Novel Protocol with Improved Outage Probability Performance for the Fading Two-Hop Half-Duplex Relay Channel

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Abstract—Based on the coding scheme recently introduced in [1], we propose new communication protocols with improved outage probability performance for the fading two-hop half-duplex (HD) relay channel. This channel is comprised of a source, a HD relay, and a destination, where a direct source-destination link does not exist. For this channel, we assume that the transmitting nodes do not have transmitter-side channel state information and therefore have to transmit with fixed rate. As a result, outages may occur. We propose protocols for the cases when feedback from receiving to transmitting nodes is not possible and restricted to one bit of feedback information per time slot, respectively. In the proposed protocols, the relay’s silent symbol intervals, when the relay receives, carry information which improves the reliability of both the source-relay and relay-destination links. In contrast, in existing protocols, the relay’s silent symbol intervals, when the relay receives, are a priori known to the destination and thereby cannot carry information. Our numerical results show that the proposed protocols achieve significant performance gains in terms of outage probability compared to existing protocols in the literature. These gains are only the result of using a different coding scheme and not of utilizing more resources than existing protocols.

I. INTRODUCTION

The two-hop relay channel is comprised of a source, a relay, and a destination, where the direct link between the source and the destination is not available. In this relay channel, the source transmits a message to the relay, which then forwards it to the destination. Generally, a relay can employ two different modes of reception and transmission, i.e., full-duplex (FD) and half-duplex (HD). In the FD mode, the relay receives and transmits at the same time and in the same frequency band. Whereas, in the HD mode, the relay receives and transmits in the same frequency band but not at the same time or at the same time but in orthogonal frequency bands, in order to avoid self-interference. Given the limitations of current radio implementations, practical FD relaying suffers from self-interference. As a result, HD relaying has been widely adopted in the literature [2]-[8].

Although extensively investigated [2]-[8], a tractable expression for evaluating the capacity of the two-hop HD relay channel was presented only recently in [1]. The authors of [1], also presented a coding scheme which achieves the capacity of the two-hop HD relay channel when fading is not present, and showed that the capacity rate is significantly higher than previously known achievable rates. The coding scheme proposed in [1] can be implemented in two different manners. In the first implementation, the relay switches between reception and transmission in a symbol-by-symbol manner. However, this approach may not be implementable in practice due to practical constraints regarding the speed of the switching. In the second implementation, the relay receives and transmits at the same time and in the same frequency band (as in FD operation), however, the simultaneous reception and transmission is performed while satisfying the HD constraint at the relay and thereby avoiding self-interference. In particular, in each symbol interval, either the transmitted or the received symbol of the relay are zero. Since the latter implementation is more practical, we adopt it in this paper.

In this paper, we use the coding scheme proposed in [1] to develop a novel communication protocol for improving the outage probability of the two-hop HD relay channel when the source-relay and relay-destination links are additive white Gaussian noise (AWGN) channels affected by fading, and source and relay do not have channel state information of their respective transmit links (CSIT). Moreover, in contrast to [9], we assume that the relay is not equipped with a buffer for long-term data storage. Hence, when the relay receives a message, it is forced to transmit it immediately in the following time slot. For the considered relay channel, we propose two protocols for the cases when feedback from receiving nodes to transmitting nodes is not possible and one bit of feedback information per time slot is possible, respectively. We will show that the proposed protocols can significantly improve the outage probability compared to the existing protocols in the literature, such as the protocol in [2]. The improved outage probability is achieved by using the coding scheme in [1], and not by utilizing more resources than existing protocols [2]. In particular, for the coding scheme used in previous protocols, the positions of the zero transmit symbols of the relay, which are the result of the silences when the relay receives, are a priori known at the destination and hence cannot be used to carry information. In contrast, in the proposed protocols, the positions of the zero transmit symbols of the relay are not known at the destination, and hence can be used to carry information. The additional encoding of the information using these zero symbols is shown to facilitate significant gains in the outage probability of the fading AWGN two-hop HD relay channel.

To the best of the authors’ knowledge, so far, the protocol achieving the best outage probability performance for the fading AWGN two-hop HD relay channel when the relay is not equipped with a buffer, and when source and relay do not have CSIT, was proposed in [2]. Thereby, in the first half of each time slot, the source transmits a codeword to the relay. The relay receives, and if it can decode the received codeword, it re-transmits the decoded information to the destination in the second half of the time slot, otherwise, it remains silent [2]. Since this protocol achieves the minimal outage probability for
the considered relay channel so far, we will use this protocol as benchmark for the proposed protocols. We note that the buffer-aided adaptive reception-transmission protocol proposed in [9] is not a fair benchmark (although it outperforms the protocol in [2]) since this protocol requires a buffer at the relay for long-term data storage, which leads to increased delays.

II. SYSTEM MODEL

The two-hop HD relay channel consists of a source, a HD relay, and a destination where a direct link between the source and the destination does not exist. Therefore, the source has to transmit its information to the destination via the relay. We assume that source and relay do not have CSIT and therefore transmit with a fixed rate $R$. As a result, outages may occur. We assume that transmission time is divided into slots of equal length and that transmission from source to destination is carried out in $N \to \infty$ time slots. Moreover, each time slot is divided into $n \to \infty$ symbol intervals.

