DYNAMIC MODELLING OF DISTRIBUTED-FEEDBACK LASERS USING SCATTERING MATRICES

A new technique for modelling DFB semiconductor lasers above threshold using scattering matrices is described. Quarter-wave-shifted grating devices are compared with unshifted devices under transient conditions.

Introduction: Distributed feedback (DFB) semiconductor lasers have a greater mode selectivity than Fabry-Perot devices, and so are preferred as sources for long-haul high-capacity fibre systems. However, dynamic single-mode (DSM) operation is still difficult. Accurate multimode representations of the optical processes which modify forward- and backward-travelling waves as they pass along the cavity, are required to represent the propagation delays of the waves and also serve to discretise time and space, allowing a digital computer to solve the network.

In Fabry-Perot TLLMs, the waves are only reflected at the facets. In DFB devices, the forward and backward waves are coupled along the entire cavity length because of a modulation of the index of refraction. This coupling can be represented by impedance discontinuities placed between the model sections (Fig. 1).

The question is: how many model sections, each with a coupling point, are required to represent this coupling accurately? A large number of sections, say two per grating period, would give a near-impossible computational task.

Studies have shown that a very small number of cross-coupling points can mimic the wavelength response of a DFB laser grating over a limited bandwidth. The condition was that the coupling per unit length \( \kappa \) must be the same as for the real device, i.e. the coupling per section must be increased to compensate for a small number of sections.

Two different matrices are required to represent the coupling: one for a high-low impedance transition (section \( n+1 \)), and one for a low-high impedance transition (section \( n+2 \)). These can be derived by assuming that the admittances of the lines are modulated by \( \pm \Delta Y \) from their mean admittance \( Y \). For small modulations, \( \Delta Y \) can be related to the coupling per unit length \( \kappa \) using:

\[
\Delta Y / Y = \kappa \Delta L \quad (1)
\]

where \( \Delta L \) is the mean length of the model sections and equal to half the model's grating period \( L \). This ensures that stop-band lies at the centre of the modelled bandwidth.

The matrix describing the low-high impedance transition and the transmission lines connecting the main scattering matrices, was derived from standard transmission-line equations, giving:

\[
\frac{A(n+1)}{B(n+1)} = \begin{bmatrix}
1 + \kappa \Delta L & -\kappa \Delta L \\
\kappa \Delta L & 1 - \kappa \Delta L
\end{bmatrix} \frac{A(n)}{B(n+1)} 
\quad (2)
\]

For a high-low impedance transition the matrix becomes:

\[
\frac{A(n+2)}{B(n+2)} = \begin{bmatrix}
1 - \kappa \Delta L & \kappa \Delta L \\
-\kappa \Delta L & 1 + \kappa \Delta L
\end{bmatrix} \frac{A(n+1)}{B(n+2)} 
\quad (3)
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10-GHz resonance of the output powers. The shifted device had a single mode at the centre of the stopband. Making the gain peak wavelength carrier dependent would show non-random mode partition effects.\(^1\)

**Fig. 3** Spectra of two lasers during first 230 ps

**Conclusions:** A simple modification to the TLLM allowed the dynamics of DFB lasers to be studied. The model could be easily extended to DRB structures, DFB amplifiers and DFBs with external feedback. Future work will concentrate on including laser chirp effects,\(^2\) allowing the study of frequency-modal interaction. This is achieved using a single DR is achieved by pin diodes.

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