Rotated Trellis Coded Lattices

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1 Introduction

Lattice sphere packings have been studied by mathematicians since the beginning of this century [1]. A lattice is a discrete group of the real N-dimensional space R^N. A finite subset of a lattice, usually called a lattice constellation, is used as a linear modulation and can be viewed as a generalized Quadrature Amplitude Modulation. The densest integer lattices are known in dimensions up to 32 [1]. These lattices are matched to additive white Gaussian noise channels and exhibit a gain of 4 to 6dB in signal-to-noise ratio. This gain, due to the lattice density, disappears on Rayleigh fading channels where the performance is essentially related to the *lattice diversity* [2]. A block code can fill up to ρ erasures if $\rho + 1 \leq d_{H,min}$, where $d_{H,min}$ is the minimum Hamming distance. In an similar manner, a lattice has an L-diversity order if the minimum Hamming distance between the lattice points is equal to L. For such a lattice constellation the bit error rate curves decrease linearly on a logarithmic plot, since $BER = K/SNR^L$. This diversity is very much increased by rotating the original lattice. The multidimensional rotation spreads the same information over many axes which creates the so called *modulation diversity* since all axes cannot be faded at the same time. Many techniques have been found recently [3] to compute multidimensional rotations with high diversity. In this paper we combine rotations with trellis coded modulation (TCM) to create a rotated trellis coded modulation (RTCM) matched to Rayleigh fading channels.

2 RTCM Encoding and Decoding

Two methods are available to map m information bits into one N-dimensional point of an integer lattice constellation. The lattice encoding is done directly by multiplying N integers with the lattice generator matrix or by partitioning a bidimensional QAM constellation and combining it with a binary encoder. To build the RTCM encoder, we start by partitioning a lattice into sublattices (cosets), we associate an encoder (a binary block encoder + a bidimensional QAM) to each coset and we add a binary convolutional code that selects the coset to be transmitted. Uncoded information bits select the point inside the coset. This encoding scheme is analog to that of Multidimensional TCMs [5]. Finally, the encoded point is rotated before beeing transmitted over a fading channel.

Partition	Index	rate	Size
$Z^4/2D_4$	32	4/5	512
$D_4/2D_4$	16	3/4	512
Z^8/E_8	16	3/4	2^{17}
E_8/RE_8	16	3/4	2^{17}
Z^{16}/H_{16}	32	4/5	2^{33}

The above Table shows five possible RTCMs. The first column gives the quotient group as Λ_2/Λ_1 where Λ_1 is a sublattice of Λ_2. The number of cosets of Λ_1 is indicated in the second column. The last columns give the convolutional encoder rate and the constellation size for a spectral efficiency of 2 bits per dimension.

For example, the 8-dimensional RTCM Z^8/E_8 exhibits a gain of $7dB$ at 10^-3 when compared to the same TCM without rotation. This RTCM has the same performance as a 16-dimensional rotation over the Rayleigh channel. The RTCM decoding combines a sphere-decoding algorithm [4] with a Viterbi algorithm. The sphere-decoder chooses the nearest lattice point and the Viterbi algorithm uses the squared distance between the received point and the decoded one to find the optimal trellis survivor.

3 Conclusions

For high spectral efficiency ($\geq$ 4 bits per symbol), a powerful trellis coded modulation designed for the additive white Gaussian noise channel has a poor performance over the Rayleigh fading channel. This performance is largely improved by a multidimensional rotation that creates a high diversity order which is essential when transmitting over fading channels.

References

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