A. Channel Model

In each time slot, source and relay transmit with powers $P_S$ and $P_R$, respectively, such that $P_S + P_R \leq P$ holds for each time slot, see also Remarks 1, 3, and 5 in Section III. The source-relay and relay-destination links are affected by independent complex circularly-invariant AWGN with variance $\sigma^2$, respectively. Moreover, we assume that the source-relay and relay-destination links are impaired by block-fading such that the fading remains constant in one time slot and changes from one time slot to the next. Let $\gamma_{SR}(i)$ and $\gamma_{RD}(i)$ denote the squared amplitudes of the complex channel gains of the source-relay and relay-destination channels in the $i$-th time slot, respectively. We assume that the channel gains of the source-relay and relay-destination links are known only at the relay and the destination, respectively, and are used for coherent decoding.

B. Half-Duplex Reception and Transmission

Due to the HD constraint of the relay, the transmit and received symbols of the relay cannot take non-zero values at the same time, i.e., either the transmit or the received symbol of the relay has to be zero (silence). More precisely, for each symbol interval, if the transmit symbol of the relay is non-zero then the received symbol of the relay has to be zero, and vice versa, if the received symbol of the relay is non-zero then the transmit symbol of the relay has to be zero. With this constraint imposed, the relay can receive and transmit at the same time, but this simultaneous reception and transmission is performed while satisfying the HD constraint and thereby self-interference does not occur.

III. CODING SCHEME AND COMMUNICATION PROTOCOLS

In this section, we introduce the proposed protocols for communication with outages over the block fading AWGN two-hop HD relay channel. We note that, for the sake of completeness, we have to review some of the results from [1]. However, the proposed communication protocols differ from the protocol in [1], which is not directly applicable to fading where outages are possible. In the following, we first explain the construction of the codewords and codebooks used for transmission. Then, we present the proposed communication protocols.

A. Construction of Codebooks and Codewords

We employ four codebooks: The source’s information codebook, the source’s auxiliary transmission codebook, the relay’s transmission codebook, and the relay’s control pattern codebook which controls the positions of the zero symbols in the source’s and relay’s transmission codewords. We assume that all four codebooks, and the corresponding mappings described below, are known at all three nodes before the start of transmission.

1) Relay’s Transmission and Control Codebooks: The relay’s transmission and control codebooks are generated by mapping each possible binary sequence comprised of $nR$ bits to a transmission codeword $x_2$ and a control vector $u$ both comprised of $n$ symbols. The $k$-th symbols, $k = 1, \ldots, n$, in codeword $x_2$ and in vector $u$ are generated in the following manner. For each symbol a coin is tossed. The coin is designed such that it produces symbol $r$ with probability $1 - P_U$ and symbol $t$ with probability $P_U$. The outcome of the coin flip is the $k$-th symbol of the relay’s control vector $u$. Furthermore, if the outcome of the coin flip is $r$, then the $k$-th symbol of the relay’s transmission codeword $x_2$ is set to zero. Otherwise, if the outcome of the coin flip is $t$, then the $k$-th symbol of codeword $x_2$ is generated independently according to the zero-mean circularly-symmetric complex Gaussian distribution with variance $P$. Each sequence of $nR$ bits is mapped to a transmission codeword $x_2$ and a corresponding control vector $u$. The $2^{nR}$ codewords $x_2$ and the corresponding $2^{nR}$ vectors $u$ form the relay’s transmission and control codebooks denoted by $C_2$ and $C_U$, respectively. We note that the codewords $u$ in $C_U$ are comprised of $n(1 - P_U) \pm \epsilon$ and $nP_U$ symbols $r$ and $t$, respectively, where $\epsilon > 0$ and $\lim_{n \to \infty} \epsilon/n = 0$ due to the law of large numbers, see [10] for details. As $n$ increases, we can ensure that the codewords $u$ in $C_U$ are comprised of exactly $n(1 - P_U)$ and $nP_U$ symbols $r$ and $t$, respectively, in the following manner. If a codeword $u$ is generated with $n(1 - P_U) - \epsilon$ and $nP_U + \epsilon$ (or with $n(1 - P_U) + \epsilon$ and $nP_U - \epsilon$) symbols $t$ in $u$ from $t$ to $r$ (or the last $\epsilon$ symbols $r$ in $u$ from $r$ to $t$), as $n \to \infty$, the number of switched symbols from $r$ to $t$ or vice versa becomes negligible compared to $n$. In this manner, we also ensure that all codewords $x_2$ in $C_2$ are comprised of $n(1 - P_U)$ and $nP_U$ zero and non-zero symbols, respectively. Moreover, note that the $2^{nR}$ vectors $u$ in $C_U$ may not be unique when $R > H(P_U)$, where $H(P_U)$ is the binary entropy function given by

$$H(P_U) = -P_U \log_2(P_U) - (1 - P_U) \log_2(1 - P_U).$$

(1)

However, this is not relevant since the vectors $u$ in $C_U$ are auxiliary vectors which are only used for determining the position of the zero symbols in relay’s transmission codewords.

