Dynamic pricing for connection-oriented services in wireless networks

Emanuele Viterbo and Carla F. Chiasserini
Dipartimento di Elettronica, Politecnico di Torino
Corso Duca degli Abruzzi 24
I-10129 Torino (Italy)
e-mail: `<name>@polito.it`

Abstract—**In this paper, we deal with dynamic pricing strategies for connection-oriented services in wireless systems. Dynamic pricing policies allow the network operator to charge a cost per time unit depending on the network usage. In this way, the users behavior can be regulated and the network management is significantly improved. We model the user demand and the call duration as functions of the service price. By using standard Markovian techniques to represent the system evolution, we devise an optimal linear pricing scheme, which can be easily computed and controlled. When compared with a flat-rate policy, where a constant price for the network services is fixed, the proposed solution is able to provide a better quality of service to the users as well as a greater revenue to the network operator. For example, when eight radio channels are available and the traffic load is equal to 0.8, we obtain a 25% improvement in the network revenue with respect to the flat-rate policy, while the blocking probability is halved.**

I. Introduction

Cellular telephony has experienced an enormous popularity all over the word, reaching up to 60% of penetration in countries such as Italy and Sweden, and 25% in the USA. The enormous growth of the cellular phone market has implied a great demand for radio resources. Since the availability of the radio bandwidth is limited, new solutions to increase the available capacity of radio systems need to be found. Different approaches can be used, from cell splitting and frequency reuse [1] to overlapping cell layers [2], [3] and dynamic channel allocation techniques [4]. However, these methods often imply either an increased system complexity or a significant degradation of the quality of service.

In this paper, we propose a solution based on dynamic pricing techniques, i.e., on pricing strategies where the cost the network operator charges per time unit depends on the network usage and is dynamically adapted to the network status. Indeed, pricing can affect users behavior and allow for an efficient control of the network operational conditions. For example, flat-rate pricing, i.e., a constant price for the network services, is very effective in stimulating new applications development, but it is no longer suitable for an environment with an increasing demand for network resources. On the contrary, by adjusting prices to the usage of the network, we can make a better use of the available bandwidth, and provide the desired quality of service to the users as well as a greater revenue to the network operator. It is intuitive that the trend of users demand during the day can be modified by imposing high rates in the correspondence of peak-traffic time periods and low rates when large radio resources are available. Thus, making prices dependent on the network usage can be an efficient solution to network congestion problems.

Dynamic pricing has been mainly used to control wired networks supporting Internet-based services [5], [6], [7]. In this case, techniques to derive the system optimal rate have been proposed, which charge users on the basis of the congestion they cause to the network. In [8], dynamic pricing has been applied to the wireless environment and simulation results have been presented for a simple pricing strategy.

We consider connection-oriented services in third-generation wireless systems and we propose a solution to dynamic pricing based on an analytical approach. We model the call duration as a function of the service price and the user demand as a decreasing exponential function of the call price per time unit and of the call blocking probability, that is the quality of service (QoS) metric we selected. Then, since customers tend to prefer transparent and easy to understand pricing policies, we impose that prices follow linear dynamics. By using standard Markovian techniques to represent the system evolution, we devise an optimal linear pricing strategy which maximizes the network revenue while providing the required quality of service to the users.

Performance of the proposed pricing strategy is compared to the results obtained through a flat-rate charging policy. When eight radio channels are available and the traffic load is equal to 0.8, we obtain an improvement with respect to the flat-rate policy of 25% in the network revenue, while the blocking probability is halved.

The paper is organized as follows. In Section II, the system model is presented and the performance metrics are defined. Section III shows an example of the improvement that can be obtained in network management by applying a linear dynamic pricing. Section IV draws some conclusions and directions for future work.

II. SYSTEM MODEL

A common econometric model assumption for the user demand function is [9]

$$D(p, Q) = \exp(-\alpha p + \beta Q) \tag{1}$$

where p is the price per time unit, Q is the *quality of service* and α, β are constant parameters related to the user population behavior. We assume as quality of service index the *call success probability*; thus $Q = 1 - P_b$, where P_b is the *call blocking probability*, i.e., the probability that the system can not accept any new call. This demand function represents the users active time expressed in seconds per time unit. This model takes into account a common user behavior: user's demand drops as the price per time unit increases and the quality of service decreases. The parameters α and β must be identified by adequate market research.

