Algebraic Construction of Good Collision Resistant Signal Sets *

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Abstract

We consider codes of rank 1 over $\mathbb{Z}_{2^m}$ in order to construct signal space codes for the collision channel. To achieve collision resistance we search among codes generated by vectors entirely consisting of units of the underlying ring. It turns out that the resulting signal sets improve over previously known collision resistant signal sets. A further advantage is that they allow for efficient decoding.

Key Words: Codes over Integer Residue Rings, Weight-Functions, Wireless Multiple Access Collision Channel, Fading.

1 Introduction

A novel class of signal space codes called Collision Resistant Modulations (CRM) for bandwidth efficient transmission on a random access collision channel, as in packet-radio applications, was proposed in [3]. There it is shown that CRMs achieve a significant gain over traditional modulation formats under a Slotted ALOHA protocol. We briefly summarize some of the key ideas presented in [3] in order to set the basis of the present paper.

We model the collision channel with AWGN and fading as the on-off vector channel

$$\mathbf{y} \;=\; C(G\mathbf{x}+\mathbf{n}) \tag{1}$$

Here $\mathbf{x}=(x_1,\ldots,x_n)$ is a transmitted word taken from the signal set S (signal-space code), $\mathbf{n}=(n_1,\ldots,n_n)$ is additive white Gaussian noise with i.i.d. components $n_i=\mathcal{N}(0,N_0/2)$ and the matrix $G=\mathrm{diag}(g_1,\ldots,g_n)$ represents the channel fading coefficients $g_i\geq 0$. The matrix $C=\mathrm{diag}(c_1,\ldots,c_n)$ represents the collision pattern with $c_i\in\{0,1\}$ for all $i=1,\ldots,n$, where $c_i=0$ indicates the presence of a collision on the i-th signal component. Finally $\mathbf{y}=(y_1,\ldots,y_n)$ is the received signal vector.

The multiple access scheme induces a certain collision statistics. For example, if the users randomly access any n slots of a common channel to transmit the n components of a signal, the c_i may be considered as Bernoulli i.i.d. random variables with $P_c=P(c_i=0)$ and if C has k collisions

$$P(C)=P_c^k(1-P_c)^{n-k} \tag{2}$$

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The fading coefficients g_i are random variables whose joint statistics depends on the physical characteristics of the propagation channel, such as the multi-path delay profile and the Doppler spectrum.

We assume that the receiver has perfect channel state information (CSI) on both the collisions and the fading, i.e., has a complete knowledge of C and G. In this case, the Maximum Likelihood (ML) detection is obtained by minimizing over all $\mathbf{x} \in S$ the metric

$$d_C^2(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} c_i (y_i - g_i x_i)^2 \qquad (3)$$

This detection criterium corresponds to the ML detection for the signal set S_C in an $(n-k)$-dimensional Euclidean space, where S_C is the projection of S on the subspace generated by the $n-k$ axes corresponding to the non zero c_i (k is the number of collisions in the pattern C). According to this detection criterium, in order to avoid systematic errors in the presence of $k < n$ collisions, we require that the points in S_C are distinct, for all C with Hamming distance $w_H(C) > 0$. Then, we have the following

Definition 1.1 A *Collision Resistant Modulation* (CRM) is a signal-space code S with the property that any projection on any coordinate subspace has the same number of distinct points, i.e., $M = |S_C| = |S|$ for all non-zero collision patterns C.

Equivalently, the words in S must have all distinct coordinates or, more precisely, the Hamming distance between any pair of words in S must be n. A similar requirement is imposed in the design of *high diversity* signal constellations for the fading channel where the number of distinct components is called *modulation diversity* [1, 2].

The symbol error probability is given by

$$P(e) = \sum_{C} P(C) P(e|C) \qquad (4)$$

where C runs over all 2^n collision patterns, $P(C)$ is given in (2) and by using the Union bound

$$P(e|C) \le \frac{1}{M} \sum_{\mathbf{x} \in S_C} \sum_{\mathbf{x} \neq \widehat{\mathbf{x}}} P(\mathbf{x} \to \widehat{\mathbf{x}}) \qquad (5)$$

where $P(\mathbf{x} \to \widehat{\mathbf{x}})$ is the pairwise error probability averaged with respect to G. $\{\mathbf{x} \to \widehat{\mathbf{x}}\}$ represents the event that the decoder chooses $\widehat{\mathbf{x}}$ when $\mathbf{x}$ was actually transmitted, as if $\mathbf{x}$ and $\widehat{\mathbf{x}}$ were the two only possible decoder outcomes. We refer to this as *pairwise error event*.