2) Source’s Information Codebook: The source’s information codebook is generated by mapping each possible binary sequence comprised of $nR$ bits to a codeword $x_1$, where each codeword is comprised of $n(1 - P_U)$ symbols. The symbols in each codeword $x_1$ are generated independently according to the zero-mean circularly-symmetric complex Gaussian distribution with variance $P$. Since in total there are $2^{nR}$ possible

\[1\] The zero-mean circularly-symmetric complex Gaussian distribution is not the optimal distribution at the relay in this case, as shown in [1]. However, we adopt this distribution since it achieves rates close to the capacity and is relatively easy to evaluate.
The second vector $u$ as the first creating all $x$ was generated, and vice versa, given a pair of codeword to a unique pair of codeword $x$. Then, the first codeword $x$ in $C$ auxiliary transmission codebook is generated from the relay's complex symbols. These we generate binary sequences comprised of $nR$ bits, with this mapping we generate $2^{nR}$ codewords $x_{1|r}$ each containing $n(1 - P_U)$ complex symbols. These $2^{nR}$ codewords form the source's information codebook, which we denote by $C_{1|r}$.

3) Source's Auxiliary Transmission Codebook: The source's auxiliary transmission codebook is generated from the relay's control codebook $C_U$ and the source's information codebook $C_{1|r}$ in the following manner. First, the source generates a codebook, denoted by $C_{U|r}$, comprised of the unique vectors in the relay's control codebook $C_U$. The number of unique vectors in $C_U$ is $2^{n \min \{R, H(P_U)\}}$. Next, using the $2^{nR}$ codewords in $C_{1|r}$ and the $2^{n \min \{R, H(P_U)\}}$ unique control vectors in $C_U$, the source generates $2^{nR \max \{R, H(P_U)\}}$ auxiliary transmission codewords $x_1$ in the following manner. To generate the first $2^{nR}$ codewords $x_1$ in $C_1$, the source takes the first vector $u$ in $C_U$, denoted by $u_1$, and sets all of its $t$ symbols to zero. Then, the first codeword $x_1$ in $C_1$ is obtained by replacing all symbols $r$ in $u_1$ with the symbols in the first codeword in $C_{1|r}$. The second codeword $x_1$ in $C_1$ is obtained by replacing all symbols $r$ in $u_1$ with the symbols in the second codeword in $C_{1|r}$. This operation is performed for all $2^{nR}$ codewords $x_{1|r}$ in $C_{1|r}$ thereby constructing the first $2^{nR}$ codewords in $C_1$. The second $2^{nR}$ codewords in $C_1$ are constructed in the same manner as the first $2^{nR}$ codewords in $C_1$ with one difference. Instead of using the first vector $u$ in $C_U$, we use the second vector $u$ in $C_U$, denoted by $u_2$. This operation is performed with all $2^{nR \min \{R, H(P_U)\}}$ codewords $u_2$ in $C_U$, thereby creating all $2^{nR \max \{R, H(P_U)\}}$ codewords $x_{1|r}$ in $C_1$. Note that for every auxiliary transmission codeword $x_1$ there is a mapping to a unique pair of codeword $x_{1|r}$, and vector $u$ from which $x_1$ was generated, and vice versa, given a pair of codeword $x_{1|r}$ and vector $u$, there is a unique mapping to a codeword $x_1$.

In Fig. 1, we show an example for vector $u$ and the corresponding codewords $x_{1|r}$, $x_1$, and $x_2$ for $n = 8$ and $P_U = 1/2$. From this example, it can be seen that $x_1$ and $x_2$ contain zero symbols for symbol intervals for which the corresponding symbols in $u$ are $t$ and $r$, respectively. By comparing $x_1$ and $x_2$ it can be observed that the HD constraint is satisfied for each symbol duration.

In the following, we present two protocols for the cases when feedback from the receiving to the transmitting nodes is not possible and restricted to one bit of feedback information per time slot, respectively. We first present the protocol when feedback is not possible.

B. Proposed Protocol Without Feedback

The transmission of information from the source to the destination via the relay is carried out in $N \to \infty$ time slots, where in each time slot the channel is used $n \to \infty$ times. We assume that the source transmits message $w(i)$, $i = 1, ..., N - 1$, to the relay in time slot $i$, where each message $w(1), ..., w(N - 1)$ is comprised of $nR$ bits. If the relay can decode message $w(i - 1)$ at the end of time slot $i - 1$, then in the $i$-th time slot, the relay retransmits message $w(i - 1)$ to the destination. Otherwise, it remains silent and listens. In the following, we describe how transmission, reception, and decoding are performed in each time slot.

