Permutation Modulation for Fast Fading Channels

Emanuele Viterbo
Politecnico di Torino, Italy – email:viterbo@polito.it

Permutation modulation has been proposed by David Slepian in 1965 [1]. A Variant I permutation modulation is the set of codewords (signals) obtained by taking all permutations of an initial vector in the n-dimensional Euclidean space. A Variant II permutation modulation is the set of codewords obtained by taking all permutations and sign changes of the components of an initial vector in the n-dimensional Euclidean space. Good permutation modulations may be designed by appropriately selecting the initial vector and may be very efficiently decoded in the AWGN channel and slow fading channel by essentially applying a sorting algorithm to the received signal vector [1]. In this talk we present a new trellis based decoder for ML detection over fast fading channels.

Let $\{\mu_1, \ldots, \mu_k\}$ be a set of distinct real numbers with $0 \le \mu_1 < \mu_2 < \cdots < \mu_k$ and let $\{m_1, \ldots, m_k\}$ be a set of positive integers such that $n = \sum_{j=1}^{k} m_j$. Consider the initial vector with components sorted in ascending order

$$\mathbf{x}_0 = [\underbrace{\mu_1, \ldots, \mu_1}_{m_1}, \underbrace{\mu_2, \ldots, \mu_2}_{m_2}, \ \cdots \ , \underbrace{\mu_k, \ldots, \mu_k}_{m_k}] \ . \qquad (1)$$

A Variant I permutation code consists of the set $\mathcal{S}$ of vectors obtained by permuting the components of the initial vector $\mathbf{x}_0$. The total number of codewords in such a code is

$$M_{\mathrm{I}} = \frac{n!}{m_1!\, m_2! \cdots m_k!} \ . \qquad (2)$$

The Variant I code with $k = 2$, $m_1 = n - 1$, $m_2 = 1$ and $\mu_1 = 0$ is the well known PPM or orthogonal modulation.

A Variant II permutation code consists of the set $\mathcal{S}$ of vectors obtained by permuting and applying all possible sign changes to the components of the initial vector $\mathbf{x}_0$.

The total number of codewords in this case is

$$M_{\mathrm{II}} = \frac{2^h n!}{m_1!\, m_2! \cdots m_k!} \qquad (3)$$

where $h = n - m_1$, if $\mu_1 = 0$ and $h = n$, if $\mu_1 > 0$. The Variant II code with $k = 2$, $m_1 = n - 1$, $m_2 = 1$ and $\mu_1 = 0$ results in the well known biorthogonal modulation. The Variant II code with $k = 1$, $m_1 = n$, $\mu_1 \neq 0$ yields an n-bit PCM. In this case the points of $\mathcal{S}$ correspond to the 2^n vertices of an n dimensional hypercube of edge length $2\mu_1$, centered at the origin.

It is clear that all codewords of both Variant I and II codes lie on a hypersphere of radius $\sqrt{n\mathcal{P}}$ centered at the origin, where $\mathcal{P} = \frac{1}{n}\sum_{j=1}^{k} m_j \mu_j^2$.

Digital transmission over the band-limited fading channel is commonly modeled in the n-dimensional Euclidean space $\mathbb{R}^n$ as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where $\mathbf{y}$ is the received signal vector, $\mathbf{x} = [x_1, \ldots, x_n]$ is the transmitted signal vector (or codeword) taken from a finite signal constellation (or codebook) $\mathcal{S}$, $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \frac{N_0}{2}\mathbf{I})$ is a real Gaussian random vector with i.i.d. components and $\mathbf{H} = \mathrm{diag}(h_1, \ldots, h_n)$ is the diagonal channel fading matrix. We distinguish two cases: i) *slow fading*, where the fading matrix is $\mathbf{H} = h\mathbf{I}$ and h changes independently from one codeword to the next; ii) *fast fading*, where the fading coefficients h_i change independently in each component. Finally, we note that when $\mathbf{H} = \mathbf{I}$ is constant, we fall back into the AWGN channel model. Assuming that the channel state information is available at the receiver, ML detection is given by

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathcal{S}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 = \arg\min_{\mathbf{x}'\in\mathcal{S}'} \|\mathbf{y} - \mathbf{x}'\|^2 \qquad (4)$$

where $\mathcal{S}'$ is the "faded signal constellation" $\mathbf{H}\mathcal{S}$. We note that the complexity of the ML receiver greatly depends on the structure of the code $\mathcal{S}$. In the worst case a total of $M = |\mathcal{S}|$ Euclidean distances must be computed. For large values of M this may be impractical, hence it is common to trade some of the performance for a reduced decoding complexity.

Let us consider ML decoding of Variant I codes in the AWGN case. We need to find the minimum of

$$\|\mathbf{y} - \mathbf{x}_i\|^2 = \|\mathbf{y}\|^2 + n\mathcal{P} - 2(\mathbf{y}, \mathbf{x}_i) \qquad (5)$$

for $i = 0, \ldots, M-1$ where $(\mathbf{y}, \mathbf{x}_i)$ denotes the scalar product of the two vectors. Since $\|\mathbf{y}\|$ is independent of i the ML decoder may simply maximize the scalar product between the received vector and the codewords, i.e.

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}\in\mathcal{S}} \sum_{k=1}^{n} x_k y_k \ . \qquad (6)$$

This maximization problem may be solved as follows. Given the received vector $\mathbf{y}$, replace the smallest m_1 components by the values μ_1, replace the smallest m_2 remaining components with μ_2, etc. Until all the components have been replaced.

The complexity of this decoding algorithm is rather small if compared to the brute force exhaustive search. In particular, it is enough to perform a sorting algorithm on the n components of the received vector and the permutation uniquely identifies the ML decoded codeword.

In the case of slow fading the coefficient h does not affect the minimization (4) but in the case of fast fading the above algorithm fails, so we resort to a new trellis description of permutation codes and perform Viterbi decoding.

References

[1] D. Slepian, Permutation modulation, *Proceedings of the IEEE*, March 1965, pp. 228–236.