Secure SWIPT Networks Based on a Non-linear Energy Harvesting Model

(Invited Paper)

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Abstract—We optimize resource allocation to enable communication security in simultaneous wireless information and power transfer (SWIPT) for internet-of-things (IoT) networks. The resource allocation algorithm design is formulated as a non-convex optimization problem. We aim at maximizing the total harvested power at energy harvesting (EH) receivers via the joint optimization of transmit beamforming vectors and the covariance matrix of the artificial noise injected to facilitate secrecy provisioning. The proposed problem formulation takes into account the non-linearity of energy harvesting circuits and the quality of service requirements for secure communication. To obtain a globally optimal solution of the resource allocation problem, we first transform the resulting non-convex sum-of-ratios objective function into an equivalent objective function in parametric subtractive form, which facilitates the design of a novel iterative resource allocation algorithm. In each iteration, the semidefinite programming (SDP) relaxation approach is adopted to solve a rank-constrained optimization problem optimally. Numerical results reveal that the proposed algorithm can guarantee communication security and provide a significant performance gain in terms of the harvested energy compared to existing designs which are based on the traditional linear EH model.

I. INTRODUCTION

In the era of the Internet of Things (IoT), it is expected that 50 billion wireless communication devices will be connected worldwide [1]. Smart physical objects equipped with sensors and wireless communication chips are able to collect and exchange information. These smart objects are wirelessly connected to computing systems to provide intelligent everyday services such as e-health, automated control, energy management (smart city and smart grid), logistics, security control, and safety management, etc. However, the limited energy storage capacity of battery-powered wireless communication devices severely limits the lifetime of wireless communication networks. Although battery replacement provides an intermediate solution to energy shortage, frequent replacement of batteries can be costly and cumbersome. This creates a serious performance bottleneck in providing stable communication services. On the other hand, a promising approach to extend the lifetime of wireless communication networks is to equip wireless communication devices with energy harvesting (EH) technology to scavenge energy from external sources. Solar, wind, tidal, biomass, and geothermal are the major renewable energy sources for generating electricity [2]. Yet, these conventional natural energy sources are usually climate and location dependent which limits the mobility of the wireless devices. More importantly, the intermittent and uncontrollable nature of these natural energy sources is a major obstacle for providing stable wireless communications via traditional EH technologies.

Recently, wireless energy transfer (WET) has attracted significant attention from both academia and industry [3]–[11], as a key to unlock the potential of IoT. Generally speaking, the WET technology can be divided into three categories: magnetic resonant coupling, inductive coupling, and radio frequency (RF)-based WET. The first two technologies rely on near-field magnetic fields and do not support any mobility of EH devices, due to the short wireless charging distances and the required alignment of the magnetic field with the EH circuits. In contrast, RF-based WET technologies [3]–[11] utilize the far-field of electromagnetic (EM) waves which enables concurrent wireless charging and data communication in WET networks over long distances (e.g. hundreds of metres). Moreover, the broadcast nature of wireless channels facilitates one-to-many wireless charging which eliminates the need for power cords and manual recharging for IoT devices. As a result, simultaneous wireless information and power transfer (SWIPT) is expected to be a key enabler for sustainable IoT communication networks. Yet, the introduction of SWIPT to communication systems has led to a paradigm shift in both system architecture and resource allocation algorithm design. For instance, in SWIPT systems, one can increase the energy of the information carrying signal to increase the amount of RF energy harvested at the receivers. However, increasing the power of the information signals may also increase their susceptibility to eavesdropping, due to the higher potential for information leakage. As a result, both communication security concerns and the need for efficient WET naturally arise in systems providing SWIPT services.

