al.* [1] have shown how to construct a family of fast-decodable, full-rate, full-rank space–time block codes (STBCs) for 2×2 multiple–input multiple–output (MIMO). The maximum-likelihood (ML) decoder can be simplified to a 4-dimensional sphere–decoder (SD) followed by an Alamouti detector [2]. The best code within this family coincides with a code originally found by Hottinen and Tirkkonen [3], under the name of *twisted space–time transmit diversity* code. Independently, the same STBCs were found in [4], and classified under the rubric of *multi–strata* space–time codes. More recently, another family of full–rate, full–diversity, fast–decodable STBCs for 2×2 MIMO was proposed in [5]. This family of STBCs employs a combination of two different rotated Alamouti codewords.

Motivated by the above, the present paper provides a unified view of the fast-decodable STBCs in [1, 3–5]. We show that these families enable the same low-complexity ML decoding procedure, and we specialize it in the form of a SD search [6, 7]. We also present general structure for full-rate, fast-decodable STBCs, and we extend this idea to MIMO 4×2. We propose a family of new STBCs. Within this family, we show a code that outperforms all previously proposed 4×2 STBCs for 4-QAM signal constellation.

Notations: Boldface letters are used for column vectors, and capital boldface letters for matrices. Superscripts ^{T}, †, and * denote transposition, Hermitian transposition, and complex conjugation, respectively. $\mathbb{Z}$, $\mathbb{C}$, and $\mathbb{Z}[j]$ denote the ring of rational integers, the field of complex numbers, and the ring of Gaussian integers, respectively, where $j^2=-1$. Also, $\mathbf{I}_n$ denotes the $n\times n$ identity matrix, and $\mathbf{0}_{m\times n}$ denotes the $m\times n$ matrix all of whose elements are 0.

Given a complex number x, we define the $\tilde{(\cdot)}$ operator from $\mathbb{C}$ to $\mathbb{R}^2$ as

$$\tilde{x} \triangleq [\Re(x), \Im(x)]$$

where $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of a complex number. The $\tilde{(\cdot)}$ operator can be extended to complex vectors $\mathbf{x}=[x_1,\dots x_n]\in\mathbb{C}^n$ as

$$\tilde{\mathbf{x}} \triangleq [\tilde{x}_1,\dots \tilde{x}_n]^T$$

The $\mathrm{vec}(\cdot)$ operator stacks the m column vectors of a $n\times m$ matrix into a mn column vector. The $\|\cdot\|$ operation denotes the Euclidean norm of a vector, and the Frobenius norm of a matrix. Finally, the Hermitian inner product of two complex column vectors $\mathbf{a}$ and $\mathbf{b}$ is denoted by

$$\langle \mathbf{a},\mathbf{b}\rangle \triangleq \mathbf{a}^T\mathbf{b}^*$$

II. System Model and Code Design Criteria

We consider a $n_t\times n_r$ MIMO system over block fading channels. At discrete time t, the received signal matrix $\mathbf{Y}\in\mathbb{C}^{n_r\times T}$ is given by

$$\mathbf{Y}=\mathbf{HX}+\mathbf{N}, \tag{1}$$

where $\mathbf{X}\in\mathbb{C}^{n_t\times T}$ is the codeword matrix, transmitted over T channel uses. Moreover, $\mathbf{N}\in\mathbb{C}^{n_r\times T}$ is a complex white Gaussian noise with i.i.d. entries $\sim\mathcal{N}_{\mathbb{C}}(0,N_0)$, and $\mathbf{H}=[h_{i\ell}]\in\mathbb{C}^{n_r\times n_t}$ is the channel matrix, assumed to remain constant during the transmission of a codeword, and varies from one transmission of a codeword to another independently. The elements of $\mathbf{H}$ are assumed to be i.i.d. circularly symmetric Gaussian random variables $\sim\mathcal{N}_{\mathbb{C}}(0,1)$. The realization of $\mathbf{H}$ is assumed to be known at the receiver, but not at the transmitter. The following definitions are relevant here:

Definition 1: ***(Code rate)*** The code rate of a STBC is defined as the number κ of independent information symbols per codeword, drawn from a complex constellation $\mathcal{S}$. If $\kappa=n_rT$, the STBC is said to have *full rate*. □

Consider now ML decoding. This consists of finding the code matrix $\mathbf{X}$ that achieves the minimum of the squared norm $m(\mathbf{X})\triangleq\|\mathbf{Y}-\mathbf{HX}\|^2$.