Expressions for the pairwise error probability for both the Gaussian and independent Rayleigh fading channels can be found in [3]. These show that in order to reduce the symbol error probability it is important to maximize different types of minimum distances of all the constellations S_C according to the type of channel being used.

Given any two signal words $\mathbf{s}$ and $\mathbf{t}$ in S we consider three types of distances:

- $d_E^2(\mathbf{s}, \mathbf{t}) := \|\mathbf{s} - \mathbf{t}\|_2^2 = \sum_{i=1}^{n} |s_i - t_i|^2$, the Squared Euclidean distance (SED). This is appropriate on the pure AWGN channel without fading.

- $d_P(\mathbf{s}, \mathbf{t}) := \prod_{i=1}^{n} |s_i - t_i|$, the Product distance. This is appropriate in the presence of independent Rayleigh fading.

- $d_L(\mathbf{s}, \mathbf{t}) := \|\mathbf{s} - \mathbf{t}\|_1 = \sum_{i=1}^{n} |s_i - t_i|$, the ℓ_1-distance. This is appropriate in the presence of Rice fading.

Note that the product distance is not a distance function in the narrow sense in that is does not satisfy the triangle inequality in general. In the following we speak about the weights w_E, w_P and w_L of the symbols in S as their distance from the origin.

In [3] some low dimensional CRMs are shown which could only be decoded by exhaustive search through the codebook. Here we are interested in higher dimensional CRMs which provide greater minimum distances in the projected constellations S_C and present a structure that can be effectively exploited for decoding.

2 The algebraic construction

Let $\mathbb{Z}_M$ denote the ring of integers modulo M where $M = 2^m$, and write $\mathbb{Z}_M = \{-M/2 + 1, \ldots M/2\}$ for convenience. Note that the usual mod M operation has to be slightly modified in order to match the symmetric representation of $\mathbb{Z}_M$.

We consider linear codes of rank 1 over $\mathbb{Z}_M$ and first note that the generator matrix of such a code is a single word $\mathbf{v} = (v_1, \ldots v_n)$ that contains at least one unit of $\mathbb{Z}_M$. The $\mathbb{Z}_M$-code generated by it is given by $\mathcal{C} = \{\beta\mathbf{v} | \beta \in \mathbb{Z}_M\}$. As corresponding signal set S we take the code words identically embedded as real-valued vectors into the Euclidean space $\mathbb{R}^n$. Under the above premise this embedding is a weight preserving mapping of $(\mathbb{Z}_M, w_{\text{Lee}})$ into $(\mathbb{R}, \ell_1)$.

In order to obtain the collision resistance property we require that the generating word entirely consists of units of $\mathbb{Z}_M$. In fact the i-th component βv_i of each codeword are all distinct as β runs over $\mathbb{Z}_M$. For $M = 2^m$ the unit group $U(\mathbb{Z}_M)$ is the set of odd numbers in $\mathbb{Z}_M$.

Proposition 2.1 *The minimum weight of a vector in S is equal to the minimum distance between any two vectors of S for all three types of distances.*

Proof: In order to see this denote by w one of the respective weight functions w_L, w_E or w_P on $\mathbb{Z}^n$ and denote by d the corresponding distance function. Then we obviously have

$$w_{\min}(S) \geq d_{\min}(S) \geq d_{\min}(\Lambda) = w_{\min}(\Lambda)$$

where $\Lambda := \mathcal{C} + M\mathbb{Z}^n$ is the lattice resulting from $\mathcal{C}$ by *Construction A* (cf. [5]). Let $\mathbf{x} := (x_1, \ldots, x_n) \in \mathbb{Z}^n$ be given such that $w(\mathbf{x}) = w_{\min}(\Lambda)$. If $\mathbf{x} \notin S$ then exchanging x_1 by $x_1' := x_1 \pmod{M}$ produces a vector $\mathbf{x}' \in \Lambda$ with $w(\mathbf{x}') \leq w(\mathbf{x})$ and hence $w(\mathbf{x}') = w(\mathbf{x})$ because of minimality. Without loss of generality we may therefore assume $\mathbf{x} \in S$, and hence conclude $w_{\min}(\Lambda) \geq w_{\min}(S)$. All in all we have shown

$$d_{\min}(S) = w_{\min}(S)$$

which proves our claim. □

We finally note that equivalent signal sets S in terms of error performance are obtained by taking permutations and sign-changed versions of the generator vector of $\mathcal{C}$.

