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Unified Theory of Nonlinear Photonic Guided-wave Coupling Systems

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A UNIFIED THEORY OF NONLINEAR PHOTONIC GUIDED-WAVE COUPLING SYSTEMS

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Abstract

A unified theoretical development of coupled wave equations is presented for nonlinear photonic guided-wave coupled systems using both scalar and vectorial approaches The theories are applicable for both generalised coupled waveguiding sytems including symmetric and asymmetric guided-wave coupling structures. An overview of the nonlinear symmetric guided-wave optical coupling systems idescribed demonstrating a generalisation of the analyses for nonlinear asymmetric coupling. General cases of studies including the nonlinear guided modes and power-nonorthogonality are considered and a full, power conserved and nonorthogonal coupled mode equations in terms of parameters directly related to the lightwave power defined in the waveguides, are proposed and analytically described. A numerical case study of the nonlinear asymmetric coupling system consisting of a slab-planar optical waveguide and a circular optical waveguide, the single mode optical fibre, is demonstrated as a proof of the developed unified theory..

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1 Overview of coupled-mode analyses for nonlinear optical guided-wave couplers

Nonlinear directional couplers offer possibilities of switching, steering and modulating of lightwaves by another control lightwaves via the nonlinear interaction in the coupling regions between waveguides. They exhibit potentials for applications in ultra-high bit rates optical communications systems in the picoseconds to sub-picoseconds switching time (ie. systems operating at bit rates >160 Gbps and beyond). The usual approach to solve such evolution of optical power coupling and switching in a nonlinear optical waveguide system is the employment of the coupled mode equations.

Jensen[1] first studied a nonlinear symmetric directional coupler (NLDC) consisting of two identical single-mode waveguides using conventional scalar coupled mode theory (CMT). Later, an analysis using a beam propagation method by Thylen et al.[2] has indicated that the conventional CMT is mainly valid for low power levels where the effects of the nonlinear perturbation on the waveguide guided modes are negligible. To include the nonlinear coupling effects, a solution to the formulation of full scalar CMT of the power-orthogonal NLDC has been developed [3,4]. The compound-mode approach, i.e. by using the expansion and superposition of symmetric-like and antisymmetric-like modes, has been used and that the coupling length has been proven to be a function of the propagating power level [4]. The power level is considered, as the average optical power required for power transferring and any temporal switching in such coupler would require an initial bias power, which must be set at this level.

Similar to the method of deriving the power-nonorthogonal vector CMT [5], a generalised vector reciprocity theorem has been used [6] in the formulation of the coupled mode equations

(CME) for a symmetric slab-slab NLDC. This differs from the previous formulations in that the nonlinear refractive index has been included in the expression of the unperturbed waveguide modes. As a result, the coupling coefficients become power-dependent. Similar numerical results for weak-coupling but improved results (in terms of accuracy) for stronger coupling in comparison. However, only TE wave propagation was considered, butt-coupling or the field-overlap has been neglected and only weak guidance was assumed in the numerical analysis. These conditions indicate that the cumbersome vector formulations may be replaced by a simple, scalar CMT without sacrificing the accuracy of the analysis. Note that these analyses have utilised parameters similar to Stokes parameters $\{s_0, s_1, s_2, s_3\}$[7] which are defined in terms of the complex modal amplitudes (a_1 and a_2) in the coupler as $s_0 = |a_1|^2 + |a_2|^2 = P_1 + P_2$, $s_1 = |a_1|^2 - |a_2|^2 = P_1 - P_2$, $s_2 = a_1 a_2^* + a_1^* a_2$ and $s_3 = \mathrm{j}(a_1 a_2^* - a_1^* a_2)$ with $j = (-1)^{1/2}$.

Furthermore graphical methods have also been introduced to assist the characterisation of the operation conditions, the power flow and evolution of the identical weakly guiding two-mode couplers [8-11]. The complex modal amplitudes were deconstructed [8] as a product of real amplitude and a phase term and, as a result, two constants of motion were derived with one representing the conventional power conservation law. In terms of the coupling coefficients and the constants of motion, the power in each mode can be analytically solved as elliptical integrals and, in addition, a few equalities can be obtained, which define the limit or range of the power operations. By analogy to a phase diagram, the graphical representation provides a quick indication of the type of the coupler operation and the range of power in each guide for certain initial conditions.

As described in Ref. [10], for a nonlinear two-mode coupler, a phase-mismatch parameter was defined and then the CMEs were expressed in terms of power in each guide, the phase-mismatch parameters, the mode and coupling constants. Based on the relative phase and the corresponding power flow between the two modes, a graphical representation can be created as a power-flow portrait of that particular coupler. In this paper the coupled-mode analysis of optical nonlinear (of third-order effects) coupling systems in both scalar approximation approach and full couple mode theory are critically summarised and compared leading to a generalised coupled mode formulation. It is shown that the full CMT offers much accurate the behaviour of the switching and modulation of the lightwaves as compared to that by a scalar analysis. Sections 2 gives a brief summary of the analyses of nonlinear symmetric optical coupling systems including a first-order, and leading to a generalised, full coupled-mode formulation in terms of parameters of power flowing in the two coupling waveguides for both cases of power-orthogonality and power-nonorthogonality of the nonlinear asymmetric guided wave slab-slab coupled systems. Symmetric and asymmetric non-linear coupling structures consisting of planar single-mode slab waveguides are summarised in identifying the orthogonality and non-orthogonality aspects in the nonlinear coupling systems, whilst Section 3 demonstrates the solutions of an asymmetric nonlinear coupling system, the nonlinear fibre-slab coupler, utilising a simplified and generalised scalar CMT formulation for coupled systems of nonlinear asymmetric fibre to slab coupling, or effectively the coupling of non-orthogonal single-mode in the fibre to multimode or a spectrum of guided modes of the planar slab guided-wave structure, and vice versa. The effects of the nonlinear medium in such asymmetric couplers are compared with the cases when the coupler is completely linear. Section 4 gives concluding remarks regarding the unified analyses for symmetric and non-symmetric nonlinear couplers.

2 Unified theoretical development for symmetric and non-symmetric nonlinear coupling systems

This section develops a generalised and unified coupled mode equations for symmetric and non-symmetric coupling systems in which both linear and nonlinear Kerr effects exist. Furthermore the scalar and vectorial coupled mode equations are described and applied to both coupling systems.