1) Time slot one: In the first time slot, the source transmits message $w(1)$, comprised of $nR$ bits, to the relay. The relay is silent and only receives during this time slot. The source transmits message $w(1)$ by mapping the $nR$ bits comprising message $w(1)$ to the corresponding information codeword for time slot one, $x_{1|r}(1)$, from codebook $C_{1|r}$. Then, codeword $x_{1|r}(1)$ is transmitted by the source to the relay in the first $n(1 - P_U)$ symbol intervals of the first time slot. Since the relay knows that codeword $x_{1|r}(1)$ is comprised of $n(1 - P_U)$ symbols, the relay also receives only during the first $n(1 - P_U)$ symbol intervals of time slot one and discards the rest of the symbols. Therefore, the relay constructs its corresponding received codeword for time slot one, which we denote by $y_{1|r}(1)$.

Lemma 1: Using $y_{1|r}(1)$, the relay can successfully decode $x_{1|r}(1)$ and thereby obtain message $w(1)$ if and only if (iff) 
\[
R < (1 - P_U) \log_2 \left(1 + \gamma_{SR}(1) \frac{P}{\sigma^2}\right)
\] (2)
is satisfied.

Proof: Since the symbols in $x_{1|r}(1)$ are independent zero-mean complex Gaussian distributed with variance $P$, and since the noise for each received symbol in $y_{1|r}(1)$ is also independent zero-mean complex Gaussian distributed with variance $\sigma^2$, the symbols in $y_{1|r}(1)$ are independent zero-mean complex Gaussian distributed with variance $\gamma_{SR}(1)P + \sigma^2$. Hence, the mutual information between the symbols in $x_{1|r}(1)$ and the corresponding symbols in $y_{1|r}(1)$ is 
\[
\log_2 (1 + \gamma_{SR}(1)P/\sigma^2)
\]
see [10]. On the other hand, the rate of codeword $x_{1|r}(1)$ is $nR/(n(1 - P_U)) = R/(1 - P_U)$. Hence, codeword $x_{1|r}(1)$ can be successfully decoded at the relay iff 
\[
R/(1 - P_U) < \log_2 (1 + \gamma_{SR}(1))
\] (2) holds. This concludes the proof.

Remark 1: Note that the average transmit powers of the source and the relay in time slot one are $P(1 - P_U)$ and zero, respectively.

2) Time slot $1 < i < N$: In time slot $1 < i < N$, the source transmits message $w(i)$ to the relay, which listens. While listening, the relay may either transmit or be silent depending on whether the relay has decoded message $w(i - 1)$ transmitted by the source in the previous (i.e., the $(i - 1)$-th) time slot. Thereby, the proposed protocol distinguishes two cases for time slot $i$. In the first case, the relay knows $w(i - 1)$ in time slot $i$, since it has successfully decoded message $w(i - 1)$ transmitted by the source in the $(i - 1)$-th time slot\(^2\). In the second case, we assume that the relay does not know $w(i - 1)$ in time slot $i$ since it was unsuccessful in decoding message $w(i - 1)$ in the

\(^{2}\)Theoretically, when the code is comprised of codewords which have $n \to \infty$ symbols and when jointly typical decoding is used, the relay knows exactly whether $w(i - 1)$ was decoded correctly or not, see [10]. In practice, for imperfect codes and finite codeword lengths, cyclic redundancy check techniques may be used.
(i - 1)-th time slot. In the following, we explain the protocol for both cases.

**Case 1 (Relay Knows w(i - 1) in Time Slot i):** In this case, for time slot i, the source and the relay both know \( w(i - 1) \) and they both use \( w(i - 1) \) to generate the control vector for time slot i, \( u(i) \). In particular, message \( w(i - 1) \) is mapped to the corresponding control codeword \( u(i) \) from codebook \( C_U \). Furthermore, using message \( w(i - 1) \), the relay also generates the corresponding transmission codeword for time slot i, \( x_2(i) \), by mapping \( w(i - 1) \) to the corresponding transmission codeword \( x_2(i) \) from codebook \( C_2 \). On the other hand, using message \( w(i) \), the source generates its information codeword for time slot i, \( x_1(i) \), by mapping \( w(i) \) to the corresponding information codeword \( x_1(i) \) from codebook \( C_1 \). Next, using the generated information codeword \( x_1(i) \) and control vector \( u(i) \), the source generates its auxiliary transmission codeword for time slot i, \( x_1(i) \). In particular, the pair \( x_1(i) \) and \( u(i) \) is mapped to the corresponding auxiliary transmission codeword \( x_1(i) \) from codebook \( C_1 \). Once \( x_1(i) \) and \( x_2(i) \) are generated, in time slot i, the source transmits \( x_1(i) \) to the relay and the relay transmits \( x_2(i) \) to the destination, while the relay is simultaneously receiving. Note that in each symbol interval either the corresponding element of \( x_1(i) \) or the corresponding element of \( x_2(i) \) is zero. Let the codeword that relay and destination receive in time slot i be denoted by \( y_1(i) \) and \( y_2(i) \), respectively. From the received codeword \( y_1(i) \), the relay selects only those symbols for which the corresponding symbol in \( u(i) \) is r and discards all other received symbols. In this way, the relay generates the received information-carrying codeword, denoted by \( y_1|r(i) \), which is comprised of received symbols corresponding to information codeword \( x_1|r(i) \). Note that the symbols in \( y_1|r(i) \) do not suffer from self-interference since when a symbol from \( y_1|r(i) \) is received at the relay, the corresponding transmit symbol of the relay is zero. In the following, we provide lemmas for the successful decoding of \( x_1|r(i) \) \( (w(i - 1)) \) from \( y_1|r(i) \), and for the successful decoding of \( x_2(i) \) \( (w(i - 1)) \) from \( y_2(i) \), respectively.