2.1 Signal sets in dimension $M/4$

In this section we show that the signal sets generated by the above codes can be easily characterized for all the three metrics when $n = M/4 = 2^{m-2}$. In the following the double factorial $(2\alpha + 1)!!$ denotes the product $(2\alpha + 1)(2\alpha - 1) \cdots 3 \cdot 1$.

Proposition 2.2 *Let $\mathbf{v} = (1, 3, \ldots, M/2 - 1)$ then for the $[M/4, 1]$ code $\mathcal{C}$ generated by $\mathbf{v}$ the following holds:*

(a) *the minimum Lee weight of $\mathcal{C}$ is $M^2/16$,*

(b) *the minimum Euclidean weight of $\mathcal{C}$ is $M^3/64$, and*

(c) *the minimum product weight of $\mathcal{C}$ is $(M/2-1)!!$.*

Proof: Let $\beta\mathbf{v}$ be a word in $\mathcal{C}$ and let $\beta = 2^k u$ for appropriate $k \le m$ and $u \in U(\mathbb{Z}_M)$. Since multiplication by u^{-1} merely applies a permutation and sign changes in $\beta\mathbf{v}$, we may without loss of generality suppose $u = 1$. For our claim concerning the Lee weight we then compute

$$\begin{aligned}
\|\beta\mathbf{v}\|_L &= \sum_{i=1}^{n} |2^k(2i-1)| \\
&= 2^{2k} \sum_{i=1}^{2^{m-1-k}} (2i-1) \\
&= 2^{2k}(2^{m-2-k}(2^{m-2-k}+1) - 2^{m-2-k}) \\
&= 2^{2m-4} = \frac{M^2}{16}
\end{aligned}$$

as claimed. Note that $\mathcal{C}\setminus\{\mathbf{0}\}$ is a constant Lee weight code. For the Euclidean weight we similarly have

$$\begin{aligned}
\|\beta\mathbf{v}\|_E &= 2^k \sum_{i=1}^{\frac{n}{2^k}} 2^{2k}(2i-1)^2 \\
&= \frac{4}{3}n^3 - \frac{4^k}{3}n = \frac{4n}{3}(n^2 - 4^{k-1})
\end{aligned}$$

which obviously assumes its minimum for $k = m-2$. Hence, in this case the resulting minimum weight is given by $8^m/64 = M^3/64$. For the product distance we have

$$\|\beta\mathbf{v}\|_P = \Big(\prod_{i=1}^{\frac{n}{2^k}} 2^k(2i-1)\Big)^{2^k}.$$

If we prove that this is an increasing function of k then the minimum product weight occurs at the unital multiples of $\mathbf{v}$ and hence on $\mathbf{v}$ itself, where it is given by $(M/2-1)!!$. Defining

$$g(k) := \Big(\prod_{i=1}^{\frac{n}{2^k}} 2^k(2i-1)\Big)^{2^k}$$

we have to show that $g(k+1)/g(k) \ge 1$ for all $0 \le k \le m-3$. An easy transformation shows that this is equivalent to

$$\frac{\prod_{i=1}^{\frac{n}{2^k}}(2i-1)}{\prod_{i=1}^{\frac{n}{2^{k+1}}}(2i-1)^2} \le 2^{\frac{n}{2^k}}$$

for all $0 \le k \le m-3$. Substituting $s := n/2^k$ and filling up the double factorials the latter holds if and only if

$$\frac{\binom{4s}{2s}}{\binom{2s}{s}} \le 2^{2s}$$

for all $s = 1, 2, 4, \ldots, 2^{m-k}$. This, however, it is easily verified because of

$$\binom{4s}{2s} = \sum_{i=0}^{2s}\binom{2s}{2s-i}\binom{2s}{i} \le \binom{2s}{s}\sum_{i=0}^{2s}\binom{2s}{i} = \binom{2s}{s}2^{2s}.$$

$\square$

In Table 1 some minimum distances of the above codes are reported.