We use the above model to derive the traffic intensity by assuming that the demand in the time span Δt is equal to the actual traffic that is generated in Δt, i.e., the traffic intensity γ weighted by the call success probability $1 - P_b$

$$\Delta t \exp[-\alpha p + \beta(1 - P_b)] = \Delta t \gamma (1 - P_b) \ . \tag{2}$$

Let N be the maximum number of available communication channels. The system may be described by a birth-death Markov chain where each state $i = 0, \ldots N$ represents the number of active calls in the network.

Let $\mathbf{p} = (p_0, p_1, \ldots p_{N-1})$ be the *price vector* representing the cost per time unit of a call started when the system is in state i, with $0 \leq i \leq N - 1$. Our main objective is to determine a convenient price vector which optimizes some network performance parameters as we describe in the following.

Let λ_i be the call arrival rate when the system is in state i ($0 \leq i \leq N - 1$) and let μ_i be the average call termination rate when the system is in state i ($1 \leq i \leq N$). According to the above econometric model the call arrival rate in state i is modeled as

$$\lambda_i = \frac{m_i}{1 - P_b} \exp[-\alpha p_i + \beta(1 - P_b)] \quad 0 \leq i \leq N - 1 \tag{3}$$

where we have set $\gamma = \lambda_i / m_i$, being m_i the average call duration of a call starting in state i.

Since we expect that users will react to the increase of price per time unit by reducing their call duration, we define T_{min} and T_{max} as the minimum and the maximum call duration times,respectively. We model the average call duration of a call starting in state i as a decreasing function of the call price per time unit in the range $[T_{min}, T_{max}]$ as

$$\frac{1}{m_i} = T_{max} \exp[-K_m(p_i - p_0)] \quad 0 \leq i \leq N - 1 \tag{4}$$

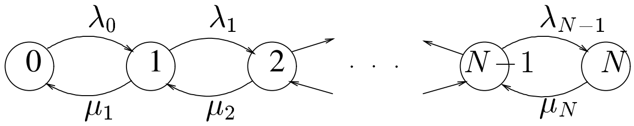

Fig. 1. Markov model.

where $K_m = \ln(T_{max}/T_{min})/(p_{N-1} - p_0)$. For example, in the case of voice traffic, a reasonable choice could be $T_{max} =$ 180s and $T_{min} =$ 60s.

Assuming that call durations are exponentially distributed with rates m_i ($0 \leq i \leq N - 1$), the average call termination rate in state i is given by

$$\mu_i = m_0 + m_1 + \cdots + m_{i-1} \quad 1 \leq i \leq N \ . \tag{5}$$

The steady state solution of the Markov chain shown in Figure 1 is the probability vector $(\pi_0, \pi_1, \ldots \pi_N)$ where

$$\begin{aligned} \pi_0 &= \left(\sum_{k=0}^{N} \prod_{j=1}^{k} \frac{\lambda_{j-1}}{\mu_j} \right)^{-1} \\ \pi_i &= \pi_0 \prod_{j=1}^{i} \frac{\lambda_{j-1}}{\mu_j} \ . \end{aligned} \tag{6}$$

The system blocking probability is $P_b = \pi_N$. Since λ_i and μ_i are also functions of P_b, the steady state solution must be computed iteratively until convergence on P_b is reached.

Let $\mathbf{g} = (g_1, \ldots g_N)$ be the revenue vector representing the total revenue per time unit of the network operator when the system is in state $1 \leq i \leq N$. We have

$$g_i = \sum_{j=0}^{i-1} p_j \tag{7}$$

since when the system is in state i each user has started a call in one of the previous states $j < i$.

The total *network revenue per time unit* is given by

$$G = \sum_{i=1}^{N} \pi_i g_i \tag{8}$$

and is of interest to the network operator.