2.1 Nonlinear symmetric optical coupled systems

2.1.1 First-order scalar CME

The well-known coupled-mode equations for a Kerr-like nonlinear, symmetric NLDC of two identical modes are given by [1]

$$ja_1' = Q_1a_1 + Q_2a_2 + (Q_3|a_1|^2 + 2\,Q_4|a_2|^2)a_1 \tag{1}$$

$$ja_2' = Q_1a_2 + Q_2a_1 + (Q_3|a_2|^2 + 2\,Q_4|a_1|^2)a_2 \tag{2}$$

where the prime $'$ represents the first derivative, a_1 and a_2 the complex normalised amplitudes of the two modes and Q_1-Q_4 are the coupling coefficients defined as

$$Q_1 = (\omega/(4\pi P_0))\int dxdy\delta|E_1|^2, \quad Q_2 = (\omega/(4\pi P_0))\int dxdy(\varepsilon+\delta)|E_1E_2^*,$$

$$Q_3 = (n_0n_2\omega/(\pi P_0))\int dxdy|E_1|^4 \quad and \quad Q_4 = (n_0n_2\omega/(\pi P_0))\int dxdy|E_1|^2|E_2|^2 \tag{3}$$

where E_1 and E_2 denote the fields of the two guided modes, ε is the unperturbed susceptibility of one guide, δ is the linear perturbing susceptibility of that guide, and n_2 is the nonlinear refractive index. Strictly speaking, the nonlinear coefficient n_2 is generally different for the core and the cladding and should be contained in the integrals for the calculation of the relevant coupling coefficients. In the above definition, the nonlinear coefficient n_2 was considered a constant for the whole cross-section of the coupler structure, which is another approximation. In addition, the expressions of the CMT in Jensen's work[1] are in Gaussian units. The relative phase between E_1 and E_2 is chosen so that Q_2 is real. Among the optical third-order nonlinear Kerr effects, i.e. the dependence of refractive index on the light intensity, has aroused the most vigorous research due to the importance of possible applications of all-optical waveguiding, coupling and signal processing devices[11]. For ideal Kerr-law media (i.e. neglecting the higher-order effects such as saturation), the nonlinear refractive index is commonly defined by $n = n_o + \frac{n_{2,E}}{2}|\boldsymbol{E}|^2 = n_o + n_{2,I}I$ where n_o is the linear (low-power) refractive index, $n_{2,E}$ and $n_{2,I}$ the nonlinear (high-power) coefficients, with respect to $|\boldsymbol{E}|^2$, the electric field intensity, and I, the local average power intensity (W/m^2). The two nonlinear coefficients, also referred to as nonlinear refractive index, are related to each other by the following relation [14]: $I = \frac{1}{2} n_0 c \varepsilon_0 |\boldsymbol{E}|^2$ with $n_{2,E} = n_0 c \varepsilon_0 n_{2,I}$ where $n_{2,E}$ is in cgs unit whilst $n_{2,I}$ is in MKS units. The real amplitudes and the phase terms, as well as to absorb the self-coupling term (i.e. terms related to Q_1) can be used as

$$a_1 = A_1 \exp\{j(\phi_1 + Q_1 z)\} \text{ and } a_2 = A_2 \exp\{j(\phi_2 + Q_1 z)\} \tag{4}$$

where $\{a_1, a_2\}$ are the complex modal amplitudes, $\{A_1, A_2\}$ and $\{\phi_1, \phi_2\}$ are real functions of z. Four subsequent equations are obtained for the four unknowns. From these resultant equations, two constants of motion were found, one is the total power P_t, and the other is Γ, that is

$$P_t = A_1^2 + A_2^2 \quad \text{and } \Gamma = 4 A_1 A_2 \cos\Psi - 2(Q_3 - 2Q_4) A_1^2 A_2^2 / Q_2 \tag{5}$$

where $\Psi = \phi_1 - \phi_2$. Let $P = A_1^2$, $P_2 = A_2^2 = P_t - P$, therefore

$$\Gamma = 4\,[P\,(P_t - P)]^{1/2} \cos\Psi - 2(Q_3 - 2Q_4) P (P_t - P)/Q_2 \tag{6}$$

The power change, P', in one waveguide along the light propagation path becomes

$$P' = \{Q_2[4Q_2 - \Gamma(Q_3 - 2Q_4)]P(P_t - P) - \Gamma^2 Q_2^2/4 - (Q_3 - 2Q_4)^2 P^2 (P_t - P)^2\}^{1/2} \quad (7)$$

The solution, expressed in elliptic function and integral, is

$$P(Z) = P_c\{1/2 + \gamma\delta(\gamma^2+\delta^2)^{-1/2}\mathrm{sd}[Z(\gamma^2+\delta^2)^{1/2} + \mathrm{F}(\phi_0|m)|m]\} \quad (8)$$

where $Z = Q_2 z$, $F(\phi_0|m)$ is an elliptic integral of the first kind, and $\mathrm{sd}(\theta|m)$ is a Jacobian elliptic function and other parameters are defined as follows

$$(\gamma P_c)^2 = -4P_t^2 + 2P_c(P_c - \Gamma) + 2P_c(P_c - 2\Gamma P_c)^{1/2} \quad (9a)$$

$$(\delta P_c)^2 = 4P_t^2 - 2P_c(P_c - \Gamma) + 2P_c(P_c - 2\Gamma P_c)^{1/2} \quad (9b)$$

$$\sin^2(\phi_0) = (\gamma^2+\delta^2)[P(0) - P_t/2]^2/\{\delta^2[P(0) - P_t/2]^2 + [\gamma\delta P_c/4]^2\} \quad (10a)$$

$$m = \delta^2/(\gamma^2+\delta^2) \quad (10b)$$

where P_c is the critical power defined by

$$P_c = 4P_2/(Q_3 - 2Q_4) \quad (11)$$

If the light is launched into one guide initially, i.e. $P_1(0) = P_t$, $\Gamma = 0$, (8) becomes

$$P_1(Z) = P_1(0)[1 + cn(2Z|m)]/2 \quad (12)$$

where $m = P_1(0)^2/P_c^2$ and cn is a Jacobian elliptic function. The elliptic function cn is periodic with a period of $4K(m)$, where $K(m)$ is a complete elliptic integral of the first kind. As the input power is increased, so are the parameter m, $K(m)$ and the period of the elliptic function $\mathrm{cn}(\phi|m)$. The critical power P_c defined in (11) is an important parameter in that it defines a boundary between two distinct types of solutions, as described below. The symmetric nonlinear coupler can thus operate in two distinct cases:

(i) Low input power, i.e. $P_1(0) < P_c$

The optical power coupling appears similar to that of a conventional phase-matched two-mode coupler, that is a cross-stage or the light can be switched from the waveguide 1 to waveguide 2, whilst the nonlinear detuning effects on waveguides 1 and 2 are closely related to a $\Delta\beta$ reversal switch[1]. In the limit of $m \approx 0$, i.e. when the input power is very small,

$$P_1(Z) = P_1(0)[1 + \cos(2Z)]/2 \tag{13}$$

which becomes a solution for a linear two-mode coupler. The power half-beat length (with power transferred back or forth between the two waveguides) is

$$z = \pi/(2Q_2) \tag{14}$$

(ii) High input power, i.e. $P_1(0) \geq P_c$

It appears that the crossed state does not occur because of the detuned phase-mismatch between the two guides induced by the nonlinear refractive index. As a result, the 50/50 power distribution point is not reached, the phase is not reversed and a crossed state is not achieved.