M	$d_{L,min}$	$d^2_{E,min}$	$d^{1/n}_{P,min}$
16	16	64	3.201
32	64	512	18.255
64	256	4096	759.473
128	1024	32768	$1.963 \cdot 10^6$
256	4096	262144	$2.524 \cdot 10^{13}$
512	16384	2097152	$1.232 \cdot 10^{28}$
1024	65536	16777216	$1.797 \cdot 10^{58}$

Table 1: Codes for $n = M/4$ generated by $\mathbf{v} = (1, 3, \ldots, M/2 - 1)$

2.2 Punctured codes

In order to improve the spectral efficiency of the previous signal sets we should consider some punctured versions of the codes presented in the previous section. The puncturing operation produces lower dimensional signal sets with the same number of points of the original signal set, due to the collision resistance property. We will consider the case $n = m$, which corresponds to 1 bit/dim spectral efficiency.

Tables 2, 3 and 4 show some optimal codes giving the greatest minimum distances. These were found by an exhaustive search through all $\mathbf{v} = (1, v_2, \ldots v_n)$ and $v_i \in U(\mathbb{Z}_M)$ all distinct since they are obtained by puncturing the codes in the previous section. We note that any vector $\beta\mathbf{v}$ with $\beta \in U(\mathbb{Z}_M)$ generates the same code. Then, without loss of generality, we can always assume the first component of $\mathbf{v}$ to be equal to 1. Due to the rapidly increasing number of generator vectors to be tested the search for these optimal codes could only be performed up to dimension $n = 7$.

M	$d_{L,min}$	$\mathbf{v}$
16	16	[1,3,5,7]
32	30	[1,3,5,11,15]
64	68	[1,3,5,17,23,31]
128	159	[1,5,17,25,27,57,63]

Table 2: Optimal punctured codes for the Lee metric $n = m$

M	$d^2_{E,min}$	$\mathbf{v}$
16	64	[1,3,5,7]
32	253	[1,3,5,7,15]
64	1222	[1,3,5,7,17,31]
128	5488	[1,3,5,23,33,49,63]

Table 3: Optimal punctured codes for the Euclidean metric $n = m$

We next consider a construction which we conjecture to produce comparatively good signal sets. These are constructed from the generating vectors $\mathbf{v} = (1, |\alpha|, |\alpha^2| \ldots |\alpha^{n-1}|)$, where α is

M	$d_{P,min}^{1/n}$	$\mathbf{v}$
16	3.201	[1,3,5,7]
32	4.638	[1,3,5,11,15]
64	8.363	[1,7,13,17,23,31]
128	15.554	[1,15,17,19,27,39,63]

Table 4: Optimal punctured codes for the Product metric $n = m$

selected among the elements of maximal order of $U(\mathbb{Z}_M)$.

Remark 2.3 Referring to [6, Chap. 4], we recall that the group of units $U(\mathbb{Z}_M)$ consists of all odd numbers in $\mathbb{Z}_M$ and is isomorphic to $C_2 \times C_{2^{m-2}}$ for all $m \geq 2$, whereas for the finite field $\mathbb{Z}_2$ it is clearly the trivial one-element group. For $M \geq 16$ the elements of maximal (multiplicative) order $M/4$ in $\mathbb{Z}_M$ are given by $(8k \pm 3) \pmod{M}$. To see this it is enough to show that $z^{M/8} \neq 1 \pmod{M}$ for all z of that form. It is clear that $(8k \pm 3)^{M/8} = 1 \pmod 4$. An induction over m, where $M = 2^m$, first proves the claim to be true for $m = 3$. In general we have $(8k \pm 3)^{2^{m+1-3}} - 1 = ((8k \pm 3)^{2^{m-3}})^2 - 1 = ((8k \pm 3)^{2^{m-3}} + 1)((8k \pm 3)^{2^{m-3}} - 1)$. If the latter were equal to 0 modulo M, then by induction hypothesis $(8k \pm 3)^{2^{m-3}} + 1 = 0 \pmod 4$, a contradiction. A similar argument shows that the elements of type $8k \pm 1$ have lower order.