Nowadays, various types of cryptographic encryption algorithms are employed at the application layer for guaranteeing wireless communication security. However, secure key management and distribution via an authenticated third party is typically required for these algorithms, which may not be realizable in future wireless IoT networks due to the expected massive numbers of devices. Therefore, a considerable amount of work has recently been devoted to information-theoretic physical (PHY) layer security as a complementary technology to the existing encryption algorithms [12]–[17]. It has been shown that in a wire-tap channel, if the source-destination channel enjoys better conditions compared to the source-eavesdropper channel [12], perfectly secure communication between a source and a destination is possible. Hence, multiple-antenna technology and advanced signal processing algorithms have been proposed to ensure secure communications. Specifically, by exploiting the extra degrees of freedom offered by multiple antennas, the information beams can be focused on the desired legitimate receivers to reduce the chance of information leakage. Besides, artificial noise can be injected into the communication channel deliberately to degrade the channel quality of eavesdroppers.
These concepts have also been extended to SWIPT systems to provide secure communication. In [17], beamforming was studied to enhance security and power efficiency in SWIPT systems. The authors of [18] proposed a multi-objective optimization framework to investigate the non-trivial tradeoff between interference, total harvested power, and energy consumption in a secure cognitive radio SWIPT network. However, the resource allocation algorithms designed for secure SWIPT systems [17], [18] were based on a linear EH model which does not capture the highly non-linear characteristics of practical end-to-end WET [19]–[22]. In particular, existing resource allocation schemes designed for the linear EH model may lead to severe resource allocation mismatches resulting in performance degradation in WET and secure communications. These observations motivate us to study the design of efficient resource allocation algorithms for secure SWIPT systems taking into account a practical non-linear EH model.

II. SYSTEM MODEL

In this section, we first introduce the notation adopted in this paper. Then, we present the downlink channel model for secure communication in SWIPT systems.

A. Notation

We use boldface capital and lower case letters to denote matrices and vectors, respectively. $A^H$, $\text{Tr}(A)$, $\text{Rank}(A)$, and $\det(A)$ represent the Hermitian transpose, trace, rank, and determinant of matrix $A$, respectively; $A \succ 0$ and $A \succeq 0$ indicate that $A$ is a positive definite and a positive semidefinite matrix, respectively; $I_N$ is the $N \times N$ identity matrix; $[q]_{m:n}$ returns a vector with the $m$-th to the $n$-th elements of vector $q$; $\mathbb{C}^{N \times M}$ denotes the set of all $N \times M$ matrices with complex entries; $\mathbb{H}^N$ denotes the set of all $N \times N$ Hermitian matrices. The circularly symmetric complex Gaussian (CSCG) distribution is denoted by $\mathcal{CN}(\mathbf{m}, \Sigma)$ with mean vector $\mathbf{m}$ and covariance matrix $\Sigma$; $\sim$ indicates “distributed as”; $\mathbb{E}\{\cdot\}$ denotes statistical expectation; $\lceil \cdot \rceil$ represents the absolute value of a complex scalar; $[x]^T$ stands for $\max\{0, x\}$ and $[\cdot]^T$ represents the transpose operation.

B. Channel Model

A frequency flat fading channel for downlink communication is considered. The SWIPT system comprises a base station (BS), an information receiver (IR), and $J$ energy harvesting receivers (ER), as shown in Figure 1. The BS is equipped with $N_T \geq 1$ antennas. The IR is a single-antenna device and each ER is equipped with $N_R \geq 1$ receive antennas for EH. In the considered system, the signal intended for the IR is overheard by the ERs due to the broadcast nature of wireless channels.

To guarantee communication security, the ERs are treated as potential eavesdroppers which has to be taken into account for resource allocation algorithm design. We assume that $N_T > N_R$ for the following study. The signals received at the IR and ER $j \in \{1, \ldots, J\}$ are modelled as

$$y = h^H(ws + v) + n, \quad (1)$$

$$y_{ERj} = G_j^H(ws + v) + n_{ERj}, \quad \forall j \in \{1, \ldots, J\}, \quad (2)$$

respectively, where $s \in \mathbb{C}$ and $w \in \mathbb{C}^{N_T \times 1}$ are the information symbol and the corresponding beamforming vector, respectively. Without loss of generality, we assume that $\mathbb{E}\{|s|^2\} = 1$. $v \in \mathbb{C}^{N_T \times 1}$ is an artificial noise vector generated by the BS to facilitate efficient WET and to guarantee communication security. In particular, $v$ is modeled as a random vector with circularly symmetric complex Gaussian distribution

$$v \sim \mathcal{CN}(0, V), \quad (3)$$

where $V \in \mathbb{H}^{N_T}$. $V \succeq 0$, denotes the covariance matrix of the artificial noise. The channel vector between the BS and the IR is denoted by $h \in \mathbb{C}^{N_T \times 1}$ and the channel matrix between the BS and ER $j$ is denoted by $G_j \in \mathbb{C}^{N_T \times N_R}$. $n \sim \mathcal{CN}(0, \sigma_n^2)$ and $n_{ERj} \sim \mathcal{CN}(0, \sigma_{ERj}^2)$ are the additive white Gaussian noises (AWGN) at the IR and ER $j$, respectively, where $\sigma_n^2$ denotes the noise power at each antenna of the receiver.