**Lemma 2:** Using \( y_1|r(i) \), the relay can successfully decode \( x_1|r(i) \) and thereby obtain message \( w(i - 1) \) iff (2) holds with \( \gamma_{SR}(1) \) replaced by \( \gamma_{SR}(i) \).

**Proof:** The proof is similar to the proof for Lemma 1, and can be obtained by replacing \( x_1|r(1), \ y_1|r(1), \) and \( \gamma_{SR}(1) \) in the proof for Lemma 1 with \( x_1|r(i), \ y_1|r(i), \) and \( \gamma_{SR}(i) \), respectively.

**Remark 2:** Note that the symbols in \( y_1(i) \) which are not contained in \( y_1|r(i) \) do not carry information since the corresponding transmit symbol of the source is zero (silence) and does not originate from the information-carrying codeword \( x_1|r(i) \).

**Lemma 3:** The destination can successfully decode \( x_2(i) \) from \( y_2(i) \) and thereby obtain message \( w(i - 1) \) iff

\[
R < -2 \int_{-\infty}^{\infty} \left( P_U f_T(y) + (1 - P_U) f_{N_R}(y) \right) dy \times \frac{\log_2 \left( P_U f_T(y) + (1 - P_U) f_{N_R}(y) \right)}{\pi \sigma^2}
\]

is satisfied, where \( f_T(y) \) and \( f_{N_R}(y) \) are real-valued zero-mean Gaussian distributions with variances \( (P\gamma_{RD}(i) + \sigma^2)/2 \) and \( \sigma^2/2 \), respectively.

**Proof:** Please refer to Appendix A.

In Fig. 2, we show a block diagram of the proposed protocol when the relay knows \( w(i - 1) \). In particular, Fig. 2 shows schematically the encoding, transmission, and decoding at source, relay, and destination. The flow of encoding/decoding in Fig. 2 is as follows. At the source, messages \( w(i - 1) \) and \( w(i) \) are encoded into \( u(i) \) and \( x_1(i) \), respectively, using encoders \( C_U \) and \( C_1 \), respectively. Then, codeword \( x_1(i) \) is constructed from the pair \( u(i) \) and \( x_1(i) \) using encoder \( C_1 \), and the source transmits \( x_1(i) \). On the other hand, the relay encodes \( w(i - 1) \) into \( u(i) \) and \( x_2(i) \) using encoders \( C_U \) and \( C_2 \), respectively. Then, the relay transmits \( x_2(i) \) while receiving \( y_1(i) \). Using \( u(i) \), the relay constructs \( y_1|r(i) \) from \( y_1(i) \) by selecting only those symbols for which the corresponding symbol in \( u(i) \) is r, see Fig. 3 for an example of extracting \( y_1|r(i) \) from \( y_1(i) \) for \( n = 8 \) symbols. The relay then decodes \( y_1|r(i) \), using decoder \( C_1^{-1} \), into \( w(i) \) and stores the decoded bits in its buffer B. On the other hand, the destination receives \( y_2(i) \) and decodes it using decoder \( C_2^{-1} \) into \( w(i - 1) \).
Case 2 (The Relay Does Not Know \( w(i-1) \) in Time Slot \( i \)):

In this case, since the source is not aware of the decoding failure at the relay, and, identical to Case 1, the source generates \( u(i) \) and \( x_{1|r}(i) \) from \( w(i-1) \) and \( w(i) \), respectively. Then, using the pair \( u(i) \) and \( x_{1|r}(i) \), the source generates its auxiliary transmit codeword \( x_1(i) \) and transmits this codeword to the relay in time slot \( i \). On the other hand, the relay does not transmit and only listens during all \( n \) symbol intervals in time slot \( i \), and thereby constructs its received codeword \( y_{1}(i) \). Now, since the relay does not know \( w(i-1) \), it cannot construct the control vector \( u(i) \), and thereby cannot extract \( y_{1|r}(i) \) from \( y_{1}(i) \) as in Case 1. Therefore, in this case, the relay decodes \( x_1(i) \) from \( y_{1}(i) \) using codebook \( C_1 \). If the relay can decode \( x_1(i) \), then it can obtain the information carrying codeword \( x_{1|r}(i) \) from which \( x_1(i) \) was generated, and thereby obtain \( w(i) \).

