Achievable Rates for the Fading Half-Duplex Single Relay Selection Network Using Buffer-Aided Relaying

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Abstract—In the half-duplex single relay selection network, comprised of a source, \( M \) half-duplex relays, and a destination, only one relay is active at any given time, i.e., only one relay receives or transmits, and the other relays are inactive, i.e., they do not receive or transmit. The capacity of this network, when all links are affected by independent time-continuous fading and additive white Gaussian noise (AWGN), is still unknown. Hence, only achievable average rates have been reported in the literature so far. In this paper, we present new achievable average rates for this network which are larger than the best known average rates in the literature. These average rates are achieved with a buffer-aided relaying protocol. Since the developed buffer-aided protocol which achieves these rates introduces unbounded delay, we also devise a buffer-aided protocol which limits the delay at the expense of decrease in rate. Moreover, we show that the proposed buffer-aided relaying protocols do not require more resources for channel state information acquisition than the existing relay selection protocols.

I. INTRODUCTION

The half-duplex single relay selection network was first proposed in [1] as a network in which only one relay is active at any given time, i.e., one relay receives or transmits, and the other relays are inactive, i.e., they do not receive or transmit. This network has recently attracted considerable interest, see [1]-[11] and references therein. The capacity of this network, when all links are affected by independent time-continuous fading and additive white Gaussian noise (AWGN), is still unknown. Hence, so far, only achievable average rates have been reported in the literature, see [10], [11]. In fact, to the best of our knowledge, the achievable average rates in [10] and [11] are the largest average rates reported in the literature. These rates are based on the relay selection protocol in [1], where, in each time slot, the relay with the strongest minimum source-to-relay and relay-to-destination channel is selected to forward the information from the source to the destination. In this paper, we will show that these rates can be surpassed. In particular, we develop a buffer-aided relay protocol which achieves average rates which are significantly larger than the rates reported in [10] and [11]. Since the buffer-aided protocol which achieves these rates introduces unbounded delay, we also devise a buffer-aided protocol which limits the average delay at the expense of decrease in rate. Moreover, we show that the proposed buffer-aided relaying protocols do not require more resources for channel state information acquisition than the existing relay selection protocols.

Buffer-aided half-duplex relay with adaptive switching between reception and transmission was first proposed in [12] for a simple three-node relay network without source-destination link. Later, buffer-aided relaying was further analyzed in [13] and [14] for adaptive and fixed rate transmission, respectively. For the considered relay selection network, relaying with buffers was investigated in [8] and [9], but for the case when all nodes transmit with fixed rates and all source-to-relay and relay-to-destination links undergo independent and identically distributed (i.i.d.) fading. These protocols were developed for improving the outage probability performance of the network. In order to use the protocols in [8] and [9] as benchmark, we modify them such that all nodes transmit with rates equal to their underlying channel capacity. However, the modified protocols are still only applicable to the case when all links are affected by i.i.d. fading and will cause data loss due to buffer overflow for independent non-identically distributed (non-i.i.d.) fading. We note however, that this is not due to our modifications since the phenomenon of buffer overflows also occurs for the protocols in [8] and [9] for fixed rate transmission when the links of the network are non-i.i.d.

This paper is organized as follows. In Section II, we introduce the system model. In Section III, we present the proposed buffer-aided protocols for the considered network. In Section IV, we discuss two implementations of the proposed protocol. In Section V, we provide some numerical examples. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