2.1.2 Full scalar coupled-mode approach

An analytical solution to the following full nonlinear coupled-mode equations for a symmetric coupler of two identical modes, based still on the unperturbed (linear) two-modes[3] as

$$j(a_1'+Na_2')=(\beta+C_{11}+Q_1|a_1|^2+2Q_2|a_2|^2)a_1+(C_{12}+N\beta+2Q_3|a_1|^2+Q_3|a_2^2|)a_2+Q_2a_1^*a_2^2+Q_3a_1^2a_2^* \tag{15a}$$

$$j(a_2'+Na_1')=(\beta+C_{11}+Q_1|a_2|^2+2Q_2|a_1|^2)a_2+(C_{12}+N\beta+2Q_3|a_2|^2+Q_3|a_1^2|)a_1+Q_2a_2^*a_1^2+Q_3a_2^2a_1^* \tag{15b}$$

where $'$ indicates differentiation with respect to z, β is the propagation constant of the unperturbed mode, C_{11} and C_{12} are the *linear* self- and *cross-coupling* coefficients, respectively, N is the butt-coupling coefficient defined by $N = \int_{A\infty}\psi_1\psi_2 dA$. The power-dependent, nonlinear coupling coefficients Q_1, Q_2 and Q_3 are defined by

$$Q_1 = kn_2\int_{A\infty}\psi_1^4 dA,\ Q_2 = kn_2\int_{A\infty}\psi_1^2\psi_2^2 dA \quad \text{and} \quad Q_3 = kn_2\int_{A\infty}\psi_1\psi_2^3 dA \tag{16}$$

Similar to the approximation made by Jensen [1], the nonlinear coefficient n_2 is outside the integrals in the above expressions. This is again due to the assumption that the nonlinear coefficients of the waveguide cores and the claddings are identical. The nonlinear coupling effect and the non-orthogonality of the fields in two guides were considered [3], as well as the usual self- and cross-phase modulation terms in Jensen's formulation [1]. These additional terms shall be necessary when the instability, amplification and more accurate power-dependent switching and phase-controlled switching features are concerned. In fact, an earlier numerical study of the N-effect was reported [14], showing a slight improvement in accuracy. Another study [17] has shown an improvement in accuracy at low power with an approximate method to include both the N-effect and the nonlinear coupling effects. The switching power for the nonlinear coupler switch, the bias power for the amplifier and the instability power are found to be lower than the results reported earlier using the scalar CMT.

A symmetric NLDC of two identical slab waveguides (denoted as '*a*' and '*b*') are considered [6] with TE mode propagation and a cladding of Kerr-like nonlinear refractive index n_g between the two slabs, that is $n_g = n_3 + n_{3NL}I$ and improved coupled-mode equations are derived utilising a reciprocity theorem [5] as

$$jA' + jP_{ab}B' = Q_1A + [Q_2 + k_c(AB^* + A^*B)]B + [(k_c - k_s)|B|^2 + k_t(AB^* + A^*B)]A \tag{17a}$$

$$jB' + jP_{ab}A') = Q_1B + [Q_2 + k_c(AB^* + A^*B)]A + [(k_c - k_s)|A|^2 + k_t(AB^* + A^*B)]B \tag{17b}$$

where A and B are the modal amplitudes in the field expansion, and the coupling coefficients are given as

$$P = \tfrac{1}{4} \iint [\boldsymbol{E}_t^{(a)} \times \boldsymbol{H}_t^{(a)*} + \boldsymbol{E}_t^{(a)*} \times \boldsymbol{H}_t^{(a)}]_z \mathrm{d}x\mathrm{d}y \; and \; P_{ab} = \tfrac{1}{4} \iint [\boldsymbol{E}_t^{(a)} \times \boldsymbol{H}_t^{(b)*} + \boldsymbol{E}_t^{(b)*} \times \boldsymbol{H}_t^{(a)}]_z \mathrm{d}x\mathrm{d}y \quad (18)$$

$$Q_1 = \omega/(4P) \iint (\Delta\varepsilon + \Delta\varepsilon_{NL}) E^{(a)} E^{(a)*} \mathrm{d}x\mathrm{d}y \; and \; Q_2 = \omega/(4P) \iint (\Delta\varepsilon + \Delta\varepsilon_{NL}) E^{(a)} E^{(b)*} \mathrm{d}x\mathrm{d}y \quad (19)$$

$$k_s = \omega\varepsilon_0/(4P) \iint \alpha |E^{(a)}|^4 \mathrm{d}x\mathrm{d}y, \; k_c = \omega\varepsilon_0/(4P) \iint \alpha |E^{(a)}|^2 |E^{(b)}|^2 \mathrm{d}x\mathrm{d}y$$

$$k_t = \omega\varepsilon_0/(4P) \iint \alpha |E^{(a)}|^2 E^{(a)} E^{(b)*} \mathrm{d}x\mathrm{d}y \quad (20)$$

$$\Delta\varepsilon = \varepsilon_0[n^2 - (n^{(a)})^2], \; \Delta\varepsilon_{NL} = \varepsilon_0(\alpha - \alpha^{(a)})|E^{(a)}|^2 \quad (21)$$

Note that, if the nonlinear effects are weak, the improved formulation (20) may be reduced to equations (22) and (23) of Ref.[3], e.g. in these cases where the second terms (i.e. the nonlinear self- and cross-coupling coefficients) in (19) can be neglected and the nonlinear guided-modes can be replaced by the linear ones[3]. The Stokes parameters s_0-s_4, using (19), become

$$s_0' = 0, \; s_1' = -2(Q_2 + k_c s_2)s_3, \quad s_2' = (k_s - k_c)s_1 s_3 \quad and \quad s_3' = 2(Q_2 + k_c s_2)s_1 - (k_s - k_c)s_1 s_2 \quad (22)$$

which indicate that the total guided power is conserved between the two coupled modes, i.e. the coupler becomes a power-orthogonal one. Note that this is not valid generally unless the weak-coupling condition of $P_{ab} << 1$ is satisfied. Experimentally this approximation can be exploited by inspection of the linearity of the behaviour of the couplers and then tune the coupling effects by increasing or decreasing the level of optical power at the quiescent operating point.