Definition 2: ***(Decoding Complexity)*** The ML decoding complexity can be measured by counting the minimum number of values of $m(\mathbf{X})$ in ML decoding. This number cannot exceed M^κ, with $M=|\mathcal{S}|$, the worst-case decoding complexity achieved by an exhaustive-search ML decoder. □

Definition 3: ***(Simplified decoding)*** We say that a STBC admits simplified decoding if ML decoding can be achieved with less than M^κ computations of $m(\mathbf{X})$. □

Assuming that the codeword $\mathbf{X}$ is transmitted, it may occur that $\|\mathbf{Y}-\mathbf{HX}\|^2>\|\mathbf{Y}-\mathbf{H}\widehat{\mathbf{X}}\|^2$, with $\widehat{\mathbf{X}}\neq\mathbf{X}$, resulting

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in a *pairwise error*. Let r denote the rank of the *codeword-difference matrix* $\mathbf{X}-\widehat{\mathbf{X}}$, with $\widehat{\mathbf{X}}\neq\mathbf{X}$, and let $\mathbf{E}\triangleq(\mathbf{X}-\widehat{\mathbf{X}})(\mathbf{X}-\widehat{\mathbf{X}})^{\dagger}$ be the *codeword-distance matrix*. Let δ denote the product of non-zero eigenvalues of the codeword distance matrix $\mathbf{E}$. The error probability of a STBC is upper-bounded by the following union bound:

$$\begin{aligned} P(e) &\leq \frac{1}{M^{\kappa}}\sum_{\mathbf{X}}\sum_{\mathbf{X}\neq\hat{\mathbf{X}}}P(\mathbf{X}\to\hat{\mathbf{X}}) \\ &= \frac{1}{M^{\kappa}}\sum_{r}\sum_{\delta}A(r,\delta)P(r,\delta) \end{aligned} \quad (2)$$

where $P(\mathbf{X}\to\hat{\mathbf{X}})$ denotes the pairwise error probability (PEP) among all distinct $(\mathbf{X},\hat{\mathbf{X}})$. The term $P(r,\delta)$ represents the PEP of the codewords with rank r and eigenvalue product δ, while $A(r,\delta)$ denotes the associated multiplicity.

Definition 4: ***(Full-diversity STBC)*** A full-diversity STBC is one with $r=n_t$ over all possible codeword-difference matrices. □

For a full-diversity STBC, the worst-case PEP depends asymptotically, for high signal-to-noise ratios, on both the rank $r=n_t$ and the *minimum determinant* of the codeword distance matrix

$$\delta_{\min}\triangleq\min_{\mathbf{X}\neq\widehat{\mathbf{X}}}\det(\mathbf{E})$$

The "rank-and-determinant criterion" (RDC) of code design requires the maximization of both r and $\delta_{\min}$. This criterion yields *diversity gain* $n_r n_t$ and *coding gain* $(\delta_{\min})^{1/n_t}$ [8].

For a non full-diversity STBC, the minimum determinant equals to zero. In such a case, we have to minimize the associated multiplicity of the *dominant pairwise terms* of rank $r\leq n_t$ independently of their product distance.

III. Fast-decodable codes for 2×2 MIMO

Consider now 2×2 STBCs. These are full-rate and full-diversity if $\kappa=4$ symbols/codeword, and $r=n_t$.

Definition 5: ***(Fast-decodable STBCs for*** 2×2 ***MIMO)*** A 2×2 STBC allows fast ML decoding if its complexity does not exceed $2M^3$. □

1st family fast–decodable STBC – Here we examine 2×2 fast-decodable STBCs endowed with the following structure [3]:

$$\mathbf{X}=\mathbf{X}_a(x_1,x_2)+\mathbf{T}\mathbf{X}_b(z_1,z_2) \quad (3)$$

where

$$\mathbf{T}=\begin{bmatrix}1&0\\0&-1\end{bmatrix} \text{ and } \mathbf{X}_a(x_1,x_2)=\begin{bmatrix}\alpha x_1&-\beta x_2^*\\\beta x_2&\alpha x_1^*\end{bmatrix} \quad (4)$$