A useful performance parameter of interest to the users is the call *average cost per time unit*, which is given by

$$\overline{c} = \sum_{i=0}^{N-1} \pi_i p_i \ . \tag{9}$$

The aim of this paper is to evaluate a pricing policy (i.e., a price vector) maximizing the total network revenue G, while reducing the call blocking probability P_b. This optimization problem in its general form may be hard to solve numerically. We consider a suboptimal approach in the sense that we assume a linearly increasing price vector which gradually discourages users from starting new calls as the network load increases. A further motivation for the linear-rate choice is the simplicity of this pricing policy, which can be better understood by the users. The optimal slope of the linearly increasing price vector is determined in order to maximize the total network revenue G. We call this price vector the optimal *linear-rate policy*. As outcome of this procedure we also obtain the corresponding P_b, which is a metric of interest to the users.

As a baseline for comparisons we consider the *flat-rate policy* where the price vector is constant and the average call duration is fixed to T_{max}. In particular, we select a flat price which lays between the minimum and the maximum value of the linear-rate prices. The fact that the initial values of the linear-rate price vector are lower than the flat price is the main incentive for the users to place calls during off-peak time.

III. Results

In this section, we show some results obtained using our analytical model for the following scenario. We consider different user populations by fixing $\alpha = 0.1$ (unless a different value of α is explicitly specified) and choosing $\beta/\alpha = 1$ and 4, in networks with $N = 8$ and $N = 16$ radio channels. The flat-rate policy is compared to the linear-rate policy on the basis of the network revenue per time unit and of the blocking probability.

In the following, all curves comparing the different strategies are given in terms of the traffic load ρ, that we have when the flat-rate policy is applied. Denoting by λ_f the call arrival rate relative to the flat-rate policy, we write

$$\rho = \lambda_f T_{max} \ . \tag{10}$$

Figure 2 shows how the network revenue can be increased by using the linear-rate strategy. For $\rho < 0.4$ there is a minor loss in network revenue when using the linear policy. This loss may be greatly compensated by the gain obtained when $\rho > 0.4$: for example for $\rho = 0.8$ we observe a 25% improvement in G of the linear-rate over the flat-rate policy. We also note that results are mildly affected by the parameter α/β.

Figure 3 shows how the blocking probability can be reduced by using the linear-rate policy. This may be a secondary desired goal of the network operator but it is of great interest to the users. By applying the linear-rate pricing scheme, P_b can be reduced by a factor ranging between 2 and 10 as ρ decreases. Here the dependence on α and β is more critical for high ρs.

Figure 4 compares on a single plot G and P_b as functions of ρ in the range $[0.1, 1]$ in steps of 0.1, for $N = 8$ and $N = 16$. Globally better performance can be noticed when the curves

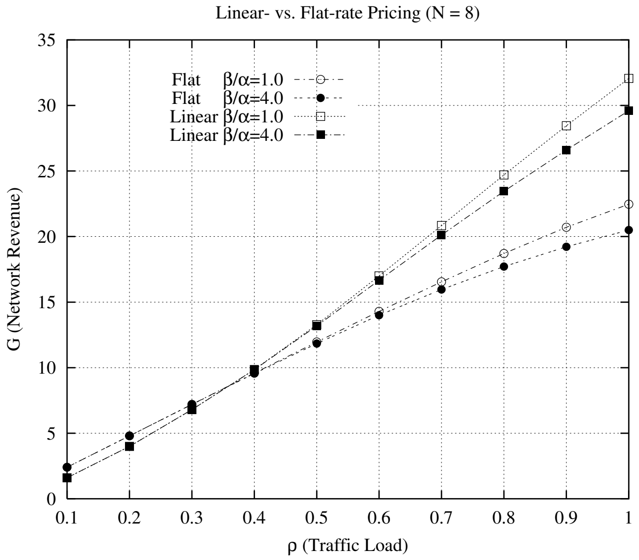

Fig. 2. Linear- vs. Flat-rate: network revenue per time unit vs. traffic load for different users' behavior ($N = 8$).

move toward the upper left corner of the plot. Only the case $\alpha/\beta = 1$ is shown since we observed only minor differences when $\alpha/\beta = 4$.

Similar curves are presented in Figure 5, where results are derived for $N = 16$ and α varying. Although the linear-rate policy still outperforms the flat-rate strategy, for $\alpha = 0.2$ the linear-rate gives remarkably worse performance in terms of G than for $\alpha = 0.1$. This is because the user demand becomes more sensitive to the price growth when higher values of α are considered. Therefore, as prices increase, the reduction in the user demand becomes more significant. Clearly, in the case of flat-rate, varying α does not affect the performance since the price is kept constant.