C. Energy Harvesting Model

Figure 2 depicts the block diagram of the ER in SWIPT systems. In general, a bandpass filter and a rectifying circuit are adopted in an RF-ER to convert the received RF power to direct current (DC) power. The total received RF power at ER $j$ is given by

$$P_{ERj} = \text{Tr}\left((ww^H + V)G_jG_j^H\right). \quad (4)$$

In the SWIPT literature, for simplicity, the total harvested power at ER $j$ is typically modelled as follows:

$$\Phi_{ERj}^{Linear} = \eta_j P_{ERj}, \quad (5)$$

where $0 \leq \eta_j \leq 1$ denotes the energy conversion efficiency of ER $j$. From (5), it can be seen that with existing models, the total harvested power at the ER is linearly and directly proportional to the received RF power. However, practical RF-based EH circuits consist of resistors, capacitors, and diodes. Experimental results have shown that these circuits [19]–[21] introduce various non-linearities into the end-to-end WET. In order to design a resource allocation algorithm for practical secure SWIPT systems, we adopt the non-linear parametric EH model from [22], [23]. Consequently, the total harvested power...
where $Q_j$ denotes the interference-plus-noise covariance matrix for $ER_j$. Hence, the achievable secrecy rate of the IR is given by [14]

$$R_{sec} = \left[ R - \max_{\psi_j} \{ R_{ER_j} \} \right]^+.$$  \hspace{1cm} (10)

### III. Optimization Problem and Solution

In the considered SWIPT system, we aim to maximize the total harvested power in the system while providing secure communication to the IR. To this end, we formulate the resource allocation algorithm design as the following non-convex optimization problem assuming that perfect channel state information is available 1:

**Problem 1. Resource Allocation for Secure SWIPT:**

\[
\begin{align*}
\text{maximize} & \quad \sum_{j=1}^{J} \psi_{ER_j} \\
\text{subject to} & \quad C1 : \|w\|^2 + \text{Tr}(V) \leq P_{max}, \\
& \quad w^H H w \leq \frac{\text{Tr}(V H) + \sigma_s^2}{\Gamma_{req}}, \\
& \quad C3 : R_{ER_j} \leq R_{ER_j}^{\text{tol}}, \forall j, \\
& \quad C4 : \text{V} \succeq 0.
\end{align*}
\]

Constants $P_{max}$ and $\Gamma_{req}$ in constraints C1 and C2 denote the maximum transmit power budget and the minimum required signal-to-interference-plus-noise ratio (SINR) at the IR, respectively. Constant $R_{ER_j}^{\text{tol}} > 0$ in C3 is the maximum tolerable data rate which restricts the capacity of $ER_j$ if it attempts to decode the signal intended for the IR. In practice, the BS sets $\log_2(1 + \Gamma_{req}) > R_{ER_j}^{\text{tol}} > 0$, to ensure secure communication. Constraint C4 and $V \in \mathbb{H}^{N_T}$ constrain matrix $V$ to be a positive semidefinite Hermitian matrix. It can be observed that the objective function in (11) is a non-convex function due to its sum-of-ratios form. Besides, the log-det function in C3 is non-convex. Now, we first transform the non-convex objective function into an equivalent objective function in subtractive form via the following theorem.