is an Alamouti 2×2 space–time block codeword [2], with $\alpha=\beta=1$ and $x_1,x_2\in\mathbb{Z}[j]$. Moreover, we have

$$\mathbf{X}_b(z_1,z_2)=\begin{bmatrix}z_1&-z_2^*\\z_2&z_1^*\end{bmatrix} \text{ and } \begin{bmatrix}z_1\\z_2\end{bmatrix}=\mathbf{U}\begin{bmatrix}x_3\\x_4\end{bmatrix} \quad (5)$$

where $z_1,z_2\in\mathbb{C}$, $x_3,x_4\in\mathbb{Z}[j]$, and $\mathbf{U}\in\mathbb{C}^{2\times2}$ is the unitary matrix

$$\mathbf{U}=\begin{bmatrix}\varphi_1&-\varphi_2^*\\\varphi_2&\varphi_1^*\end{bmatrix}$$

with $|\varphi_1|^2+|\varphi_2|^2=1$. Vectorizing, and separating real and imaginary parts, the matrices $\mathbf{X}$ yield

$$\widetilde{\mathrm{vec}(\mathbf{X})}=\mathbb{G}_1[\tilde{x}_1,\tilde{x}_2]^T+\mathbb{G}_2[\tilde{x}_3,\tilde{x}_4]^T$$

where $\mathbb{G}_1,\mathbb{G}_2\in\mathbb{R}^{8\times4}$ are the *generator matrices* of $\mathbf{X}_a$ and $\mathbf{T}\mathbf{X}_b$, respectively. Note that the matrix $\mathbf{T}$ is chosen in order to guarantee that the matrix $\mathbb{G}=[\mathbb{G}_1|\mathbb{G}_2]$ is an orthogonal matrix, i.e., $\mathbb{G}^T\mathbb{G}=\mathbf{I}_{2\kappa}$. This implies that the code has *cubic shaping* (or that is *information lossless*).

The matrix $\mathbf{U}$ is chosen to achieve full rank and maximize the minimum determinant, characterized by the unitary matrix [1, 3, 4]:

$$\mathbf{U}=\frac{1}{\sqrt{7}}\begin{bmatrix}1+j&-1+2j\\1+2j&1-j\end{bmatrix}$$

This code has $\delta_{\min}=16/7$ for 4-QAM signalling, which is smaller than that of the Golden code ($\delta_{\min}=16/5$) [9].

2nd family fast–decodable STBC – In the second family of fast-decodable STBCs [5], both $\mathbf{X}_a$ and $\mathbf{X}_b$ have the Alamouti structure (4), but with different coefficients α,β, and $\mathbf{T}=\mathbf{I}$ yielding $\delta_{min}=1.9973$ for 4-QAM signalling. If we compare with the 1st Family, we can see that in the 2nd Family of STBCs, the generator matrix $\mathbb{G}$ is not an orthogona matrix. This implies that the 2nd family STBCs do not have cubic shaping.

Low–Complexity MLD – At the receiver, due to the linearity of the code, a sphere decoder can be employed. It was pointed out in [1, 4, 5] that both family STBCs admits a low-complexity decoder thanks to orthogonality properties of the two component codes in (3). The relevant performance comparison in terms of codeword error rate (CER) is given in [10]. Let $\mathbf{Y}=[y_{\ell n}]\in\mathbb{C}^{2\times2}$, $\mathbf{H}=[h_{\ell n}]\in\mathbb{C}^{2\times2}$, and $\mathbf{N}=[n_{\ell n}]\in\mathbb{C}^{2\times2}$. After vectorization, we obtain

$$\mathbf{y}=\mathcal{H}\mathbf{x}+\mathbf{n} \quad (6)$$

where

$$\begin{aligned}\mathbf{y}&\triangleq[y_{11},y_{21},y_{12}^*,y_{22}^*]^T \qquad \mathbf{n}\triangleq[n_{11},n_{21},n_{12}^*,n_{22}^*]^T\\ \mathbf{x}&\triangleq[x_1,x_2,x_3,x_4]^T\end{aligned}$$

and explicitly for 1st family STBC, we have

$$\mathcal{H}\triangleq[\mathbf{f}_1|\mathbf{f}_2|\mathbf{f}_3\mid\mathbf{f}_4]=\begin{bmatrix}\alpha h_{11}&\alpha h_{12}&A&-B\\\alpha h_{21}&\alpha h_{22}&C&-D\\\beta^* h_{12}^*&-\beta^* h_{11}^*&-B^*&-A^*\\\beta^* h_{22}^*&-\beta^* h_{21}^*&-D^*&-C^*\end{bmatrix} \quad (7)$$