The fading half-duplex relay selection network consists of a source \( S \), \( M \) half-duplex decode-and-forward relays \( R_k \), \( k = 1, ..., M \), and a destination \( D \). The source transmits its information to the destination only through the relays, i.e., because of high attenuation there is no direct link between the source and the destination, and therefore, all the information that the destination receives is first processed by the relays. The relays in the network are half-duplex nodes, i.e., they cannot transmit and receive at the same time. Furthermore, in each time slot only one relay is active, i.e., it receives or transmits, and the other relays are inactive, i.e., they do not receive and transmit. Each relay is equipped with a buffer of unlimited size in which it stores the information that it receives from the source and extracts the information that it transmits to the destination. We assume that all nodes transmit their codewords with constant power \( P \) and that the noise at all receivers is independent additive white Gaussian noise with variance \( \sigma_n^2 \). We assume transmission with capacity achieving codes. Hence, the transmitted codewords are Gaussian distributed, comprised of \( n \to \infty \) symbols, and span one time slot. Moreover, we assume that each source-to-relay and relay-to-destination channel is affected by independent time-continuous slow fading such that the fading remains constant during a single time slot and changes from one time slot to the next. We assume that the fading is an ergodic and stationary random process. Let \( |h_{Sk}(i)|^2 \) and \( |h_{kD}(i)|^2 \) denote the squared channel gains of the source-to-\( k \)-th-relay and \( k \)-th-relay-to-destination channels in the \( i \)-th time slot, respectively. Then, the signal-to-noise ratios (SNRs) of the source-to-\( k \)-th-relay and \( k \)-th-relay-to-destination channels are given by

\[
\gamma_{Sk}(i) = \frac{P}{\sigma_n^2} |h_{Sk}(i)|^2 \quad \text{and} \quad \gamma_{kD}(i) = \frac{P}{\sigma_n^2} |h_{kD}(i)|^2,
\]

respectively. On the other hand, the capacities of the source-to-\( k \)-th-relay and \( k \)-th-relay-to-destination channels in the \( i \)-th time
has stored in its buffer. When the active relay transmits, the received codeword is needed as the relay cannot transmit more information than what it stores in its buffer. Let \( Q_k(i) \) denote the number of bits/symbols in the buffer of the \( k \)-th relay at the end of time slot \( i \). Then, with this transmission, \( Q_k(i) \) is updated as \( Q_k(i) = Q_k(i-1) + C_{Sk}(i) \). On the other hand, if the active relay is selected to transmit, it extracts information from its buffer, maps it to a Gaussian distributed codeword with rate \( R_{Sk}(i) = C_{Sk}(i) \), where \( C_{Sk}(i) \) is given by (2). The selected relay can successfully decode this codeword and stores the corresponding information. The destination decodes the received codeword and stores the corresponding information in its buffer. Let \( Q_k(i) \) denote the number of bits/symbols in the buffer of the \( k \)-th relay at the end of time slot \( i \). Then, with this transmission, \( Q_k(i) \) is updated as \( Q_k(i) = Q_k(i-1) + C_{Sk}(i) \).

### III. BUFFER-AIDED RELAYING PROTOCOLS AND ACHIEVABLE AVERAGE RATES

In this section, we develop buffer-aided relaying protocols without and with imposing a delay constraint and derive the corresponding achievable average rates.

#### A. Buffer-Aided Relaying Protocol without a Delay Constraint

Assume that the \( k \)-th relay has been selected to be active in the \( i \)-th time slot. Then, if the active relay is selected to receive, the source sends to the selected relay one Gaussian distributed codeword with rate \( R_{Sk}(i) = C_{Sk}(i) \), where \( C_{Sk}(i) \) is given by (2). The selected relay can successfully decode this codeword and stores the corresponding information in its buffer. Let \( Q_k(i) \) denote the number of bits/symbols in the buffer of the \( k \)-th relay at the end of time slot \( i \). Then, with this transmission, \( Q_k(i) \) is updated as \( Q_k(i) = Q_k(i-1) + C_{Sk}(i) \). On the other hand, if the active relay is selected to transmit, it extracts information from its buffer, maps it to a Gaussian distributed codeword with rate \( R_{Dk}(i) = \min\{Q(i-1), C_{Sk}(i)\} \), and transmits it to the destination. The minimum in the expression for rate \( R_{Dk}(i) \) is needed as the relay cannot transmit more information than what it has stored in its buffer. When the active relay transmits, \( Q_k(i) \) is updated as \( Q_k(i) = Q_k(i-1) - R_{Dk}(i) \). The destination decodes the received codeword and stores the corresponding information. In order to model the reception and transmission of the \( k \)-th relay, we introduce two binary indicator variables \( r_k^R(i) \in \{0,1\} \) and \( r_k^T(i) \in \{0,1\} \), which indicate whether, in the \( i \)-th time slot, the \( k \)-th relay is receiving and transmitting, respectively. More precisely, \( r_k^R(i) \) and \( r_k^T(i) \) are defined as

\[
\begin{align*}
    r_k^R(i) &\triangleq \begin{cases} 
    1 & \text{if the } k \text{-th relay receives} \\
    0 & \text{if the } k \text{-th relay does not receive}, 
    \end{cases} \\
    r_k^T(i) &\triangleq \begin{cases} 
    1 & \text{if the } k \text{-th relay transmits} \\
    0 & \text{if the } k \text{-th relay does not transmit}.
    \end{cases}
\end{align*}
\]