**Theorem 1.** Suppose \(\{w^*, V^*\}\) is the optimal solution to (11), then there exist two column vectors $\mu^* = [\mu_1^*, \ldots, \mu_J^*]^T$ and $\beta^* = [\beta_1^*, \ldots, \beta_J^*]^T$ such that \(\{w^*, V^*\}\) is an optimal solution to the following optimization problem

\[
\begin{align*}
\text{maximize} & \quad \sum_{j=1}^{J} \mu_j^* \left[ M_j - \beta_j^* \left( 1 + \exp \left( -a_j (P_{ER_j} - b_j) \right) \right) \right], \\
\text{subject to} & \quad F \text{ is the feasible solution set of (11). \(w^*, V^*\) also satisfies the following system of equations:}
\end{align*}
\]

which $Q_j$ denotes the interference-plus-noise covariance matrix for $ER_j$. Hence, the achievable secrecy rate of the IR is given by [14]

$$R_{sec} = \left[ R - \max_{\psi_j} \{ R_{ER_j} \} \right]^+.$$  \hspace{1cm} (10)

**Proof:** Please refer to [24] for a proof of Theorem 1. \[\square\]

In the sequel, since $\Omega_j$ does not affect the design of the optimal resource allocation policy, with a slight abuse of notation, we will directly use $\psi_{ER_j}$ to represent the harvested power at $ER_j$ for simplicity of presentation.

In the considered problem formulation, we can guarantee that the achievable secrecy rate is bounded below by $R_{sec} \geq \log_2(1 + \Gamma_{req}) - R_{ER_j}^{\text{tol}} > 0$ if the problem is feasible.
Algorithm

1: Initialize the maximum number of iterations $L_{\text{max}}$, iteration index $n = 0$, $\mu^\ast$, and $\beta^\ast$.  
2: repeat (Outer Loop)  
3: Solve the inner loop problem in (18) via semidefinite program relaxation for given $(\mu^\ast, \beta^\ast)$ and obtain the intermediate beamformer $w^\ast$ and artificial noise covariance matrix $V^\ast$. 
4: if (20) is satisfied then  
5: return Optimal beamformer $w^\ast = w'$ and artificial noise covariance matrix $V^\ast = V'$. 
6: else  
7: Update $\mu$ and $\beta$ according to (19) and $n = n + 1$  
8: end if  
9: until (13) and (14) are satisfied or $n = L_{\text{max}}$. 

Therefore, for (11), we have an equivalent optimization problem in (12) with an objective function in subtractive form with extra parameters $(\mu^\ast, \beta^\ast)$. More importantly, the two problems have the same optimal solution $\{w^\ast, V^\ast\}$. Besides, the optimization problem in (12) can be solved by an iterative algorithm consisting of two loops [24]. In the inner loop, the optimization problem in (12) for given $(\mu, \beta)$, $\mu = [\mu_1, \ldots, \mu_J]^T$ and $\beta = [\beta_1, \ldots, \beta_J]^T$, is solved. Then, in the outer loop, we find the optimal $(\mu^\ast, \beta^\ast)$ satisfying the system of equations in (13) and (14), cf. Algorithm 1 in Table I.

A. Solution of the Inner Loop Problem

In each iteration, in line 3 of Algorithm 1, we solve an inner loop optimization problem in its hypograph form:

**Problem 2. Inner Loop Problem**

$$\max_{W, V \in \mathbb{R}^{M \times T}, \tau} \sum_{j=1}^{J} \mu_j^\ast \left[ M_j - \beta_j^\ast \left(1 + \exp \left(-a_j (\tau_j - b_j)\right)\right) \right]$$

subject to

- $C1 : \text{Tr}(W + V) \leq P_{\text{max}}$,
- $C2 : \frac{\text{Tr}(WH)}{\Gamma_{\text{req}}} \geq \text{Tr}(VH) + \sigma_s^2$,
- $C3, C4$,
- $C5 : \text{Tr}\left((W + V)G_j G_j^H\right) \geq \tau_j, \forall j$,
- $C6 : \text{Rank}(W) = 1, C7 : W \succeq 0$, (15)

where $W = WW^H$ is a new optimization variable matrix and $\tau = [\tau_1, \tau_2, \ldots, \tau_J]$ is a vector of auxiliary optimization variables. The extra constraint $C5$ represents the hypograph of the inner loop optimization problem.