with

$$\begin{aligned}A&=h_{11}\varphi_1-h_{12}\varphi_2 \qquad B=h_{11}\varphi_2^*+h_{12}\varphi_1^*\\ C&=h_{21}\varphi_1-h_{22}\varphi_2 \qquad D=h_{21}\varphi_2^*+h_{22}\varphi_1^*\end{aligned} \quad (8)$$

We conduct the QR decomposition of $\mathcal{H}$, i.e., $\mathcal{H}=\mathbf{QR}$, where $\mathbf{Q}\in\mathbb{C}^{4\times4}$ is an unitary matrix and $\mathbf{R}\in\mathbb{C}^{4\times4}$ is an upper-triangular matrix. Here $\mathbf{Q}$ and $\mathbf{R}$ are given by

$$\mathbf{Q} = [\mathbf{e}_1|\mathbf{e}_2|\mathbf{e}_3|\mathbf{e}_4]$$

$$\mathbf{R} = \begin{bmatrix} \|\mathbf{d}_1\| & \langle \mathbf{f}_2, \mathbf{e}_1 \rangle & \langle \mathbf{f}_3, \mathbf{e}_1 \rangle & \langle \mathbf{f}_4, \mathbf{e}_1 \rangle \\ 0 & \|\mathbf{d}_2\| & \langle \mathbf{f}_3, \mathbf{e}_2 \rangle & \langle \mathbf{f}_4, \mathbf{e}_2 \rangle \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \|\mathbf{d}_4\| \end{bmatrix}$$

The QR decomposition is related to the Gram–Schmidt orthogonalization algorithm through the following equations:

$$\mathbf{u}_i = \mathbf{f}_i - \sum_{j=1}^{i-1} \mathrm{Proj}_{\mathbf{e}_j} \mathbf{f}_i, \quad \mathbf{e}_i = \frac{\mathbf{u}_i}{\|\mathbf{u}_i\|}, \quad i = 2, \cdots, 4$$

where $\mathrm{Proj}_{\mathbf{u}} \mathbf{v} \triangleq \frac{\langle \mathbf{v}, \mathbf{u} \rangle}{\langle \mathbf{u}, \mathbf{u} \rangle} \mathbf{u}$ and $\mathbf{u}_1 = \mathbf{f}_1$. Direct computation shows that $\mathbf{R}$ has the following properties:

1) $\langle \mathbf{f}_2, \mathbf{e}_1 \rangle = 0, \qquad \langle \mathbf{f}_4, \mathbf{e}_3 \rangle = 0$
2) $\|\mathbf{d}_1\|^2 = \|\mathbf{d}_2\|^2 \triangleq \mu, \qquad \|\mathbf{d}_3\|^2 = \|\mathbf{d}_4\|^2 \triangleq \gamma$
3) $[\mathbf{e}_1 \mid \mathbf{e}_2] = \frac{1}{\sqrt{\mu}} [\mathbf{f}_1 \mid \mathbf{f}_2] = \frac{1}{\sqrt{\mu}} \mathbf{F}_1$
4) We obtain

$$\begin{bmatrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{\mu}} \phi_1 & -\frac{1}{\sqrt{\mu}} \phi_2 \\ \frac{1}{\sqrt{\mu}} \phi_2^* & \frac{1}{\sqrt{\mu}} \phi_1^* \end{bmatrix}$$

where $\phi_1 \triangleq \frac{1}{\sqrt{2}}(\rho\varphi_1 - 2c\varphi_2)$, $\phi_2 \triangleq \frac{1}{\sqrt{2}}(\rho\varphi_2^* + 2c\varphi_1^*)$, with $\rho \triangleq |h_{11}|^2 - |h_{12}|^2 + |h_{21}|^2 - |h_{22}|^2$, and $c \triangleq h_{11}^* h_{12} + h_{21}^* h_{22}$. We have $\Phi_1 \Phi_2^* + \Phi_3 \Phi_4^* = 0$