Since exactly one relay is active in each time slot, \( r_k^R(i) \) and \( r_k^T(i) \), \( \forall i, k \), must satisfy

\[
\sum_{k=1}^{M} r_k^R(i) + r_k^T(i) = 1. 
\]

Using \( r_k^R(i) \) and \( r_k^T(i) \), the average rate received at and transmitted by the \( k \)-th relay, denoted by \( \bar{R}_{Sk} \) and \( \bar{R}_{Dk} \), respectively, can be expressed as

\[
\begin{align*}
\bar{R}_{Sk} &= \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} r_k^R(i) R_{Sk}(i) \\
&= \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} r_k^R(i) C_{Sk}(i), \\
\bar{R}_{Dk} &= \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} r_k^T(i) R_{Dk}(i) \\
&= \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} r_k^T(i) \min\{Q_k(i-1), C_{Sk}(i)\}.
\end{align*}
\]

Using \( \bar{R}_{Dk}, \forall k \), the average rate received at the destination, denoted by \( \bar{R}_{BA} \), can be expressed as

\[
\bar{R}_{BA} = \sum_{k=1}^{M} \bar{R}_{Dk} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{M} r_k^T(i) \min\{Q_k(i-1), C_{Sk}(i)\}. 
\]
problem by relaxing the binary constraints. In particular, we first convert the problem to a linear optimization problem, i.e., the solution of the relaxed problem is shown to be at the boundaries, $\forall k$. We are now ready to present the solution to (12). Later, for a given time slot $i$, and $C_4$, respectively. Fortunately, (12) can be solved analytically.

Theorem 1 reveals that the optimal values of $r_k^R(i)$ and $r_k^F(i)$ depend only on the CSI of the $i$-th time slot, and are independent of the CSI of past and future time slots. The above theorem gives the maximum achievable rate of the $k$-th equation, $k = 1, ..., M$, is given by

$$\int_0^\infty \log_2(1 + x) f_{\Gamma_k i j}(x) dx = \int_0^\infty \log_2(1 + x) f_{\Gamma_k D}(x) dx.$$  

\[ \text{(16)} \]

$$f_{\Gamma_k i j}(x) = f_{\Gamma k i}(x) f_{\Gamma j}(x) \left( (1 + x)^{\frac{\lambda_k}{\gamma}} - 1 \right) \left( (1 + x)^{\frac{\lambda_j}{\gamma}} - 1 \right), \quad \text{where } \alpha = kD, \beta = Sk, \text{ and } \gamma = \mu_k. \]  

\[ \text{(17)} \]

By solving this system of $M$ equations, we obtain the optimal values of $\mu_k$, $\forall k$, denoted by $\mu_k^*$, which maximize $\bar{R}_{BA}$. When the fading on all source-to-relay and relay-to-destination links is i.i.d., then, $\mu_k^* = 1/2$, $\forall k$.

Proof: Due to lack of space, we only provide a sketch of the proof. The distribution in (17) can be found using ordered statistics theory. In particular, random variable (RV) $\Gamma_{k i j}$ becomes identical to RV $\gamma$, when $\lambda_k \gamma$ is the maximum over all RVs in the set $A$, and is zero otherwise, where $\alpha \in \{Sk, kD\}$. Hence, for $\Gamma_{k i j} > 0$, $f_{\Gamma_k i j}(x) = f_{\Gamma_k}(x) \Pr\{\lambda_k \gamma \text{ is max } A\}$. On the other hand, $\Pr\{\lambda_k \gamma \text{ is max } A\}$ is the probability that, except for element $\lambda_k \gamma$, all other elements in $A$ are smaller than $(1 + x)^{\lambda_k}$. From this, (17) follows.