Lemma 4: The relay can successfully decode \( x_1(i) \) from \( y_{1}(i) \) and thereby obtain message \( w(i) \) iff

\[
R + \max\{R, H(P_U)\} < -2 \int_{\infty}^{1-P_R} \left( (1-P_U) f_R(x) + P_U f_{N_R}(x) \right) \log_2 \left( (1-P_U) f_R(x) + P_U f_{N_R}(x) \right) dx - \log_2(2\pi\sigma^2)
\]

is satisfied, where \( f_R(x) \) and \( f_{N_R}(x) \) are real-valued zero-mean Gaussian distributions with variances \((P\gamma_{SR}(i) + \sigma^2)/2 \) and \( \sigma^2/2 \), respectively.

Proof: Please refer to Appendix B.

In Fig. 4, we show a block diagram of the proposed protocol when the relay does not know \( w(i-1) \). Since this block diagram differs from the block diagram in Fig. 4 only with regard to the relay, we only show the block diagram of the relay. Thereby, we show schematically the decoding at the relay, which is as follows. The relay receives \( y_{1}(i) \) and since it does not know \( u(i) \) it decodes \( x_{1|r}(i) \) form \( y_{1}(i) \) using decoder \( C_{1}^{-1} \) and stores the decoded message, \( w(i) \), in its buffer. On the other hand, the destination does not receive anything since the relay is silent.

Remark 3: For Case 1, the average transmit powers of source and relay in time slot \( i \) are \( P(1-P_U) \) and \( PP_U \), respectively, whereas, for Case 2, the average transmit powers of source and relay in time slot \( i \) are \( P(1-P_U) \) and zero, respectively.

3) Time slot \( N \): In time slot \( N \), the source is silent since it has transmitted all \( N-1 \) messages in the previous \( N-1 \) time slots. On the other hand, the relay acts in the same way as in time slots \( 1 < i < N \). Hence, if Case 1 holds, the relay transmits \( w(N-1) \) to the destination iff (3) holds for \( i = N \). Otherwise, if Case 2 holds, the relay is silent since it does not have anything to transmit.

In the following, we propose an alternative protocol for the case when the relay can transmit one bit of feedback information per time slot to the source.

C. Proposed Protocol With One Bit of Feedback Information

The coding and the communication protocol when feedback of one bit of information is possible is almost identical to the case when feedback is not possible, but with two important differences. First, an additional codebook has to be generated. This codebook is comprised of \( 2^{nR} \) codewords, denoted by \( x_{1F} \), each comprised of \( n \) independently generated zero-mean complex Gaussian distributed symbols with variance \( P \). We denote this codebook by \( C_{F} \). We assume a unique mapping from each sequence of \( nR \) bits to a unique codeword in \( C_{F} \).

The second difference concerns the case when the relay cannot decode \( w(i-1) \). In this case, the relay informs the source about this event with one bit of feedback before the start of time slot \( i \). As a result, since the source is aware of the decoding failure at the relay, instead of mapping message \( w(i) \) to \( x_{1|r}(i) \) in \( C_{1|r} \), the source maps \( w(i) \) to the corresponding codeword \( x_{1F}(i) \) in \( C_{F} \) and transmits it to the relay. In this way, instead of encoding the \( nR \) bits to a codeword \( x_{1|r}(i) \) comprised of \( n(1-P_U) \) symbols, the \( nR \) bits are encoded into a codeword \( x_{1F}(i) \) comprised of \( n \) symbols. Hence, when feedback is possible, we use an additional \( nP_U \) symbols for encoding the \( nR \) bits. This additional encoding improves the reliability of the transmitted information. On the other hand, the relay only listens, since it does not have anything to transmit, and constructs the received codeword, which we denote by \( y_{1F}(i) \).

Lemma 5: Using \( y_{1F}(i) \), the relay can successfully decode \( y_{1F}(i) \) and thereby obtain message \( w(i) \) iff

\[
R < \log_2 \left( 1 + \gamma_{SR}(i) \frac{P}{\sigma^2} \right)
\]

is satisfied.

Proof: Since \( x_{1F}(i) \) and \( y_{1F}(i) \) are comprised of symbols which are independent complex Gaussian distributed with variances \( P \) and \( \gamma_{SR}(i) \) \( P \) and \( \sigma^2 \), respectively, the mutual information between the symbols in \( x_{1F}(i) \) and the corresponding symbols in \( y_{1F}(i) \) is \( \log_2 \left( 1 + \gamma_{SR}(i) \frac{P}{\sigma^2} \right) \). Now, since the rate of \( x_{1F}(i) \) is \( R \), \( x_{1F}(i) \) can be decoded iff (5) holds.

All other steps are exactly the same as for the protocol without feedback described in Section III-B.