We note that the inner loop problem in (15) is still a non-convex optimization problem. In particular, the non-convexity arises from the log-det function in $C3$ and the combinatorial rank constraint $C6$. To circumvent the non-convexity, we first introduce the following proposition to handle constraint $C3$.

**Proposition 1.** For $R_{\text{ER}}^{\text{req}} > 0, \forall j$, and $\text{Rank}(W) \leq 1$, constraint $C3$ is equivalent to constraint $C3^\ast$, i.e.,

$$C3 \Leftrightarrow C3^\ast : G_j^H W G_j \preceq \alpha_{\text{ER}} Q_j, \forall j,$$ (16)

where $\alpha_{\text{ER}} = 2R_{\text{ER}}^{\text{req}} - 1$ is an auxiliary constant and $C3^\ast$ is a linear matrix inequality (LMI) constraint.

Proof: Please refer to Appendix A in [25] for the proof. ■

Now, we apply Proposition 1 to Problem (15) by replacing constraint $C3$ with constraint $C3^\ast$ which yields:

**Problem 3. Equivalent Formulation of Problem (15)**

$$\max_{W, V \in \mathbb{R}^{M \times T}, \tau} \sum_{j=1}^{J} \mu_j^\ast \left[ M_j - \beta_j^\ast \left(1 + \exp \left(-a_j (\tau_j - b_j)\right)\right) \right]$$

subject to

- $C1, C2, C4, C5, C7$, $C3^\ast : G_j^H W G_j \preceq \alpha_{\text{ER}} Q_j, \forall j$,
- $C6 : \text{Rank}(W) = 1$. (17)

The non-convexity of (17) is now only due to the rank constraint in $C6$. We adopt semidefinite programming (SDP) relaxation to obtain a tractable solution. Specifically, we remove the non-convex constraint $C6$ from (17) which yields:

**Problem 4. SDP Relaxation of Problem (17)**

$$\max_{W, V \in \mathbb{R}^{M \times T}, \tau} \sum_{j=1}^{J} \mu_j^\ast \left[ M_j - \beta_j^\ast \left(1 + \exp \left(-a_j (\tau_j - b_j)\right)\right) \right]$$

subject to

- $C1, C2, C4, C5, C7$, $C3^\ast : G_j^H W G_j \preceq \alpha_{\text{ER}} Q_j, \forall j$,
- $C6 : \text{Rank}(W) = 1$. (18)

In fact, (18) is a standard convex optimization problem which can be solved by numerical convex program solvers such as Sedumi or SDPT3 [26]. Now, we study the tightness of the adopted SDP relaxation in (18).

**Theorem 2.** For $\Gamma_{\text{req}} > 0$ and if the considered problem is feasible, we can construct a rank-one solution of (17) based on the solution of (18).

Proof: Please refer to the Appendix. ■

Therefore, the non-convex optimization problem in (15) can be solved optimally.

B. Solution of the Outer Loop Problem

In this section, an iterative algorithm based on the damped Newton method is adopted to update $(\mu, \beta)$ for the outer loop problem. For notational simplicity, we define functions $\varphi_j(\beta) = \beta_j \left(1 + \exp \left(-a_j (P_{\text{ER}} - b_j)\right)\right) - M_j$ and $\varphi_{j+i}(\mu_i) = \mu_i \left(1 + \exp \left(-a_i (P_{\text{ER}} - b_j)\right)\right) - 1, i \in \{1, \ldots, J\}$. It is shown in [24] that the unique optimal solution $(\mu^\ast, \beta^\ast)$ is obtained if and only if $\varphi(\mu, \beta) = \varphi_1, \varphi_2, \ldots, \varphi_J = 0$. Therefore, in the $n$-th iteration of the iterative algorithm, $\mu^{n+1}$ and $\beta^{n+1}$ can be updated as, respectively,

$$\mu^{n+1} = \mu^n + \zeta^n q_{1+2}^{n+1}$$

and $\varphi(\mu, \beta)$ is the Jacobian matrix of $\varphi(\mu, \beta)$. $\zeta^n$ is the largest $\epsilon$ satisfying

$$\|\varphi(\mu^n + \epsilon l q_{1+2}^{n+1}, \beta^n + \epsilon l q_{1+2}^{n+1})\| \leq (1 - \eta^n)\|\varphi(\mu, \beta)\|,$$ (19)