Remark 2: For i.i.d. links, since $\mu_k^* = 1/2$, $\forall k$, the proposed protocol, given by (14), always utilizes for transmission the link with the largest instantaneous channel gain among all $2M$ links. Hence, for i.i.d. links this protocol becomes identical to the protocol proposed in [8]. However, for non-i.i.d. links, the protocol in [8] will cause data loss due to buffer overflow.

Lemma 3: Let us insert the optimal $\mu_k^*$, $\forall k$, found using Lemma 2, into $f_{\Gamma_k D}(x)$ given by (17), and denoted it by $f_{\Gamma_k D}^*(x)$. Then, the maximum achievable average rate of the protocol in Theorem 1, is given by

$$\bar{R}_{BA} = \sum_{k=1}^{M} \int_0^\infty \log_2(1 + x) f_{\Gamma_k D}^*(x) dx.$$  

\[ \text{(19)} \]

\[ \text{For i.i.d. fading on all links, (19) simplifies to} \]

$$\bar{R}_{BA} = M \int_0^\infty \log_2(1 + x) f_{\Gamma_{k D}}(x) \left( f_{\Gamma_{k D}}(x) \right)^{2M-1} dx.$$  

\[ \text{(20)} \]

Proof: Eq. (19) is obtained from (16) using $f_{\Gamma_{k D}}^*(x)$, whereas (20) is obtained by simplifying (19) for $\mu^* = 1/2$.
B. Buffer-Aided Protocol for Delay Constrained Transmission

The protocol in (14) with the $\mu_k^c, \forall k$, from Lemma 2, gives the maximum average achievable rate but introduces unbounded delay. To bound the delay, in the following, we propose a buffer-aided relaying protocol for delay limited transmission. To this end, we first limit the size of the buffer at all $M$ relays to at most $Q_{max}$ bits/symbol. Using $Q_{max}$, we now define $\hat{C}_{Sk}(i), \hat{C}_{KD}(i)$, and $A(i)$ as

$$\hat{C}_{Sk}(i) = \begin{cases} C_{Sk}(i), & \text{if } Q_{max} - Q_k(i-1) > C_{Sk}(i) \\ 0, & \text{otherwise} \end{cases}$$

$$\hat{C}_{KD}(i) = \min\{Q_k(i-1), C_{KD}(i)\}$$

$$A(i) = \{\mu_1\hat{C}_{St}(i), \mu_2\hat{C}_{St}(i), ..., \mu_M\hat{C}_{St}(i), (1-\mu_1)\hat{C}_{ID}(i), (1-\mu_2)\hat{C}_{ID}(i), ..., (1-\mu_M)\hat{C}_{ID}(i)\}.$$ (23)

Buffer-Aided Protocol for Delay Constrained Transmission:

Exploiting the above definitions, the values of $r^T_k(i)$ and $r^B_k(i)$ for the protocol with a constraint on the average delay are given by

$$\begin{cases} r^T_k(i) = 1 & \text{if } (1-\mu_k)\hat{C}_{KD}(i) = \max A(i) \\ r^T_k(i) = 0 & \text{otherwise} \end{cases}$$

$$\begin{cases} r^B_k(i) = 1 & \text{if } \mu_k\hat{C}_{Sk}(i) = \max A(i) \\ r^B_k(i) = 0 & \text{otherwise} \end{cases}$$

where $\mu_k, \forall k$, are the same as in the buffer-aided protocol without delay a constraint and are obtained using Lemma 2. The corresponding average rate achieved by this protocol is given by

$$\hat{R}_{BA} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{M} r^T_k(i)\hat{C}_{KD}(i).$$ (25)

The average delay of this protocol is controlled via $Q_{max}$.