Finally, Figure 6 shows the average cost per time unit as a function of ρ. The flat-rate price was selected so that for low traffic load ($\rho < 0.13$), the linear policy becomes more attractive to the users. Note that in the flat-rate case the average cost per time unit decreases when traffic load is high; this is explained by looking at (9) and considering that for high traffic load the blocking probability becomes relevant.

We observe that the results given here are a simple example of the possibilities offered by our dynamic pricing scheme. More realistic parameters should be evaluated by the network operator according to their user population behavior analysis. The proposed method is an extremely flexible and simple tool to estimate the performance of dynamic pricing policies.

IV. Conclusions and Future Work

In this paper, a dynamic pricing scheme for connection-oriented services in wireless communication systems was

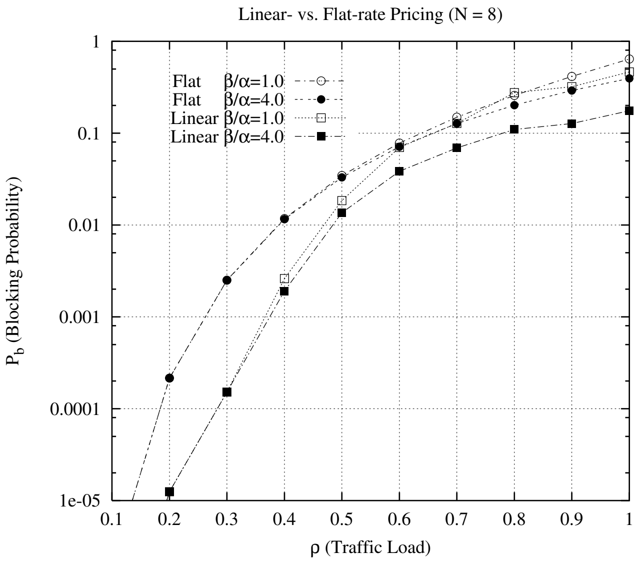

Fig. 3. Linear- vs. Flat-rate: call blocking probability vs. traffic load for different users' behavior ($N = 8$).

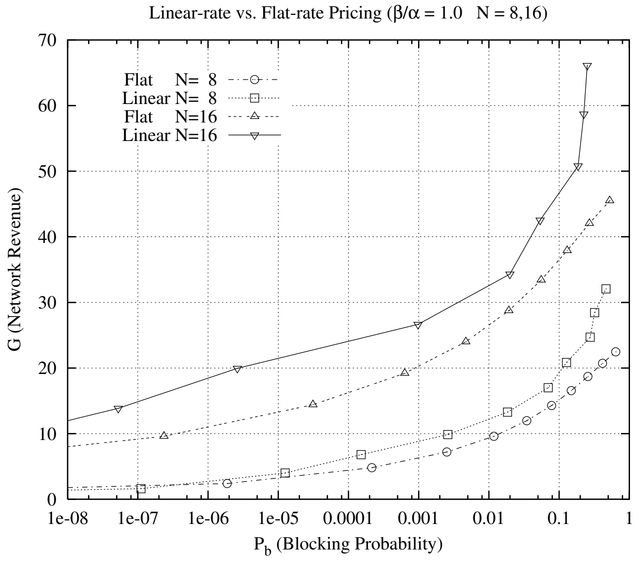

Fig. 4. Linear- vs. Flat-rate: network revenue per time unit vs. call blocking probability for $N = 8, 16$.