Remark 4: Comparing the case when feedback is not possible to the case when feedback is possible, we note that the two protocols differ only when the relay does not know \( w(i-1) \) in time slot \( i \). In this case, when feedback is not possible, the source transmits codeword \( x_1 \), of which only \( n(1-P_U) \) symbols carry information, and which can be decoded iff (4) holds. On the other hand, when feedback is possible, the source transmits codeword \( x_{1F} \), of which all \( n \) symbols carry information, and which can be decoded iff (5) holds. Since the event in (5) is more probable than the event in (4), we expect the outage probability of the protocol with feedback to be smaller than the outage probability of the protocol without feedback, see Fig. 5. However, we also expect this difference in the outage probabilities to decrease with increasing signal-to-noise ratio (SNR) since the probability of the relay not knowing \( w(i-1) \) decreases with increasing SNR, cf. Fig. 5.
Algorithm 1 Computation of the Outage Probability.

\begin{verbatim}
if feedback is possible then
    F = 1
else
    F = 0
end if

do generate RVs \( \gamma_{SR}(i) \) and \( \gamma_{RD}(i) \) for \( i = 1, ..., N \).
for \( i = 1 \) to \( N \) do
    set \( O_{SR}(i) = 0 \) if \( O_{RD}(i) = 0 \)
    if \( i = 1 \) and (2) holds then
        \( O_{SR}(i) = 1 \)
    elseif \( O_{SR}(i-1) = 1 \) and (2) holds with \( \gamma_{SR}(1) \) replaced by \( \gamma_{SR}(i) \) or \( O_{SR}(i-1) = 0 \) and \( F = 0 \) and (4) holds
        \( O_{SR}(i) = 1 \)
    elseif \( O_{SR}(i-1) = 0 \) and \( F = 1 \) and (5) holds then
        \( O_{SR}(i) = 1 \)
    end if
    if \( i > 1 \) and (3) holds then
        \( O_{RD}(i) = 1 \)
    end if
end for
return \( P_{out} = 1 - \frac{1}{N-1} \sum_{i=2}^{N} O_{SR}(i-1) \cdot O_{RD}(i) \)
\end{verbatim}

**Remark 5:** For the case of feedback, the average transmitted powers of the source and the relay in time slot \( i \) are either \( P(1 - P_U) \) and \( PP_U \), respectively, or \( P \) and zero, respectively.

D. Outage Probability

In order to derive the outage probability, we define the following random variables (RVs)

\[
O_{SR}(i) = \begin{cases} 
0 & \text{if relay cannot decode the source’s message at the end of time slot } i, \\
1 & \text{if relay can decode the source’s message at the end of time slot } i,
\end{cases}
\]

\[
O_{RD}(i) = \begin{cases} 
0 & \text{if destination cannot decode the relay’s message at the end of time slot } i, \\
1 & \text{if destination can decode the relay’s message at the end of time slot } i.
\end{cases}
\]

Hence, an outage does not occur for message \( w(i-1) \) iff \( w(i-1) \) is decoded successfully at the relay and the destination at the end of time slots \( i-1 \) and \( i \), respectively, i.e., \( \text{iff } O_{SR}(i-1) = 1 \) and \( O_{RD}(i) = 1 \). Thereby, the outage probability, denoted by \( P_{out} \), is found as

\[
P_{out} = 1 - \Pr\{O_{SR}(i-1) = 1 \text{ AND } O_{RD}(i) = 1\}. \tag{6}
\]

Due to the page limit, we do not provide analytical expressions for the outage probabilities in this paper. Instead, we resort to simulations to evaluate the outage probabilities, cf. Algorithm 1.

IV. NUMERICAL RESULTS

In this section, we compare the outage probability achieved by the proposed protocols and the benchmark protocol in [2]. For the protocol in [2], we assume that the source and the relay transmit with rate \( 2R \). In this way, \( R \) bits/symb can be received at the destination in each time slot if an outage does not occur, which is identical to the \( R \) bits/symb that can be received at the destination for the proposed protocols if an outage does not occur, i.e., both the benchmark and the proposed protocols have identical throughputs in the absence of outages.

For the numerical example, we assume that the fading gains of the source-relay and relay-destination links follow unit-power independent identically distributed Nakagami-\( m \) RVs with \( m = 4 \). Hence, \( \gamma_{SR}(i) \) and \( \gamma_{RD}(i) \) have the following distribution

\[
f_{\gamma_{SR}}(x) = f_{\gamma_{RD}}(x) = \frac{4^m}{3^m} e^{-4x} \tag{7}
\]

In the simulations, we set the value of \( P_U \) such that

\[
E_{\gamma_{SR}} \{ \log_2(1 + \gamma_{SR}(i))(1 - P_U) \} = E_{\gamma_{RD}} \left\{ -2 \int_{-\infty}^{\infty} (P_U f_T(y) + (1 - P_U)f_{N^R}(y)) \log_2 (P_U f_T(y) + (1 - P_U)f_{N^R}(y)) dy \right\} - \log_2(\pi e \sigma^2) \tag{8}
\]

holds, where \( E_x\{\cdot\} \) denotes expectation with respect to \( x \). In this way, the average mutual information of the source-relay channel is identical to the average mutual information of the relay-destination channel. The value of \( P_U \) that satisfies (8) can be easily computed using a numerical software package such as Mathematica.

The outage probabilities of the considered protocols are shown in Fig. 5 for different values of \( R \). As can be seen from Fig. 5, for high SNRs and \( R \) equal to 1, 2, 3, and 4 bits/symb, the proposed protocols achieve SNR gains of 1, 2, 3, and 4 dB compared to the benchmark protocol, respectively. We note that if in this example \( R \) is increased further, the SNR gain will saturate at 5 dB. The saturation occurs since all considered protocols have different coding but identical diversity gains. From Fig. 5, we also observe that the outage probabilities of the protocols with and without feedback converge to the same value for high SNR, as predicted in Remark 4.