The average delay for a system with $M$ parallel queues is well known, and given by [17, Eq. 11.69]

$$E\{T(i)\} = \sum_{k=1}^{M} E\{Q_k(i)\}$$

where $E\{Q_k(i)\}$ and $E\{A_k(i)\}$ are the average queue length and the average arrival rate in the buffer of the $k$-th relay, respectively, which are given by $E\{Q_k(i)\} = \lim_{N \to \infty} \sum_{i=1}^{N} Q_k(i)/N$ and $E\{A_k(i)\} = \lim_{N \to \infty} \sum_{i=1}^{N} r^B_k(i)\hat{C}_{Sk}(i)/N$, respectively. The average delay is studied in Section V via simulations.

IV. Implementation of the Proposed Protocol

In this section, we discuss the implementation of the protocol proposed in Theorem 1. The proposed protocol for transmission with a delay constraint, given in Section III-B, can be implemented in a similar manner.

The proposed protocol can be implemented in a centralized or in a distributed manner. A centralized implementation assumes a central node which selects the active relay in each time slot and decides whether it should receive or transmit. On the other hand, in the distributed implementation there is no central node and the relays themselves negotiate which relay should be active in each time slot. In the following, we discuss both implementations.

A. Centralized Implementation

For the centralized implementation, we assume that the destination is the central node. Hence, in each time slot, the destination has to obtain the CSI of all links. To this end, at the beginning of each time slot, the source transmits pilot symbols from which all relays acquire their respective source-to-relay channels. Then, each relay broadcasts orthogonal pilots, from which the source and destination learn all source-to-relay and relay-to-destination channels, respectively. Next, each relay feedback\(^3\) the CSI of its respective source-to-relay channel to the destination. With the acquired CSI, the destination computes $C_{Sk}(i)$ and $C_{KD}(i), \forall k$. In order to select the active relay according to the protocol in Theorem 1, the destination has to construct the set $A(i)$, given by (13). This requires the computation of the constants $\mu_k, \forall k$. These constants can be computed using Lemma 2, but this requires knowledge of the PDFs of the fading gains of all links before the start of transmission. Such a priori knowledge may not be available in practice. Therefore, the destination has to estimate $\mu_k, \forall k$, in real-time using only the CSI knowledge until time slot $i$. Since $\mu_k, \forall k$, are actually Lagrange multipliers obtained by solving the linear optimization problem in (12), an accurate estimate of $\mu_k, \forall k$, can be obtained using the gradient descent method [18]. In particular, using $\hat{C}_{Sk}(i)$ and $\hat{C}_{KD}(i)$, the destination computes an estimate of $\mu_k$, denoted by $\hat{\mu}_k(i)$, as

$$\hat{\mu}_k(i) = \mu_k^c(i-1) + \delta_k(i)(R^{e}_{KD}(i-1) - \hat{R}^e_{Sk}(i-1)),$$ (27)

where $R^{e}_{Sk}(i-1)$ and $R^{e}_{KD}(i-1)$ are real-time estimates of $R_{Sk}$ and $R_{KD}$, respectively, computed as

$$R^{e}_{\alpha}(i-1) = \frac{i-2}{i-1} R^{e}_{\alpha}(i-2) + \frac{i}{i-1} C_{\alpha}(i-1), i \geq 2.$$ (28)

where $\alpha = Sk, \beta = R$, and if $\alpha = kD, \beta = T$, and where $R^{e}_{\alpha}(0)$ is set to zero $\forall \alpha$. In (27), $\delta_k(i)$ is an adaptive step size which controls the speed of convergence of $\hat{\mu}_k(i)$ to $\mu_k$. The step size $\delta_k(i)$ can be some properly chosen monotonically decaying function of $i$ with $\delta_k(1) < 1$.

Once the destination has $C_{Sk}(i), C_{KD}(i)$, and $\mu_k^c(i), \forall k$, it constructs the set $A(i)$, and selects the active relay according to Theorem 1. Next, the destination broadcasts a packet to the relays which contains information regarding which relay is selected and whether it will receive or transmit. If the selected relay is scheduled to transmit, it transmits the information codeword intended for the destination. Otherwise, if the selected relay is scheduled to receive, it sends a packet to the source which informs the source which relay is selected. Then, the source transmits the information codeword intended for the selected relay.