Tables 5, 6 and 7 show the codes generated by some of these elements giving the best minimum distance. The generator vector is given only for the first α in the list. In order to show the interest in optimizing the choice of α, we also report in parentheses the distance obtained by the worst choice of α. It is an unsolved problem to relate the minimum weight of these codes to their generator vector and the choice of α.

M	$d_{L,min}$	α	$\mathbf{v}$
16	16	3,5	[1,3,7,5]
32	29	3,5,11,13	[1,3,9,5,15]
64	(60) 66	19,27	[1,19,23,11,17,3]
128	(107) 145	11,19,27,35	[1,11,7,51,49,27,41]
256	(256) 324	35,117	[1,35,55,123,47,109,25,107]
512	(489) 679	109,155	[1,109,105,181,239,61,7,251,223]
1024	(932) 1576	293,339	[1,293,167,221,241,43,311,13,287,123]

Table 5: Good punctured codes for the Lee metric $n = m$

3 Comparison with rotated hypercube

In order to better understand the improvements given by the new signal sets we will compare, as an example, the 4-dimensional rotated hypercube signal set [2, 3] with the $\mathbb{Z}_{16}$ signal set generated by $\mathbf{v} = [1, 3, 5, 7]$. Tables 8,9,10 and 11 show the Euclidean and the product distances of all the projections S_C of both signal sets. Figures 1 and 2 show the two dimensional projections on the six coordinate planes of both signal sets. Finally, Figures 3 and 4 show some approximate curves of the symbol error probabilities $P(e)$ (solid lines) and some error

M	$d^2_{E,min}$	α	$\mathbf{v}$
16	64	3,5	[1,3,7,5]
32	(237) 253	5,13	[1,5,7,3,15]
64	(942) 1118	11,29	[1,3,9,11,17,3]
128	(3023) 4444	3,43	[1,3,9,27,47,13,39]
256	(12392) 18600	5,51	[1,5,25,125,113,53,9,45]
512	(39657) 85897	45,91	[1,45,23,11,17,253,121,187,223]
1024	(159106) 354370	293,339	[1,293,167,221,241,43,311,13,287,123]

Table 6: Good punctured codes for the Euclidean metric $n = m$

M	$d^{1/n}_{P,min}$	α	$\mathbf{v}$
16	3.201	3,5	[1,3,7,5]
32	(4.237) 4.309	3,11	[1,3,9,5,15]
64	(6.289) 7.911	19,27	[1,19,23,11,17,3]
128	(8.287) 14.581	19,27	[1,19,23,53,17,61,7]
256	(14.987) 25.545	35,117	[1,35,55,123,47,109,25,107]
512	(22.383) 49.926	35,117	[1,35,201,133,47,109,231,107,161]
1024	(46.491) 95.459	251,461	[1,251,487,381,399,203,247,467,481,101]

Table 7: Good punctured codes for the Product metric $n = m$

probabilities conditioned on the number of collisions k (dashed curves from left to right $k = 0, 1, 2, 3$). In particular, we write

$$P(e) \leq \sum_{k=0}^{4} \binom{n}{k} P_c^k (1 - P_c)^{n-k} P_{\max}(e|k) \tag{6}$$

where $P_{\max}(e|k)$ is the largest error probability for given a collision pattern with k collisions. In the case of an AWGN channel we simply approximated $P_{\max}(e|k)$ with $\frac{1}{2}\mathrm{erfc}(d_{k,\min}/(2\sqrt{N_0}))$ where $d_{k,\min}$ is the smallest entry in the k-th row of Tables 8 and 9. Comparing Figures 3 and 4 we observe an improvement in the symbol error probability in the range 10–35dB of E_b/N_0 for the $\mathbb{Z}_{16}$ signal set.

k	C with k collisions					
0	86					
1	32.88	32.88	32.88	32.88		
2	12.59	9.08	4.41	4.41	9.08	12.59
3	0.06	0.06	0.06	0.06		

Table 8: Minimum SED of all the projections S_C of the rotated 4D-hypercube signal set arranged according to the number of collisions k.

k	C with k collisions					
0	64					
1	35	35	35	35		
2	10	10	8	8	10	10
3	1	1	1	1		

Table 9: Minimum squared Euclidean distances of all the projections S_C of the $\mathbb{Z}_{16}$ signal set arranged according to the number of collisions k.

k	C with k collisions					
0	184.9					
1	12.48	12.48	12.48	12.48		
2	2.33	1.98	2.21	2.21	1.98	2.33
3	0.24	0.24	0.24	0.24		

Table 10: Minimum product distances of all the projections S_C of the rotated 4D-hypercube signal set arranged according to the number of collisions k.