With the input power initially launched into only guide 'a' of the NLDC, i.e. $A(0) = 1$ and $B(0) = 0$ and, from (1), $s_0(0) = s_1(0) = 1$, and $s_2(0) = s_3(0) = 0$, we have[6]

$$s_2' = 4Q_2[\eta(\zeta - \eta)(s_2 - \alpha_1)s_2(s_2 - \alpha_2)(s_2 - \beta_2)]^{1/2} \quad (23)$$

where $\zeta = (k_s - k_c)/(4Q_2)$, $\eta = k_c/(2Q_2)$, $\alpha_1 = -[1+(1+4\zeta\eta)^{1/2}]/(2\eta) \cong -1/\eta$, $\alpha_2 = -[1-(1+4\zeta\eta)^{1/2}]/(2\eta) \cong \zeta$, $\beta_2 = 1/(\zeta-\eta)$ and $k_c << Q_2$ and $k_c << k_s$ are generally valid, leading to

$$|\zeta| >> |\eta|,\ |\eta| << 1 \text{ and } \zeta\eta << 1\ ,\ |\alpha_1| >> |\alpha_2| \text{ and } |\alpha_1| >> |\beta_2| \tag{24}$$

Using the above constants and approximations, (20) can be analytical solved and from (1) and (21), the mode power in guide a as [6]

$$P_a = \begin{cases} \dfrac{P}{2}\{1 + \mathrm{cn}[2Q_2(4\zeta\eta + 1)^{1/4} z \mid m]\} & m \le 1 \\ \dfrac{P}{2}\{1 + \mathrm{dn}[2Q_2(\zeta(\zeta - \eta))^{1/2} z \mid m^{-1}]\} & m \le 1 \end{cases} \tag{25}$$

where cn and dn are Jacobian elliptic functions and m their modulus given by

$$m = \alpha_2(\beta_2 - \alpha_1)/[\beta_2(\alpha_2 - \alpha_1)] \approx \zeta(\zeta-\eta) \tag{26}$$

Because the elliptic functions have a period of $4k(m)$, with $k(m)$ as a complete elliptic integral of the first kind[13], the coupling length L_c (i.e. half beat length) of the NLDC is given by

$$L_c = \begin{cases} k(m)/[Q_2(4\zeta\eta + 1)^{1/4}] & m \le 1 \\ k(m^{-1})/[2Q_2(\zeta(\zeta - \eta))^{1/2}] & m \le 1 \end{cases} \tag{27}$$

In the case of weak coupling, $\eta \cong 0$ and the nonlinear effects are negligible, which leads to a power-independent Q_2. The full coupled-mode solutions of (24) and (26) are reduced to the first-order results of (9), respectively. When the input power is low, i.e. $m << 1$ and $k(m) \cong 0.5\pi(1 + 0.25m + ...) \cong 0.5\pi$. That is, the value of $k(m)$ is nearly a constant and L_c is therefore only dependent on Q_2. As the input power increases, Q_2 increases for self-focusing nonlinearity or decreases for self-defocusing nonlinearity in the NLDC. When m is increased, the contribution of $k(m)$ to the value of L_c becomes more and more significant. When $m = 1$, the input power is defined as the critical power P_c (which coincides with the first-order definition of (9) in the special case of $\eta \cong 0$) and L_c becomes infinite. In other words, the power transfer between the two modes (i.e. a crossed state) cannot be achieved. This special feature makes the NLDC a potential candidate as an all-optical power switch [6].

2.2 Nonlinear asymmetric slab-slab couplers

Important nonlinear coupled-mode formulations have been introduced in Section 2.1. In brief, the solutions of first-order coupled-mode equations (CME) for identical, power-orthogonal two-mode NLDC with Kerr-like nonlinearity can be expressed in standard elliptic integrals, whereas those for non-identical modes cannot, due to the mode asymmetry [1]. The full, nonlinear CME [3] have been solved with the aid of the two compound-mode amplitudes (a linear combinations of the linear waveguide-mode amplitudes) and the power orthogonality was enforced upon the formulation and was later improved [6] to replace the linear individual guided modes with the nonlinear correspondents. As a result, all the coupling coefficients in their CME become power dependent. However, after introducing the Stokes parameters in an attempt to analytically solve the CME, the power-nonorthogonality was totally ignored because only the case with negligible butt-coupling coefficient was considered. In this section, a generalised, full scalar CMT is proposed for symmetric two-mode NLDC with Kerr nonlinearity. Firstly, the coupled-mode equations are given, with the definitions of all the coupling coefficients, and reformulated in terms of the power parameters. Then the constants of motion (i.e. the z-invariants along the z-axis in Figure 1) are included satisfying the power conservation law, and analytical solutions are attempted and described. Our analysis is based on the power (i.e. guided-mode and cross-mode power) parameters. As an example, they are first applied to the simple, power-orthogonal two-mode NLDC and the law of power conservation and redistribution are given. Then a generalised, full coupled-mode formulation is proposed for power-nonorthogonal two-mode NLDC and, in terms of the power parameters, analytical solutions of the NLDC are obtained. In particular, the formulations in the power parameters are self-contained in that the total power is conserved and our full, power-nonorthogonal formulation generalises the previously published power-orthogonal results [1,3, 6].

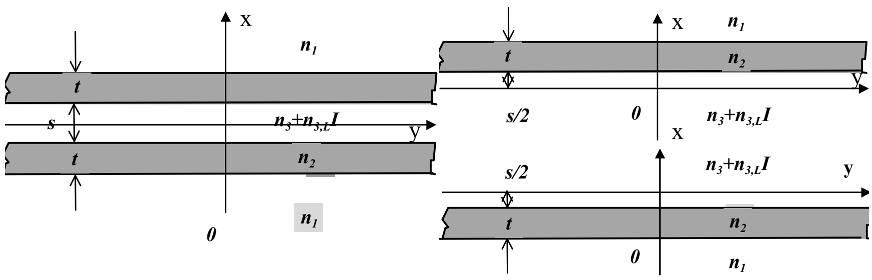

Figure 1. A cross-sectional schematic of the third-order nonlinear two-mode couplers: (a) The coupler; (b) The constituent waveguides

2.2.1 Power Parameters

The following four power parameters are defined

$$P_a(z) = a^*(z)a(z),\ P_b(z) = b^*(z)b(z),\quad P_r(z) = \mathrm{Re}[a^*(z)b(z)]\ \text{ and } P_i(z) = \mathrm{Im}[a^*(z)b(z)] \tag{28}$$