The destination may receive the information bits in an order different than the order transmitted by the source. However, using the acquired CSI, the destination can keep track of the amount of information received and transmitted by each relay in each time slot. This information is sufficient for the destination to perform successful reordering of the received information bits.

B. Distributed Implementation

We now outline the distributed implementation of the proposed protocol using timers, similar to the scheme in [1].

At the beginning of time slot $i$, source and destination transmit pilots in successive pilot time slots. This enables the relays to acquire the CSI of their respective source-to-relay and relay-to-destination channels, respectively. Using the acquired CSI, the $k$-th relay computes $C_{Sk}(i)$ and $C_{KD}(i)$. Next, using $C_{Sk}(i)$ and $C_{KD}(i)$, the $k$-th relay computes the estimate of $\mu_k, \hat{\mu}_k(i)$, using (27) and (28). Using $C_{Sk}(i), C_{KD}(i)$, and $\hat{\mu}_k(i)$, the $k$-th

\(^3\)This feedback can also be done using pilots. In particular, since the destination already knows the channel from each relay to itself, the relay can broadcast pilots identical to the channel gains of the channel from the source to the selected relay.
relay turns on a timer proportional to \(1/\max\{\mu_k(i)C_{Sk}(i), (1-\mu_k(i))C_{kD}(i)\}\). This procedure is performed by all \(M\) relays. If \(\max\{\mu_k(i)C_{Sk}(i), (1-\mu_k(i))C_{kD}(i)\} = \mu_k(i)C_{Sk}(i)\) and \(\max\{\mu_k(i)C_{Sk}(i), (1-\mu_k(i))C_{kD}(i)\} = (1-\mu_k(i))C_{kD}(i)\), the \(k\)-th relay knows that if it is selected, then it will receive and transmit, respectively. The relay whose timer expires first, broadcasts a packet containing pilot symbols and information about which relay is selected and whether the selected relay receives or transmits. From the packet broadcasted by the selected relay both source and destination learn the channels from the selected relay to the source and destination, respectively, and learn which relay is selected and whether it is scheduled to receive or transmit. If the selected relay is scheduled to transmit, then it transmits the information codeword to the destination and no additional feedback is required in this time slot. However, if the relay is scheduled to receive, then the relay has to feedback its source-to-relay channel to the destination. This information is needed by the destination to keep track of the amount of information that each relay receives and transmits in each time slot so that the destination can perform successful reordering of the received information bits.

We note that distributed relay selection protocols based on timers may suffer from long waiting times before the first timer expires. Moreover, collisions are possible when two or more relay nodes declare that they are the selected node at approximately the same time. However, by choosing the timers suitably, as proposed in [19], these negative effects can be minimized.

C. Comparison of Resource Requirements of Conventional Protocols and the Proposed Buffer-Aided Protocol

In conventional relay selection protocols [1]-[7], the relay with the maximum \(\min\{C_{Sk}(i), C_{kD}(i)\}\) is selected to forward information from the source to the destination in the \(i\)-th time slot. Moreover, for both fixed and adaptive rate transmission, the selected relay and the destination have to know the CSI of the source-to-selected-relay and selected-relay-to-destination channels, respectively, in order to perform coherent detection. Furthermore, for adaptive rate transmission, both source and destination have to acquire \(\min\{C_{Sk}(i), C_{kD}(i)\}\) in order for the source to adapt its transmission rate and for the destination to know the size of the codebook used in time slot \(i\).

In Table I, we compare the number of pilot symbol transmissions and feedbacks of conventional relay selection protocols with fixed and adaptive rate transmission, respectively, and the proposed buffer-aided (BA) relaying protocol. In Table I, \(K(i)\) denotes the number of relays which can successfully decode the fixed rate codeword transmitted by the source in time slot \(i\), see [6]. As can be seen, the proposed buffer-aided protocol does not require more resources than the conventional relay selection protocols with adaptive rate transmission. We note however, that the proposed protocols require the computation of \(\mu_k(i)\), \(\forall i\), which are not required for the conventional protocols.