4 The Decoder

We show here how to perform efficient ML detection in the presence of collisions for the signal sets we have constructed. Let us consider the lattice $\Lambda = \mathcal{C} + M\mathbb{Z}_M^n$ obtained by applying Construction A to a $[n, 1]$ code over $\mathbb{Z}_M$ generated by the vector $\mathbf{v} = (1, v_2, \ldots v_n)$

$$\Lambda = \mathcal{C} + M\mathbb{Z}_M^n$$

This lattice possesses a generator matrix of the form

$$G = \begin{pmatrix} 1 & v_2 & \cdots & v_n \\ 0 & M & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & M \end{pmatrix}$$

Given the collision pattern C with k collisions, the resulting projected lattice Λ_C has a generator matrix G_C which is obtained from G by removing the k rows and the columns corresponding to the collided components.

We conclude that all the signal sets S_C are lattice constellations hence we are able to decode them efficiently by applying the Universal Lattice decoder proposed in [7] and [8]. The search radius for the decoder can be adjusted according to the number of collisions in order to increase the decoding speed.

k	C with k collisions					
0	105					
1	15	15	15	15		
2	3	3	4	4	3	3
3	1	1	1	1		

Table 11: Minimum product distances of all the projections S_C of the $\mathbb{Z}_{16}$ signal set arranged according to the number of collisions k.