In comparison with the standard Stokes parameters and under power conservation, we have the following correspondence

$$P_a \Leftrightarrow (s_0+s_1)/2, \qquad P_b \Leftrightarrow (s_0-s_1)/2, \qquad P_r \Leftrightarrow s_2/2 \qquad and \qquad P_i \Leftrightarrow s_3/2 \tag{29}$$

2.2.2 Simplified CME for nonlinear asymmetric slab-slab coupled systems

The scalar, first-order coupled-mode equations for two non-identical guided modes can be obtained as

$$a' = -j(Q_a+Q_{aa}|a|^2+2Q_{ba}|b|^2)a\ -jK_{ab}b \tag{30a}$$

$$b' = -j(Q_b+Q_{bb}|b|^2+2Q_{ab}|a|^2)b -jK_{ba}a \tag{30b}$$

and in term of power parameters, they become

$$P_a' = 2K_{ab}P_i,\ P_b' = -2K_{ba}P_i,\ P_r' = [Q_b-Q_a+(2Q_{ab}-Q_{aa})P_a-(2Q_{ba}-Q_{bb})P_b]P_i$$

$$\text{and } P_i' = K_{ab}P_b-K_{ba}P_a+[Q_a-Q_b+(2Q_{ba}-Q_{bb})P_b-(2Q_{ab}-Q_{aa})P_a]P_r \tag{31}$$

The power conservation can be expressed as

$$P' = P_a'+P_b'+P_{ab}' = 0 \tag{32}$$

we have, from (31)

$$P_{ab}' = 2(K_{ba}-K_{ab})P_i \tag{33}$$

and the constant of motion Γ

$$\Gamma = K_{ba}P_a+K_{ab}P_b \tag{34}$$

Without losing generality, the new, power-nonorthogonal coupled-mode equations are given for an asymmetric two-mode NLDC consisting of two non-identical asymmetric slab waveguides. Application of the resultant formulation to other symmetric, nonlinear two-mode coupler configurations are straightforward. Figure 1(a) shows the cross-sectional view of a third-order NLDC whilst Figure 1(b) illustrates the constituent asymmetric waveguides a and b, respectively. The parameter t represents the thickness of the slab guides, s the separation distance between the two slab guides, n_1 and n_2 the refractive indices of the outside claddings and slab guides, respectively. The inside cladding with a refractive index of $n_3+n_{3NL}I$ is optically nonlinear, where n_3 is the linear component (as n_1 and n_2) whilst $n_{3NL}I$ is the nonlinear component with a third-order nonlinear coefficient n_{3NL} and local light intensity of I.

The influence of the nonlinearity of the waveguide materials on the coupled-modes in a waveguide coupler can be described by the nonlinear polarisation [6] related to the index change or, equivalently, by a perturbation to the refractive-index profiles. The perturbed index profiles of the two-mode coupler, waveguide *a* and *b*, i.e. $n(x)$, $n_a(x)$ and $n_b(x)$, can be expressed as a superposition of a linear (low power, with a sub-index of *L*) and Kerr-like nonlinear (high power, with a sub-index of *3NL*) refractive indices, respectively

$$n = n_L + n_{3NL,I} \tag{35a}$$

$$n_\nu = n_{L\nu} + n_{3NL,I\nu} \qquad (\nu = a, b) \tag{35b}$$

$$n_L = \begin{cases} n_1 \\ n_3 \\ n_2 \end{cases} \text{and } n_{3NL,I} = \begin{cases} 0 & for \quad |x| > s/2+t \\ n_{3,I} I & for \quad s/2 \le |x| \le s/2+t \\ 0 & for \quad |x| \le s/2 \end{cases} \tag{36a}$$

$$n_{La} = \begin{cases} n_1 \\ n_2 \\ n_3 \end{cases} \text{and } n_{3NL,I_a} = \begin{cases} 0 & for \quad x > s/2+t \\ 0 & for \quad s \le x \le s/2+t \\ n_{3,I_a} I_a & for \quad x < s/2 \end{cases} \tag{36b}$$

$$n_{Lb} = \begin{cases} n_3 \\ n_2 \\ n_1 \end{cases} \text{and } n_{3NL,I_b} = \begin{cases} n_{3,I_b} I_b & for \quad x > -s/2 \\ 0 & for \quad -s/2-t \le x \le -s/2 \\ 0 & for \quad x < s/2 \end{cases} \tag{36c}$$

where I is the local light intensity in W/m^2 and $\{n_{3,I\mu}\}(\mu = I, I_a, I_b)$ is a set of third-order nonlinear coefficients of the coupler system. Alternatively, in terms of the electric fields, we have

$$n^2 = n_L^2 + n_{3NL,E} \qquad and \qquad n_\nu^2 = n_{L\nu}^2 + n_{3NL,E\nu} \tag{37}$$

$$n_{3NL,E} = \begin{cases} 0 & for \quad |x| > s/2+t \\ n_{3,E} \, |\boldsymbol{E}|^2 & for \quad s/2 \le |x| \le s/2+t \\ 0 & for \quad |x| \le s/2 \end{cases} \tag{38a}$$

$$n_{3NL,E_a} = \begin{cases} 0 & for \quad x > s/2+t \\ 0 & for \quad s/2 \le x \le s/2+t \\ n_{3,E_a} \mid \boldsymbol{E_a} \mid^2 & for \quad x < s/2 \end{cases} \tag{38b}$$

$$n_{3NL,E_b} = \begin{cases} n_{3,E_b} \mid \boldsymbol{E_b} \mid^2 & for \quad x > -s/2 \\ 0 & for \quad -s/2-t \le x \le -s/2 \\ 0 & for \quad x < s/2 \end{cases} \tag{38c}$$

where the nonlinear coefficients $n_{3,E}$ and n_{3,E_i} $(i = 1, 2)$ are related to the nonlinear coefficients $n_{3,I}$ and ${}^2n_{3,I\nu}$ respectively, as given in Section 2, by

$$n_{3,E} = c\varepsilon_0 n_L{}^2 n_{3,I} \qquad and \qquad n_{3,E\nu} = c\varepsilon_0 n_{L\nu}{}^2 n_{3,I\nu} \qquad (\nu = a, b) \tag{39}$$