<table>
<thead>
<tr>
<th></th>
<th>Con., fixed rate</th>
<th>Con., adaptive rate</th>
<th>BA protocol</th>
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<tr>
<td>Centralized</td>
<td>(2K(i) + 1)</td>
<td>(2M + 2)</td>
<td>(2M + 1) or (2M + 2)</td>
</tr>
<tr>
<td>Distributed</td>
<td>3</td>
<td>4</td>
<td>3 or 4</td>
</tr>
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V. NUMERICAL EXAMPLES

We assume that all source-relay and relay-destination links are impaired with Rayleigh fading, where \(\Omega_{Sk}\) and \(\Omega_{kD}\) are the mean power of the fading of the source-to-\(k\)-th-relay and \(k\)-th-relay-to-destination channels, respectively. Throughout this section, we use the abbreviation “BA” to denote “buffer-aided”.

In Fig. 1, we plot the theoretical maximum average rate obtained from Theorem 1, and Lemmas 2 and 3, for \(M = 5\) relays and non-i.i.d. fading. We have also included simulation results for the proposed buffer-aided protocol for the considered network, where \(\mu_k, k = 1, \ldots, 5\), are found using the estimation method given in (27) with \(\delta_k(i) = 0.1/\sqrt{i}, \forall k\). Moreover, in Fig. 2, we show the corresponding estimated parameters \(\mu_k(i)\) and \(\mu_k(i)\) as functions of time for \(P/\sigma_n^2 = 0 \text{ dB}\). In Fig. 1, the simulated average rate coincides perfectly with the theoretical average rate. As benchmark, in Fig. 1, we have also plotted the average rate given in [10] and [11]. We note that we cannot use the protocols in [8] and [9] as benchmarks in this case since these protocols are not applicable to cases with non-i.i.d. fading as the buffers would become unstable. In particular, applying the protocols in [8] and [9], the buffers at relays with \(\Omega_{Sk} = \Omega_{kD}\) suffer from overflow and receive more information than they transmit. Hence, a fraction of the source’s data is trapped inside the buffers and does not reach the destination, i.e., data loss occurs.

In Fig. 3, we plot the theoretical achievable average rates for BA relaying for i.i.d. fading with \(\Omega_{Sk} = \Omega_{kD} = 1, \forall k\).
and $P/\sigma_n^2 = 10$ dB, as a function of the number of relays $M$. As can be seen from this numerical example, the growth rate of the maximum average rate is inversely proportional to $M$, i.e., the growth rate of the average data rate decreases as $M$ increases. In particular, the largest increase in data rate is observed when $M$ increases from one to two relays, whereas the increase in the maximum average rate when $M$ increases from 29 to 30 relays is almost negligible. As benchmarks, we also show the average rate given in [10] and [11], and the average rates achieved with the protocols in [8] and [9]. For i.i.d. links, as explained in Remark 1, the protocol in [8] is identical to the protocol presented in Theorem 1, thereby leading to the same rate.

In Fig. 4, we plot the achievable average rate for BA relaying without and with a delay constraint, as a function of $P/\sigma_n^2$, for i.i.d. fading and different numbers of relays $M$. This numerical example shows that, as the number of relays increases, the permissible average delay has to be increased in order for the rate with a delay constraint to approach the rate without a delay constraint. More precisely, for a single relay network, an average delay of five time slots is sufficient for the rate with a delay constraint to approach the rate without a delay constraint. However, for a network with two and four relays, the required delay for this is 7.5 and 11 time slots, respectively. In Fig. 4, we have also plotted the average rate given in [10] and [11], which have a delay of one time slot. Fig. 4 shows that the average rate of the buffer-aided relaying protocol with five time slots delay for a single relay network surpasses the average rate given in [10] and [11] achieved for a network with four relays.

VI. CONCLUSION

We have devised buffer-aided relaying protocols for the fading half-duplex relay selection network and derived their achievable average rates. We have proposed a buffer-aided protocol which maximizes the achievable average rate but introduces an unbounded delay, and a buffer-aided protocol which bounds the average delay at the expense of decrease in rate. We have shown that the new achievable rates are larger than the rates achieved with existing relay selection protocols. We have also provided centralized and distributed implementations of the proposed buffer-aided protocol, which do not require more resources for CSI acquisition than conventional relay selection protocols with adaptive rate transmission.

REFERENCES