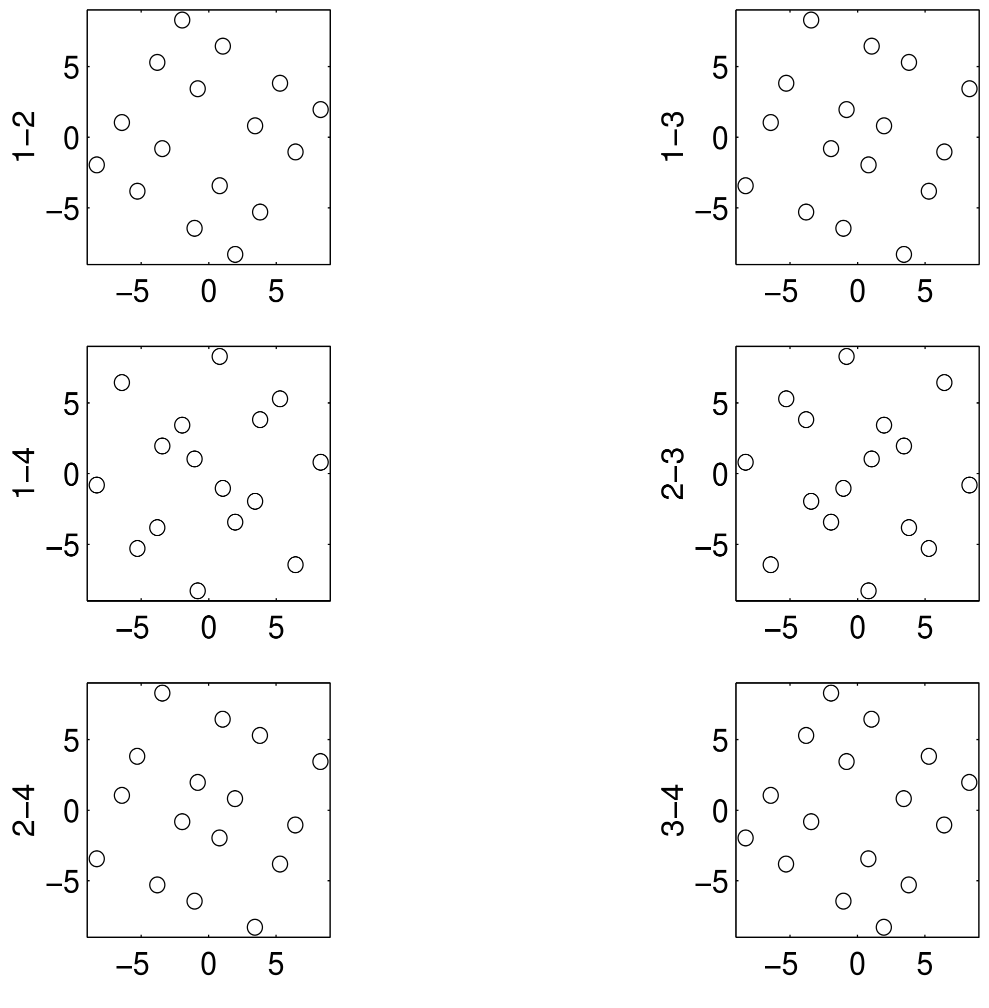

Figure 1: The 6 two-dimensional projections of the rotated 4D-hypercube signal set.

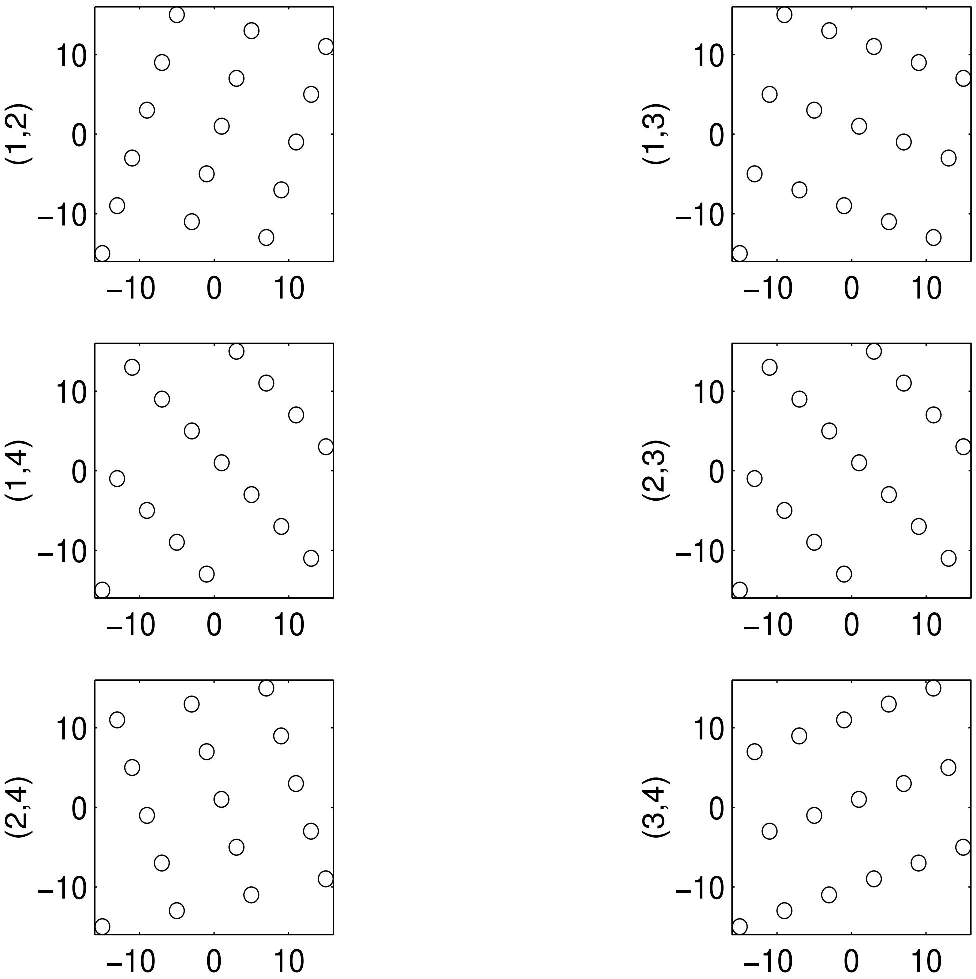

Figure 2: The 6 two-dimensional projections of the $\mathbb{Z}_{16}$ signal set.