Assuming TE wave propagation, and utilising the vector reciprocity principle, the following full, Kerr-like nonlinear and power-conserved coupled-mode equations can be obtained as

$$a(z)' + P_{ab} b(z)' = -jQ_{az} a(z) - jK_z b(z) \tag{40a}$$

$$b(z)' + P_{ab} a(z)' = -jQ_{bz} b(z) - jK_z a(z) \tag{40b}$$

with the prime sign $'$ representing, again, the first-order derivative with respect to z, and Q_{1z}, Q_{2z} and K_z equivalent to

$$Q_{az} = Q + (K_c - K_s)|b(z)|^2 + K_t[a(z)b(z)^* + a(z)^* b(z)] \tag{41a}$$

$$Q_{bz} = Q + (K_c - K_s)|a(z)|^2 + K_t[a(z)b(z)^* + a(z)^* b(z)] \tag{41b}$$

$$K_z = K + K_c[a(z)b(z)^* + a(z)^* a(z)] \tag{41c}$$

where Q, K and P_{ab} are the new power-dependent self-, cross- and butt-coupling coefficients, whilst K_s, K_c and K_t are the self, cross phase-modulation and nonlinear coupling coefficients can be obtained as

$$Q = \frac{\varepsilon_0 \omega}{4P_0} \int_{A\infty} (n^2 - n_\nu^2) |\boldsymbol{E}_\nu|^2 dA \qquad (\nu = a \text{ or } b) \tag{42a}$$

$$K = \frac{\varepsilon_0 \omega}{4P_0} \int_{A\infty} (n^2 - n_\mu^2) \boldsymbol{E}_\mu \boldsymbol{E}_\nu^* dA \qquad (\mu, \nu = a, b \text{ but } \mu \neq \nu) \tag{42b}$$

$$K_s = \frac{\varepsilon_0 \omega}{4P_0} \int_{A\infty} n_{3NL,E} |\boldsymbol{E}_\nu|^4 dA \qquad (\nu = a \text{ or } b) \tag{43a}$$

$$K_c = \frac{\varepsilon_0 \omega}{4P_0} \int_{A\infty} n_{3NL,E} |\boldsymbol{E}_\mu|^2 |\boldsymbol{E}_\nu|^2 dA \qquad (\mu, \nu = a, b \text{ but } \mu \neq \nu) \tag{43b}$$

$$K_t = \frac{\varepsilon_0 \omega}{4P_0} \int_{A\infty} n_{3NL,E} |\boldsymbol{E}_\mu|^2 \boldsymbol{E}_\mu \boldsymbol{E}_\nu^* dA \qquad (\mu, \nu = a, b \text{ but } \mu \neq \nu) \tag{43c}$$

$$P_{ab} = \frac{1}{4P_0} \int_{A\infty} (\boldsymbol{E}_\mu \times \boldsymbol{H}_\nu^* + \boldsymbol{E}_\nu^* \times \boldsymbol{H}_\mu) \cdot \hat{\boldsymbol{z}}\, dA \qquad (\mu, \nu = a, b \text{ but } \mu \neq \nu) \tag{44a}$$

$$P_0 = \frac{1}{4} \int_{A\infty} (\boldsymbol{E}_\nu \times \boldsymbol{H}_\nu^* + \boldsymbol{E}_\nu^* \times \boldsymbol{H}_\nu) \cdot \hat{\boldsymbol{z}}\, dA \qquad (\nu = a \text{ or } b) \tag{44b}$$

where P_0 is the power of the ν^{th} guided-mode and $\hat{\boldsymbol{z}}$ is the unit vector along the z axis. Alternatively, (40.a) and (40.b) can be transformed into the following set of differential equations

$$a(z)' = -j(\underline{Q}_{az} - P_{ab}\underline{K}_z)a(z) - j(\underline{K}_z - P_{ab}\underline{Q}_{bz})b(z) \tag{45a}$$

$$b(z)' = -j`(\underline{Q}_{bz} - P_{ab}\underline{K}_z)b(z) - j(\underline{K}_z - P_{ab}\underline{Q}_{az})a(z) \tag{45b}$$

where the underline-sign '_' represents an operation of having the variable or constant divided by $(1-P_{ab})$. For example, we define

$$\underline{A} \equiv A/(1-P_{ab}) \tag{46}$$

With the above-defined coupling coefficients, the coupled-mode equations can be reformulated in power parameters as follows,

$$P_a' = (\underline{Q}_{ba}+2\underline{K}_{ct}P_r+P_{ab}\underline{K}_{sc}P_a)P_i, \qquad P_b' = -(\underline{Q}_{ba}+2\underline{K}_{ct}P_r+P_{ab}\underline{K}_{sc}P_b)P_i, \qquad P_r' = -\underline{K}_{sc}(P_a-P_b)P_i$$

$$\text{and} \qquad P_i' = (-\underline{Q}_{ba}+2\underline{K}_{st}P_r)(P_a-P_b) \tag{47}$$

where the following constants are defined in relation to the above coupling coefficients

$$\underline{K}_{sc} = \underline{K}_s - \underline{K}_c, \qquad \underline{Q}_{ba} = \underline{Q}_{bz} - P_{ab}\underline{Q}_{az}, \qquad \underline{K}_{ct} = \underline{K}_c - P_{ab}\underline{K}_t$$

$$\text{and} \qquad \underline{K}_{st} = \underline{K}_{sc} - 2\underline{K}_{ct} = \underline{K}_s - 3\underline{K}_c + 2P_{ab}\underline{K}_t \tag{48}$$

2.2.2.1 Constants of motion

From the above reformulated coupled-mode equations (45a&b), a few constants of motion can be readily derived. They include the law of power conservation (in the new formulation), and other equations in the power parameters, which are very helpful in achieving possible analytical solutions to the coupled-mode equations.

From (47), the following relationship for the conservation of the total guided power can be readily obtained as

$$P_a' + P_b' = -2P_{ab}P_r' \qquad \text{and} \qquad \Gamma_1 = P_a + P_b + 2P_{ab}P_r = P \tag{49}$$

where Γ_1 is a constant of motion and P represents the total power guided along the waveguide (coupler) axis. In fact, in comparison with the standard definition of the total guided power in the Poynting vectors, it can be found that, P_a and P_b represent the guided power in the mode a and b, respectively, whilst P_r represents the cross-power term that accounts for the mode (power) non-orthogonality of the coupler system. From (47), the constants of motion can also be found as

$$\Gamma_2 = (P_a-P_b)^2 - [1-4(2\underline{Q}_{21}/\underline{K}_{sc}+P_{ab})P_r-4(2\underline{K}_{ct}/\underline{K}_{sc}-P_{ab}^2)P_r^2] \tag{50a}$$

$$\Gamma_3 = P_i^2 - [2(\underline{Q}_{ba}/\underline{K}_{sc})P_r - (\underline{K}_{st}/\underline{K}_{sc})P_r^2] \qquad (50b)$$

A number of advantages of the reformulated coupled-mode equations (47) in power parameters, compared to those formulated in modal amplitudes (40) or (41) can be deduced. *Firstly,* these constants of motion are z-invariant, namely, they remain constant during the light propagation and coupling in the coupler system. In other words, during the operation of the coupler, these special constants of motion, which are functions of the power parameters, may completely reveal and describe the operation conditions of the whole coupler system. *Secondly*, these constants of motion can be determined by the initial launching conditions of the coupler and can therefore be used to search for the trajectories of the motion or to conduct stability analysis [7]. Finally, these special relations among the power parameters are very useful in the search for possible analytical solutions in terms of the mode or cross-mode powers.