V. CONCLUSION

We have proposed new protocols for improving the outage probability of the fading two-hop HD relay channel based on the coding scheme in [1]. Thereby, the relay receives and transmits at the same time and in the same frequency band while satisfying the HD constraint at the relay and avoiding...
distribution, it follows that $N$ distribution, and since information between $X(2)$, we introduce the following notation. Let $X_2$ and $Y_2$ denote the RVs that model the transmitted and received complex symbols at relay and destination, respectively. Furthermore, let $N_2$ denote the RV that models the complex AWGN at the destination. Note that $X_2$ can be represented as $X_2 = X_2^R + jX_2^I$, where $X_2^R$ and $X_2^I$ are independent real-valued RVs that model the real and imaginary parts of the symbols transmitted by the relay, similarly. $N_2$ can be represented as $N_2 = N_2^R + jN_2^I$, where $N_2^R$ and $N_2^I$ are independent real-valued Gaussian RVs having variance $\sigma^2/2$ and modeling the real and imaginary parts of the complex AWGN at the destination, respectively. As a result, and since the destination performs coherent decoding, $Y_2$ can be expressed as

$$Y_2 = Y_2^R + jY_2^I = \left(\sqrt{\gamma_{RD}(i)}X_2^R + N_2^R\right) + j\left(\sqrt{\gamma_{RD}(i)}X_2^I + N_2^I\right),$$

where $Y_2^R = \sqrt{\gamma_{RD}(i)}X_2^R + N_2^R$ and $Y_2^I = \sqrt{\gamma_{RD}(i)}X_2^I + N_2^I$ are independent real-valued RVs that model the real and imaginary parts of the symbols received at the destination, respectively. Exploiting the mutual independence of all RVs modelling the real and imaginary parts, the mutual information between $X_2$ and $Y_2$ is obtained as $I(X_2; Y_2) = I(X_2^R; Y_2^R) + I(X_2^I; Y_2^I)$. Since $X_2^R$ and $X_2^I$ have the same distribution, and since $N_2^R$ and $N_2^I$ also have the same distribution, it follows that $I(X_2^R; Y_2^R) = I(X_2^I; Y_2^I)$, and therefore $I(X_2; Y_2) = 2I(X_2^R; Y_2^R)$. Hence, it is sufficient to consider only $I(X_2^R; Y_2^R)$. Now, note that $X_2^R$ is a mixture distribution, where the probability of $X_2^R$ having an outcome which belongs to a real-valued zero-mean Gaussian distribution with variance $P/2$ is $P_U$ and the probability of $X_2^R$ having outcome zero is $1 - P_U$. Hence, the distribution of $X_2^R$ is given by

$$f_{X_2^R}(x) = P_U f_G(x) + (1 - P_U) \delta(x),$$

where $f_G(x)$ is the real zero-mean Gaussian distribution with variance $P/2$ and $\delta(x)$ is the Dirac delta function. Since the distributions of $X_2^R$ and $N_2^R$ are known, and since $\gamma_{RD}(i)$ is a constant known at the destination, we can obtain the distribution of $Y_2^R = \sqrt{\gamma_{RD}(i)}X_2^R + N_2^R$ as

$$f_{Y_2^R}(y) = P_U f_T(y) + (1 - P_U)f_{N_2^R}(y),$$

where $f_T(y)$ and $f_{N_2^R}(y)$ are real zero-mean Gaussian distributions with variances $(P\gamma_{RD}(i) + \sigma^2)/2$ and $\sigma^2/2$, respectively. Now, the mutual information between $X_2^R$ and $Y_2^R$ is obtained as

$$I(X_2^R; Y_2^R) = h(Y_2^R) - h(Y_2^R | X_2^R),$$

where $h(Y_2^R) = h(N_2^R) = h(N_2^R)$ is the differential entropy of a real-valued zero-mean Gaussian RV with variance $\sigma^2/2$, which is given by $h(Y_2^R) = \log_2(\pi e \sigma^2/2)$. On the other hand, the differential entropy of $Y_2^R$, $h(Y_2^R)$ is obtained using the standard formula $[10]$}

$$h(Y_2^R) = -\int_{-\infty}^{+\infty} f_{Y_2^R}(y) \log_2(f_{Y_2^R}(y)) \, dy.$$

Inserting $f_{Y_2^R}(y)$ from (11) into (13), and then inserting (13) into (12), and using $I(X_2^R; Y_2^R) = 2I(X_2^R; Y_2^R)$, we obtain $I(X_2^R; Y_2^R)$ as the right hand side of (3). Hence, any rate $R$ that is smaller than the mutual information $I(X_2^R; Y_2^R)$, i.e., the right hand side of (3), can be successfully decoded at the destination using e.g. jointly typical decoding $[10]$. This concludes the proof.

**References**