Thus, $\mathbf{R}$ has the form

$$\mathbf{R} = \begin{bmatrix} \sqrt{\mu} & 0 & \Phi_1 & \Phi_2 \\ 0 & \sqrt{\mu} & \Phi_3 & \Phi_4 \\ 0 & 0 & \sqrt{\gamma} & 0 \\ 0 & 0 & 0 & \sqrt{\gamma} \end{bmatrix} \tag{9}$$

For 2nd family STBCs, we obtain the same structure of the above $\mathbf{R}$ matrix but with different components. Examination of the structure of $\mathbf{R}$ discloses the simplified-decoding property of both families of codes, with complexity $2M^3$. In fact, the ML metric turns out to be a sum of four quadratic functions, depending on (x_1, x_3, x_4), (x_2, x_3, x_4), x_3, and x_4, respectively. The low–complexity MLD using SD searching algorithm is given as follows.

1) We first choose a pair (x_3, x_4) using a 4-dimensional real SD (M^2 branch metric computation complexity).
2) For every such pair, using Alamouti symbol-by-symbol decoding, we choose in parallel x_1 and x_2, resulting in $2M$ branch metric computation complexity.
3) The worst-case decoding complexity of fast-decodable STBCs is $2M^3$, as compared to a standard SD complexity M^4.
4) The zero elements in the upper triangular matrix $\mathbf{R}$ allows faster branch metric computation.

Design Criteria of Fast–decodable STBCs – In summary, a STBC of the above form (see 1st, 2nd family STBCs) has low–complexity decoding if $\mathbf{X}_a$ has an Alamouti structure and $\mathbf{X}_b$ has an orthogonal generator matrix. If cubic shaping is required, $\mathbf{T}$ should be chosen such that $\mathbb{G}^T\mathbb{G} = \mathbf{I}$. If $\mathbf{X}_b$ has the Alamouti structure, extra savings of computation complexity are available in the SD.

Codes	$\delta_{\min}$	Multiplicity
New STBC	0	$\sum_\delta A(2,\delta) = 160$
Perfect Code $\mathbf{U}$ matrix	0	$\sum_\delta A(2,\delta) = 560$
DjABBA	0.8304	$A(4, 0.8304) = 770$
Two-Layers Perfect Code	0.0016	$A(4, 0.0016) = 128$

TABLE I

MINIMUM DETERMINANTS OF 4×2 STBCS WITH 4-QAM SIGNALING

IV. NEW STBC FOR 4×2 MIMO SYSTEMS

Here we design a fast-decodable 4×2 STBC based on the concepts elaborated upon in the previous sections. We first introduce the relevant definitions.

Definition 6: ***(Quasi-orthogonal structure)*** [11] A code such that

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* & -x_3^* & x_4 \\ x_2 & x_1^* & -x_4^* & -x_3 \\ x_3 & -x_4^* & x_1^* & -x_2 \\ x_4 & x_3^* & x_2^* & x_1 \end{bmatrix}$$

where $x_i \in \mathbb{C}$, $i = 1, \dots, 4$, is said to have a quasi-orthogonal structure. Note that quasi-orthogonal STBCs are not full rank, and $r = 2$. □

Definition 7: ***(Full-rate, fast-decodable STBC for*** 4×2 ***MIMO)*** A full-rate, fast-decodable STBC for 4×2 MIMO, denoted $\mathcal{G}'$, has $\kappa = 8$ symbols/codeword, and can be decoded by a 12-dimensional real SD algorithm (rather than the standard 16-dimensional SD). □

The 4×4 codeword matrix $\mathbf{X} \in \mathcal{G}'$ encodes eight QAM symbols $\mathbf{x} = [x_1, \dots, x_8] \in \mathbb{Z}[j]$, and is transmitted by using the channel four times, i.e., $T = 4$. Following the idea of the previous section, we choose the following codeword structure:

$$\mathbf{X} = \mathbf{X}_a(x_1, x_2, x_3, x_4) + \mathbf{T}\mathbf{X}_b(z_1, z_2, z_3, z_4) \tag{10}$$

where

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_2 & 0 \\ 0 & -\mathbf{I}_2 \end{bmatrix} \tag{11}$$

is used to preserve the orthogonality between the two components of the code (similarly to the codes of previous section), $\mathbf{X}_a(x_1, x_2, x_3, x_4)$ and $\mathbf{X}_b(z_1, z_2, z_3, z_4)$ follow the quasi-orthogonal STBC structure (see Definition 6), where $x_1, x_2, x_3, x_4 \in \mathbb{Z}[j]$ and