2.2.2.2 Analytical solutions

In this section, an analytical solution is presented for the generalised and optical nonlinear coupled-mode equations (47) and the results are expressed in terms of the power parameters. Without losing generality, the practical one-guide launching conditions of a two-mode couplers are used in the following analysis, namely $a(0) = 1$ *and* $b(0) = 0$. Then the initial conditions for the power parameters become, by their definition

$$P_a(0) = 1 \qquad and \qquad P_b(0) = P_r(0) = P_i(0) = 0 \qquad (51)$$

and, utilising the coupled-mode equations (47 a-d), we have

$$P_a'(0) = P_b'(0) = P_r'(0) = 0 \qquad and \qquad P_i'(0) = -\underline{Q}_{ba} \qquad (52)$$

where $\underline{Q}_{ba}$ is defined in (48). From (47) - (50), the following relations can be derived

$$(P_a - P_b) = [-(\underline{\eta}/\underline{\zeta})(2P_r-\underline{\alpha}_1)(2P_r-\underline{\alpha}_2)]^{1/2} \quad \text{and} \quad P_i = [\underline{\gamma}P_r(\underline{\beta}_2-2P_r)]^{1/2} \tag{53}$$

in which the following constants are used

$$\underline{\zeta} = (4\underline{Q}_{ba}/\underline{K}_{sc}+2P_{ab})^{-1} \quad \text{and} \quad \underline{\eta} = (2\underline{K}_{ct}/\underline{K}_{sc}-P_{ab}^2)(4\underline{Q}_{ba}/\underline{K}_{sc}+2P_{ab})^{-1} \tag{54a\&b}$$

$$\underline{\alpha}_1 = -[1+(1+4\underline{\zeta}\underline{\eta})^{1/2}]/(2\underline{\eta}) \cong -1/\underline{\eta} \quad \text{and} \quad \underline{\alpha}_2 = -[1-(1+4\underline{\zeta}\underline{\eta})^{1/2}]/(2\underline{\eta}) \cong \underline{\zeta} \tag{54c\&d}$$

$$\underline{\beta}_2 = 4\underline{Q}_{ba}/\underline{K}_{st} = \{1-P_{ab}[2(\underline{\zeta}-\underline{\eta})+P_{ab}]/\underline{\gamma}\}/(\underline{\zeta}-\underline{\eta}) \cong (1-2P_{ab}\underline{\zeta})/(\underline{\zeta}-\underline{\eta}) \tag{54e}$$

$$\underline{\gamma} = \underline{K}_{st}/\underline{K}_{sc} = (\underline{\zeta}-\underline{\eta})/\underline{\zeta}+P_{ab}^2 \cong (\underline{\zeta}-\underline{\eta})/\underline{\zeta} \tag{54f}$$

where in (54c)-(54f), the condition of $P_{ab}^2 << 1$ is used, which is equivalent to that the two modes are not strongly coupled. From definition of (42) and (43), $K_c << K$ and $K_c << K_s$ are generally valid, which leads to the following approximations

$$|\underline{\zeta}| >> |\underline{\eta}|,\ |\underline{\eta}| << 1 \text{ and } \underline{\zeta}\underline{\eta} << 1 \quad \text{and} \quad |\underline{\alpha}_1| >> |\underline{\alpha}_2| \text{ and } |\underline{\alpha}_1| >> |\underline{\beta}_2| \tag{55}$$

Note that if the effects of P_{ab} are totally neglected, the above full coupled-mode equations, coupling coefficients and constants, i.e. those of (40)-(54) containing the underlined constants, are reduced to those defined in Ref.[6]. For example, when $P_{ab} << 1$, $\underline{\zeta} \to \zeta$, $\underline{\eta} \to \eta$, $\underline{\alpha}_1 \to \alpha_1$, $\underline{\alpha}_2 \to \alpha_2$, $\underline{\beta}_2 \to \beta_2$, respectively. Therefore, our formulation generalises the previous ones[1,3,6]. From (47) and (53), we have

$$2P_r' = \underline{K}_{sc}[(\underline{\eta}\underline{\gamma}/\underline{\zeta})2P_r(2P_r-\underline{\alpha}_1)(2P_r-\underline{\alpha}_2)(2P_r-\underline{\beta}_2)]^{1/2} \tag{56a}$$

$$\int_0^{2P_r(z)} [2P_r(2P_r-\underline{\alpha}_1)(2P_r-\underline{\alpha}_2)(2P_r-\underline{\beta}_2)]^{-1/2}\mathrm{d}(2P_r) = \underline{K}_{sc}(\underline{\eta}\underline{\gamma}/\underline{\zeta})^{1/2}z \tag{56b}$$

Utilising (54f), we have

$$K_{sc}(\eta\gamma/\zeta)^{1/2}z \cong 4Q_{ba}[\eta(\zeta-\eta)]^{1/2}/(1 - 2P_{ab}\zeta) \tag{57}$$

Without the loss of generality, under the assumption of the self-focusing Kerr nonlinearity, that is

$$n_{3,E} > 0,\ \alpha_1 < 0,\ \alpha_2 > 0,\ \text{and}\ n_{3,E} > 0,\ \beta_2 > 0 \tag{58}$$

The solution of the elliptic integral in (56) can be first expressed in the odd and even Jacobian elliptic functions of sn and cn[12, 6], respectively, and, utilising the approximations of (55), leading to

$$P_r = \begin{cases} \dfrac{P}{2}[1+\mathrm{cn}(\kappa_+ z \mid m)] & m \le 1 \quad (\alpha_2 \le \beta_2) \\ \dfrac{P}{2}[1+\mathrm{dn}(\kappa_- z \mid m^{-1})] & m > 1 \quad (\alpha_2 > \beta_2) \end{cases} \tag{59}$$

where $\{\kappa_+, \kappa_-\}$ and m are constants and the modulus of the elliptic functions {cn, dn}, respectively, defined as

$$\kappa_+ = 2\underline{Q}_{ba}(1 + 4\underline{\zeta}\underline{\eta})^{1/4}/(1 - 2P_{ab}\underline{\zeta}) \quad \text{and} \quad \kappa_- = 2\underline{Q}_{ba}[\underline{\zeta}(\underline{\zeta} - \underline{\eta})]^{1/2}/(1 - 2P_{ab}\underline{\zeta}) \tag{60}$$

$$m = \underline{\alpha}_2(\underline{\beta}_2 - \underline{\alpha}_1)/[\underline{\beta}_2(\underline{\alpha}_2 - \underline{\alpha}_1)] \cong \underline{\alpha}_2/\underline{\beta}_2 \cong \underline{\zeta}(\underline{\zeta} - \underline{\eta})/(1-2P_{ab}\underline{\zeta}) \tag{61}$$

