Multidimensional Signal Sets Generalizing PSK Modulations

Michele Elia, Giorgio Taricco, and Emanuele Viterbo
Dipartimento di Elettronica [file elia.tex]
Politecnico di Torino – I-10129 Torino - Italy

Summary

Point constellations, generated by commutative groups of order 2^n in n-dimensional spaces, are used to produce signals that generalize iterated 2-PSK modulation signals preserving their main advantageous features. A digital modulation with an underlying group structure consists of a set $\mathcal{S} = \{s_j(t) = \sum_{i=1}^{n} x_{ij}\psi_i(t) : \ t \in [0,T]\}_{j=1}^{M}$ of M signals, which is obtained by using a set $\Psi = \{\psi_i(t) : \ t \in [0,T]\}_{i=1}^{n}$ of n base signals and an n-dimensional point constellation $\mathcal{X} = \{\mathbf{X}_j = (x_{1j}, \ x_{2j}, \ \ldots, \ x_{nj})\}_{j=1}^{M}$, generated by the action, over an initial vector $\mathbf{X}_0$, of the matrices $D(\cdot)$ representing a group $\mathcal{G}$, i.e. $\mathcal{X} = \{\mathbf{X}_j = D(g_j)\mathbf{X}_0, \ g_j \in \mathcal{G}\}$ [5].
Signal sets of large dimension are interesting for at least two reasons. First, at fixed energy per bit, the minimum distance between points of a constellation increases as the dimension n increases. Second, there are time-limited orthogonal signals, conveying through a filter the maximum energy over a time interval $[0,T]$, which preserve, after filtering, the orthogonality over $[0,T]$ and have a computable energy loss [2]. This property lets us overcome one of the major problem in the use of signal spaces of larger dimension: the linear channel filtering distortion which usually destroys both base-signal normalization and orthogonality.
We consider commutative group representations of dimension n. Thus, the computation of the optimal initial vector yielding the maximum minimum distance is performed for every abelian group of order 2^n [4], and for a given group over every non-equivalent faithful representation [3]. For each representation we can solve the initial vector problem as a linear programming problem [1]. In the following Table we show the main parameters of the optimal configurations for dimension up to 6. Here, $N(n)$ is the total number and $N_a(n)$ is the number of abelian nonisomorphic groups of order 2^n respectively. $N_{ra}(n)$ is the number of non-equivalent n-dimensional representations of the abelian groups (trivial representations are not counted). The gain γ for a constellation with respect to the antipodal configuration, is evaluated as $10\log_{10}(d_{min}^2/4)$. Lastly, some inferences on the asymptotic behavior as n goes to infinity are elaborated.

Group parameters					Optimal Constellations			
n	$\|\mathcal{G}\|$	$N(n)$	$N_a(n)$	$N_{ra}(n)$	M	$\mathcal{G}$	d_{min}^2	γ (dB)
3	8	5	3	3	8	$\mathcal{C}_8$	$\frac{12(4-\sqrt{2})}{7}$	0.4
4	16	14	5	12	16	$\mathcal{C}_2 \times \mathcal{C}_8$	$8(2-\sqrt{2})$	0.7
5	32	51	7	36	32	$\mathcal{C}_4 \times \mathcal{C}_8$	$20(3\sqrt{2})-4)$	0.8
6	64	294	11	2004	64	$\mathcal{C}_{64}$	5.08	1.0

References

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