where $l \in \{1, 2, \ldots\}$, $\epsilon l \in (0, 1)$, and $\eta \in (0, 1)$. The damped Newton method converges to the unique solution $(\mu^\ast, \beta^\ast)$ satisfying the system of equations (13) and (14), cf. [24].
Table II: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>System bandwidth</td>
<td>200 kHz</td>
</tr>
<tr>
<td>Carrier center frequency</td>
<td>915 MHz</td>
</tr>
<tr>
<td>Transceiver antenna gain</td>
<td>10 dBi</td>
</tr>
<tr>
<td>Number of receive antennas</td>
<td>3</td>
</tr>
<tr>
<td>Noise power $\sigma^2$</td>
<td>$-95$ dBm</td>
</tr>
<tr>
<td>Maximum transmit power $P_{\text{max}}$</td>
<td>36 dBm</td>
</tr>
<tr>
<td>BS-to-ER fading distribution</td>
<td>Rician with Rician factor 3 dB</td>
</tr>
</tbody>
</table>

Fig. 4. Average total harvested power (dBm) versus the minimum required SINR (dB).

IV. Results

In this section, simulation results are presented to illustrate the performance of the proposed resource allocation algorithm. We summarize the most important simulation parameters in Table II. In the simulation, the IR and the $J = 10$ ERs are located at 50 meters and 10 meters from the BS, respectively. The maximum tolerable data rate at the potential eavesdropper is set to $R_{\text{ER}}^{\text{Tot}} = 1$ bit/s/Hz. For the non-linear EH circuit, we set $M_j = 20$ mW which corresponds to the maximum harvested power per ER. Besides, we adopt $a_j = 6400$ and $b_j = 0.003$.

In Figure 4, we study the average total harvested power versus the minimum required receive SINR, $\Gamma_{\text{req}}$, at the IR for different numbers of transmit antennas and resource allocation schemes. As can be observed, the average total harvested power decreases with increasing $\Gamma_{\text{req}}$. Indeed, to satisfy a more stringent SINR requirement, the direction of information beam has to be steered towards the IR which yields a smaller amount of RF energy for EH at the ERs. On the other hand, a significant EH gain can be achieved by the proposed optimal scheme when the number of antennas equipped at the BS increases. In fact, additional transmit antennas equipped at the BS provide extra spatial degrees of freedom which facilitate a more flexible resource allocation. In particular, the BS can steer the direction of the artificial noise and the information signal towards the ERs accurately to improve the WET efficiency. For comparison, we also show the performance of a baseline scheme. For the baseline scheme, the resource allocation algorithm is designed based on an existing linear EH model, cf. (5). Specifically, we optimize $\mathbf{w}, \mathbf{V}$ to maximize the total harvested power subject to the constraints in (11). Then, this baseline scheme is applied for resource allocation in the considered system with non-linear ERs. We observe from Figure 4 that a substantial performance gain is achieved by the proposed optimal resource allocation algorithm compared to the baseline scheme. This is due to the fact that resource allocation mismatch occurs in the baseline scheme as it does not account for the non-linear nature of the EH circuits.

Figure 5 illustrates the average system secrecy rate versus the minimum required SINR $\Gamma_{\text{req}}$ of the IR for different numbers of transmit antennas, $N_T$. The average system secrecy rate, i.e., $C_{\text{sec}}$, increases with increasing $\Gamma_{\text{req}}$. This is because the maximum achievable rate of the ERs for decoding the IR signal is limited by the resource allocation to be less than $R_{\text{GR}}^{\text{Tot}} = 1$ bit/s/Hz. Besides, although the minimum SINR requirement increases in Figure 5, the proposed optimal scheme is able to fulfill all QoS requirements due to the proposed optimization framework.

V. Conclusions

In this paper, a resource allocation algorithm enabling secure SWIPT in IoT communication networks was presented. The algorithm design based on a practical non-linear EH model was formulated as a non-convex optimization problem for the maximization of the total energy transferred to the ERs. We transformed the resulting non-convex optimization problem into two nested optimization problems which led to an efficient iterative approach for obtaining the globally optimal solution. Numerical results unveiled the potential performance gain in EH brought by the proposed optimization and its robustness against eavesdropping for IoT applications.