$$\begin{bmatrix} z_1 & z_2 & z_3 & z_4 \end{bmatrix}^T = \mathbf{U} \begin{bmatrix} x_5 & x_6 & x_7 & x_8 \end{bmatrix}^T \tag{12}$$

where $z_i \in \mathbb{C}$, $x_k \in \mathbb{Z}[j]$, $i = 1, \dots, 4$, $k = 5, \dots, 8$ and $\mathbf{U}$ is a 4×4 unitary matrix. Note that 1) The matrix $\mathbf{T}$ guarantees cubic shaping, and 2) Since the matrix $\mathbf{X}_a$ has a quasi-orthogonal structure, the code is not full rank: in fact, it has $r = 2$. As a consequence, we conduct a search over the matrices $\mathbf{U}$ leading to the minimum of $\sum_\delta A(2, \delta)$, i.e., the total multiplicity of all rank 2 terms in (2). The term $A(2, \delta)$ represents the total number of codeword difference matrices of rank 2 and product distance δ. Since an exhaustive search through all 4×4 unitary matrices is too complex, we focus on those with the form $\mathbf{U} = \mathbf{DF}$, where $\mathbf{F} \triangleq [\exp(j2\pi\ell n/4)]_{\ell,n=1,\ldots,4}$ is a 4×4 discrete-Fourier-transform matrix, and $\mathbf{D} \triangleq \text{diag}(\exp(j2\pi n_\ell/N)$ for some integers N, ℓ, with $0 \le n_\ell < N$ and $\ell = 1, \ldots, 4$.

For 4-QAM signaling, taking $N = 7$ and $n_\ell = 1, 2, 5, 6$, we have obtained

$$\mathbf{U}=\begin{bmatrix} 0.31+0.39i & 0.31+0.39i & 0.31+0.39i & 0.31+0.39i \\ -0.11+0.49i & -0.49-0.11i & 0.11-0.49i & 0.49+0.11i \\ -0.11-0.49i & 0.11+0.49i & -0.11-0.49i & 0.11+0.49i \\ 0.31-0.39i & -0.39-0.31i & -0.31+0.39i & 0.39+0.31i \end{bmatrix}$$

which yields the minimum $\sum_\delta A(2, \delta)$.

Under 4-QAM signaling, we compare the minimum determinants δ_{min} and their associated multiplicities, as well as CERs of the above STBC to the following 4×2 codes:

1) Code (10), with $\mathbf{U}$ the 4×4 "perfect" rotation [12].
2) The best DjABBA code of [3].
3) The "perfect" two-layer code of [13].

Determinant and multiplicity values are shown in Table I. The CERs are shown in Fig. 1. The proposed code achieves the best CER up to 10^{-5}. Due to diversity loss, the performance curve of the new code and the one of DjABBA cross over at CER$= 10^{-5}$.

Low–Complexity MLD – Following the same decoding process as that of 2×2 MIMO, we obtain the following $\mathbf{R}$ matrix

$$\mathbf{R} = \begin{bmatrix} \mathbf{S}_1 & \mathbf{S}_2 \\ \mathbf{0}_{4\times4} & \mathbf{S}_3 \end{bmatrix} \quad (13)$$

where

$$\mathbf{S}_1 = \begin{bmatrix} \sqrt{\mu} & 0 & 0 & \Phi_1 \\ 0 & \sqrt{\mu} & -\Phi_1 & 0 \\ 0 & 0 & \sqrt{\gamma} & 0 \\ 0 & 0 & 0 & \sqrt{\gamma} \end{bmatrix} \quad (14)$$

$\mathbf{S}_2$ is a 4×4 non-zero matrix, $\mathbf{S}_3$ is a 4×4 upper triangular matrix, $\mu = \sum_{i=1}^{2}\sum_{j=1}^{4} |h_{ij}|^2$, and $\Phi_1 \triangleq 2\Re(h_{11}^* h_{14} + h_{21}h_{24}^* - h_{12}h_{13}^* - h_{22}h_{23}^*)$

Summarizing, we use the following MLD where the search is based on SD algorithm:

- We use a 12-dimensional real SD, requiring M^6 branch metric computations, to find $\mathbf{u}_5^{16} \triangleq [u_5, \ldots, u_{16}]$. Then, we subtract the interference terms from $\mathbf{u}_5^{16}$ and the partial symbol vector $\mathbf{u}_1^4 \triangleq [u_1, \ldots, u_4]$ can be computed directly using Alamouti symbol–by–symbol decoding with decoding complexity $2M$.
- The worst-case decoding complexity of fast-decodable STBCs is $2M^7$ rather than a standard SD complexity M^8.
- The zero entries in the matrix $\mathbf{R}$ allow simple branch metric calculation in the decoding.