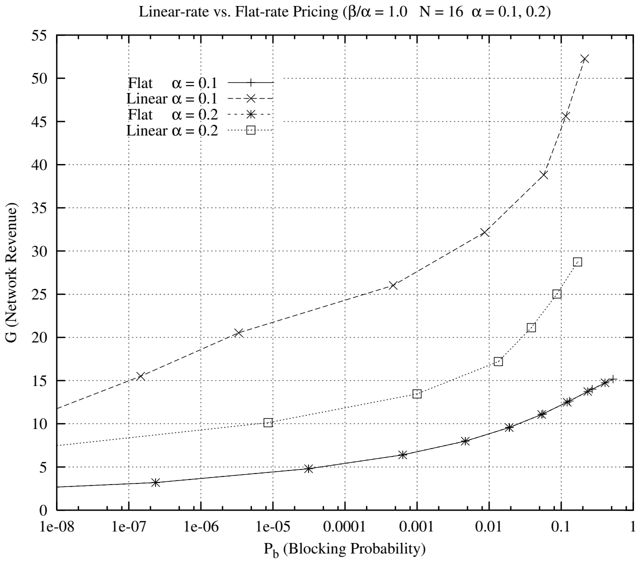

Fig. 5. Linear- vs. Flat-rate: network revenue per time unit vs. call blocking probability for $\alpha = 0.1, 0.2$ and $N = 16$.

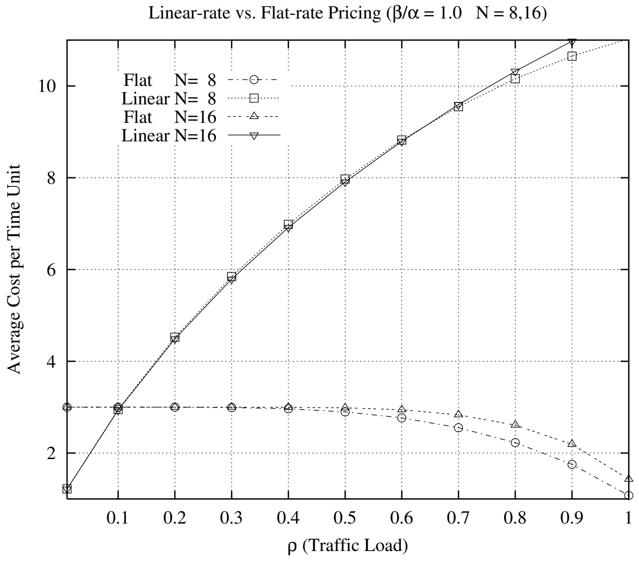

Fig. 6. Linear- vs. Flat-rate: user average cost per time unit vs. traffic load for $N = 8, 16$.

presented. We modeled the user demand and call duration as functions of the service price. Then, by using standard Markovian techniques, an optimal linear pricing policy was derived in order to obtain a transparent and easily controllable pricing strategy. The performance of the proposed solution was derived and compared to the results obtained when a flat-rate policy is applied. It was shown that by adjusting prices to the usage of the network, we can make a better use of the available bandwidth, and provide a greater revenue to the network operator as well as an improved quality of service to the users.

Future work will study the performance of dynamic pricing when connection-oriented and connection-less services share the same pool of network resources. Alternative dynamic pricing strategies and different models of user demand will be analyzed.

REFERENCES

[1] T.S. Rappaport, *Wireless communications: Principles and practice,* Prentice Hall, Upper Saddle River, NJ, 1996.

[2] R. Beraldi, S. Marano, C. Mastroianni, "A reversible hierarchical scheme for microcellular systems with overlaying macrocells," *IEEE INFOCOM'96*, pp. 51–58, 1996.

[3] A. Grimaldi, S. Marano, C. Mastroianni, M.T. Scarpelli, "A macro-microcellular system with mobile user speed estimation," *IEEE PIMRC'97,* pp. 381–385, 1997.

[4] J. Gibson, *The Mobile Communications Handbook,* IEEE Press, 1996.

[5] D. Songhurst, F.P. Kelly, "Charging schemes for multiservice networks," *IEE Colloquium on Charging for ATM,* 1996.

[6] R.J. La, V. Anantharam, "Charge-sensitive TCP and rate control in the Internet," *IEEE INFOCOM 2000,* Tel Aviv, Israel, pp. 1166–75, March 2000.

[7] I.C. Paschalidis, J.N. Tsitsiklis, "Congestion-dependent pricing of network services," *IEEE/ACM Transactions on Networking,* Vol. 8, No. 2, pp. 171–84, April 2000.

[8] E.D. Fitkov-Norris, A. Khanifar, "Dynamic pricing in mobile communication systems," *3G Communication Technologies Conference,* pp. 416–20, 2000.

[9] L.D. Taylor, *Telecommunications demand in theory and practice,* Kluwer Academic Publishers, Boston, 1994.