Appendix: Proof of Theorem 2

To start with, we first define $\tau^*$ as the optimal objective value of (18). When $\text{Rank}(\mathbf{W}) > 1$ holds after solving (18), an optimal rank-one solution for (18) can be constructed as follows [27]. For a given $\tau^*$, we solve an auxiliary optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \mathbf{w}, \mathbf{V} \in \mathbb{H}^{N_T} \quad \text{Tr}(\mathbf{W}) \\
\text{subject to} & \quad C_1, C_2, C_3, C_4, C_5, C_7.
\end{align*}
\]

It can be observed that the optimal solution of (21) is also an optimal resource optimal resource allocation policy for (18) when $\tau^*$ is fixed in (21). Therefore, in the remaining part of the proof, we show that solving (21) returns a rank-one
beamforming matrix $W$. Therefore, we study the Lagrangian of problem (21) which is given by:

$$
L = \text{Tr}(W) + \lambda (\text{Tr}(W + V) - P_{\text{max}}) - \text{Tr}(WR) \\
- \sum_{j=1}^{J} \rho_j \left( \tau_j^* - \text{Tr}(W(V)G_jG_j^H) \right) - \text{Tr}(VZ) \\
+ \sum_{j=1}^{J} \text{Tr}(D_{C,j}(G_j^HWG_j - \alpha \text{ER}Q_j)) \\
+ \alpha \left( \text{Tr}(VH) + \frac{a_s^2}{\Gamma_{\text{req}}} - \frac{\text{Tr}(WH)}{\Gamma_{\text{req}}} \right) + \Delta,
$$

where $\lambda \geq 0$, $\alpha \geq 0$, $D_{C,j} \geq 0$, $\forall j \in \{1, \ldots, J\}$, $Z \geq 0$, $\rho_j \geq 0$, and $\Gamma \geq 0$ are the dual variables for constraints C1, C2, C3, C4, C5, and C7, respectively. $\Delta$ is a collection of variables and constants that are not relevant to the proof.

Then, we exploit the following Karush-Kuhn-Tucker (KKT) conditions which are needed for the proof:\n
$$
R^*, V^*, D_{C,j}^* \geq 0, \quad \lambda^*, \alpha, \rho_j^* \geq 0,
$$

$$
R^*W = 0, \quad Z^*V = 0,
$$

$$
R^* = (\lambda^* + 1) I_{N_T} - \sum_{j=1}^{J} \rho_j G_jG_j^H + \sum_{j=1}^{J} G_jD_{C,j}^*G_j^H \\
- \alpha^* \frac{H}{\Gamma_{\text{req}}},
$$

$$
Z^* = \lambda^* I_{N_R} - \sum_{j=1}^{J} \rho_j G_jG_j^H \\
- \sum_{j=1}^{J} \alpha_{\text{ER}} G_jD_{C,j}^*G_j^H + \alpha^* H.
$$

Then, subtracting (26) from (25) yields

$$
R^* = I_{N_T} + Z^* + \sum_{j=1}^{J} (1 + \alpha_{\text{ER}}) G_jD_{C,j}^*G_j^H \\
- \alpha^* \frac{H}{\Gamma_{\text{req}} + 1}.
$$

Besides, constraint C2 in (21) is satisfied for equality for the optimal solution and we have $\alpha^* > 0$. From (27), we have

$$
\text{Rank}(R^*) + \text{Rank}(\alpha^* \frac{H}{\Gamma_{\text{req}} + 1}) \\
\geq \text{Rank}(R^*) + \alpha^* \text{Tr}(I_{N_T}) \\
= \text{Rank}(A) = N_T \\
\Rightarrow \text{Rank}(R^*) \geq N_T - 1.
$$

As a result, Rank($R^*$) is either $N_T - 1$ or $N_T$. Furthermore, since $\Gamma_{\text{req}} > 0$ in C2, $W^* \neq 0$ is necessary. Hence, Rank($R^*$) = $N_T - 1$ and Rank($W^*$) = 1 hold and the rank-one solution for (18) is constructed.

REFERENCES