V. Conclusion

In this paper we study, under a unified framework, two families of full-rate, full-diversity 2×2 STBCs. We examine how both families allow low-complexity ML decoding. We also derive design criteria of fast-decodable STBCs for 2×2 MIMO. We then extend this design to the MIMO 4×2. We propose a family of new STBCs. Within this family, a new code is found that outperforms any known 4×2 code for 4-QAM signaling. A reduced-complexity SD algorithm enables using only a 12-dimensional real SD, rather than the standard 16-dimensional one. Moreover the branch metric computation cost can be reduced thanks to the quasi–orthogonal STBC structure.

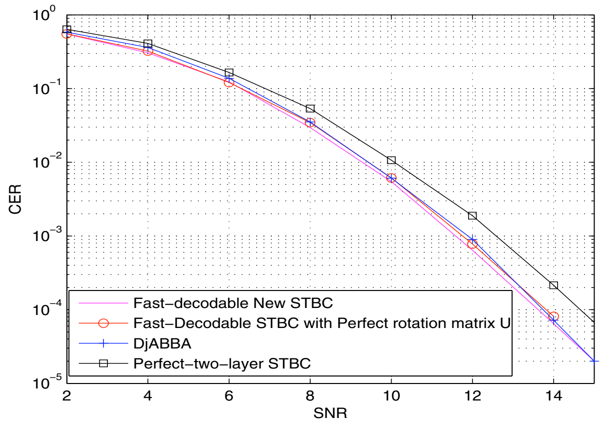

Fig. 1. Comparison of the CER of different 4×2 STBCs with 4-QAM signaling.

References

[1] J. Paredes, A. B. Gershman, and M. G. Alkhanari, "A 2×2 space–time code with non-vanishing determinants and fast maximum likelihood decoding," in *Proc. ICASSP*, Honolulu, Hawaii, USA, April 2007.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, October 1998.

[3] A. Hottinen and O. Tirkkonen, "Precoder designs for high rate space–time block codes," *Proc. CISS 2004*, Princeton, March 17-19, 2004.

[4] M. Samuel and M. P. Fitz, "Reducing the detection complexity by using 2×2 multi-strata space–time codes," *Proc. IEEE ISIT 2007*, pp. 1946–1950, Nice, France, June 2007.

[5] S. Sezginer and H. Sari, "A full-rate full-diversity 2×2 space–time code for mobile Wimax Systems," in Proc. *IEEE International Conference on Signal Processing and Communications*, Dubai, July 2007.

[6] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading chanel," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1639–1642, July 1999.

[7] M. O. Damen, H. El Gamal, and G. Caire, "On maximum-likelihood detection and the search for the closest lattice point," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2389–2402, October 2003.

[8] V. Tarokh, N. Seshadri and A. R. Calderbank, "Space–time codes for high data rate wireless communications: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744–765, March 1998.

[9] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The Golden Code: A 2×2 full-rate space–time code with non-vanishing determinants," *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 1432–1436, Apr. 2005.

[10] E. Biglieri, Y. Hong, and E. Viterbo, "On fast-decodable space–time block codes," submitted to *IEEE Trans. Inform. Theory*, also available in arXiv: 07082804, Aug 2007.

[11] H. Jafarkhani, "A quasi-orthogonal space–time block code," in *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 1–4, January 2001.

[12] F. Oggier, G. Rekaya, J.-C. Belfiore, and E. Viterbo, "Perfect space–time block codes," *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp. 3885–3902, September 2006.

[13] Y. Hong, E. Viterbo, and J.-C. Belfiore, "A space–time block coded multiuser MIMO downlink transmission scheme," *Proc. IEEE ISIT*, pp. 257–261, Seattle, WA, June–July 2006.