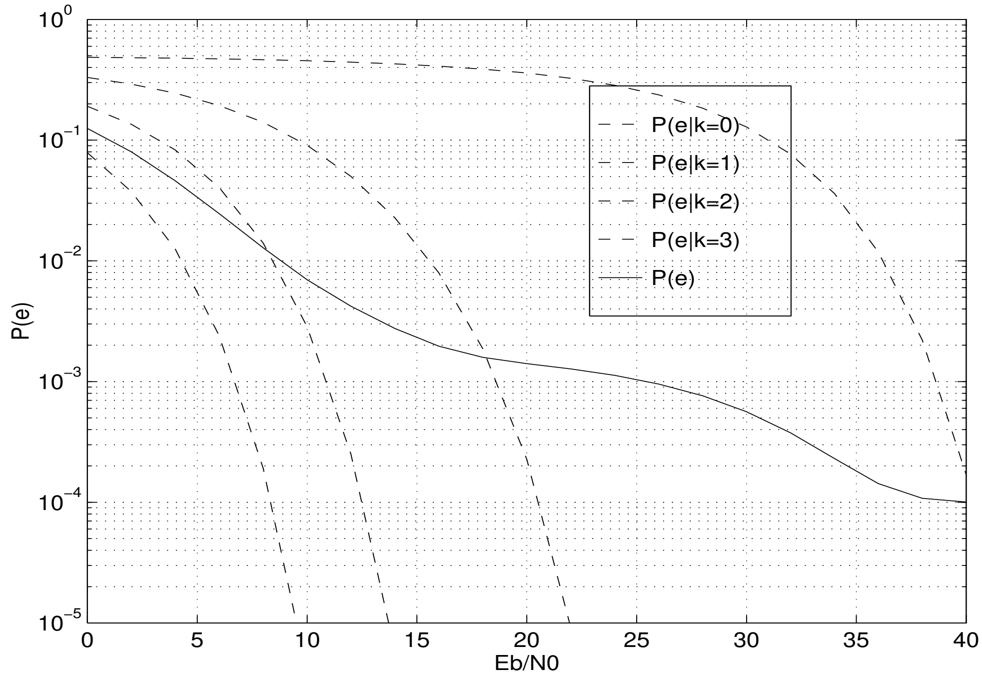

Figure 3: Rotated 4D-hypercube signal set $P_c = 0.1$.

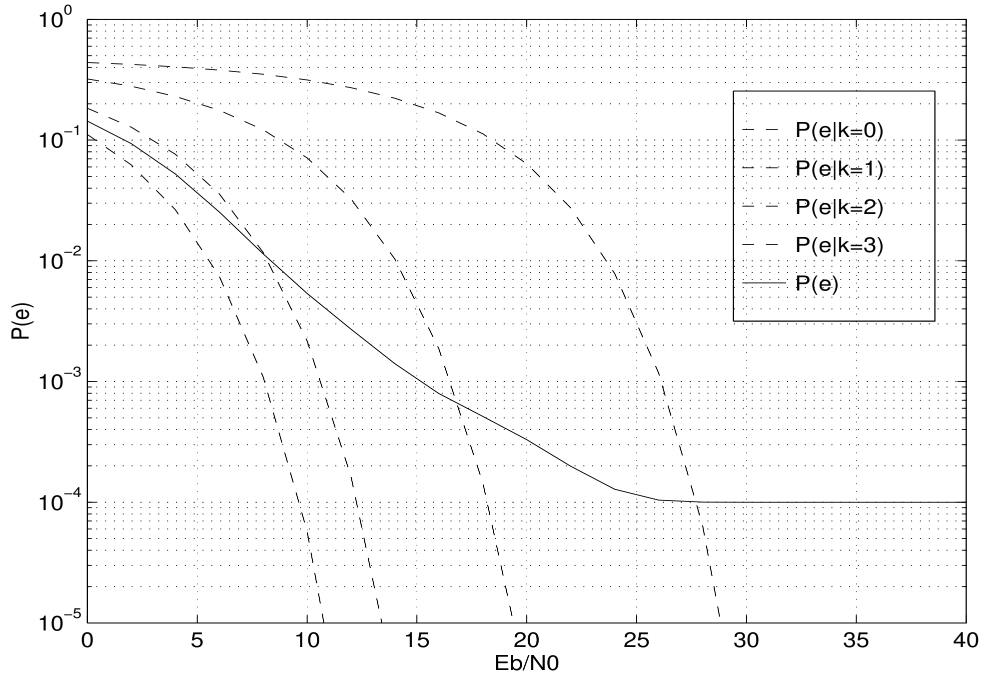

Figure 4: The $\mathbb{Z}_{16}$ signal set $P_c = 0.1$.

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