In (60), the conditions of (55) and $P_{ab}^2 << 1$ are used. Due to the periodic properties of the elliptic functions of cn and dn, the half-beat length of the nonlinear coupler are given by

$$L_c = \begin{cases} (1 - 2P_{ab}\underline{\zeta})k(m)/[Q_2(4\underline{\zeta}\underline{\eta}+1)^{1/4}] & m \le 1 \quad (\alpha_2 \le \beta_2) \\ (1 - 2P_{ab}\underline{\zeta})k(m^{-1})/[2Q_2(\underline{\zeta}(\underline{\zeta}-\underline{\eta}))^{1/2}] & m \le 1 \quad (\alpha_2 > \beta_2) \end{cases} \tag{62}$$

The analytical solutions (59) and (62) therefore generalise those of Ref.[6], in that the field-overlap effects (i.e. butt-coupling coefficients and power-nonorthogonality) are accounted for in our CME and solutions without fully represented formulation. The special case of weak coupling with negligible field-overlap integrals can be derived from the formulation presented in this section and solution by applying the approximation $P_{ab} << 1$ and the above results are reduced to those of Ref.[6].

3 Case studies: Nonlinear asymmetric fibre-slab couplers

The reformulation technique adapting the power parameters demonstrated for the optical two-mode coupler systems is not completely applicable to the optical composite fibre-slab guided-wave coupling system. This is because that the fibre-slab mode-coupling involves *multimodes* and, therefore, an analytical solution to a set of large number of differential equations seems intractable. In this section, weak guidance, weak linear and nonlinear couplings are assumed. The fibre-slab nonlinear coupler with third-order nonlinear index distribution can thus be considered as a linear coupler with a nonlinear perturbation superimposed on its index profile. The cross sectional structure of such fibre-slab coupler is shown in Figure 2. Such fibre-slab structure has been analysed under complete linear coupling systems as described in Refs. [15, 16] and its main CME are briefly given in the Appendix. The problems of coupling between a fiber and an infinite slab in a completely linear system has been extensively investigated[15,18] with some corrections of the coupling coefficients of such systems[19]. The effect of the nonlinear coupling is expressed through the index modulation (perturbation) and thus the induced additional nonlinear coupling coefficients based on the linear, coupled fibre-slab guided modes. Furthermore a set of scalar, first-order coupled-mode equations is derived for a typical composite fibre-slab coupler configuration under linear operation and given in the appendix while the complete coupled equations incorporating both the linear and the additional nonlinear coupling coefficients are given analytically and examined in the following sub-sections.

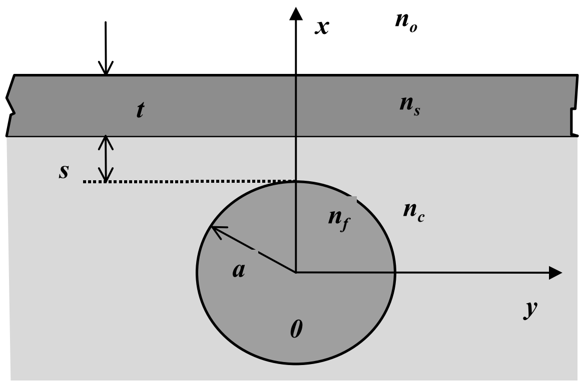

Figure 2. *Cross-sectional view of a fibre-slab waveguide coupler.*

3.1 Simplified Scalar CME for Nonlinear Asymmetric fibre-slab Couplers

The refractive indices of the coupler $n(x, y)$ (i.e. over the whole coupler cross-section A_∞) and of its different cross-sections $\{n_\nu(x, y)\}$ (i.e. over the cross-sectional areas of A_o, A_s, A_c and A_f, with $A_\infty \equiv A_o + A_s + A_c + A_f$) can be expressed as

$$n^2 = n_L^2 + n_2|\boldsymbol{E}|^2 \text{ for } (x, y \in A_\infty), \qquad n_\nu^2 = n_{L\nu}^2 + n_{2\nu}|\boldsymbol{E}_\nu|^2 \quad (x, y \in A_\nu)$$

$$\text{and} \qquad n_2 \equiv \{n_{2o}, n_{2s}, n_{2c}, n_{2f}\} \; (x, y \in A_\infty) \tag{63}$$

where $\nu = \{o, s, c\ f\}$ represents the overlay of the slab (A_o), the slab (A_s), the fibre cladding (A_c) and core (A_f), the first and second terms correspond to the linear and nonlinear index profiles, respectively. $\{\boldsymbol{E}, n_2\}$ and $\{\boldsymbol{E}_\nu, n_{2\nu}\}$ represent the electric fields and the third-order refractive coefficient of the coupler in each cross-section, respectively. Under scalar approximations, i.e. assuming weak guidance and coupling of the fibre and slab guided-modes, the following simplified, first-order CME and the above expression of the nonlinear index perturbation in (63)

$$a_0^{'} = -\mathrm{j}(\beta_{f0} + Q_{f00} + I_{f0}|a_0|^2 + \sum_{m,n}^{N} I_{fsmn} b_m b_n{}^*) a_0 - \mathrm{j}\sum_{n}^{N} K_{f0n} b_n \tag{64a}$$

$$b_m^{'} = -\mathrm{j}\sum_{n} (\delta_{mn}\beta_{sn} + Q_{smn} + \sum_{p,l}^{N} I_{mnpl} b_p b_l{}^* + I_{sfmn}|a_0|^2) b_n - \mathrm{j}K_{sm0} a_0 \tag{64b}$$

where a_0 and $\{b_m\}$ are the fibre and slab modal amplitudes in the total-field expansion, the prime sign ′ represent the first derivative with respect to the propagation distance z, $\{\beta_{f0}, Q_{f00}, K_{f0n}\}$ and $\{\beta_{sn}, Q_{smn}, K_{sm0}\}$ are the propagation constants, self- and cross-coupling coupling coefficients of the fibre and slab, respectively. The coupling coefficients under case when the coupler is under complete linear operating region, consisting of a circular fibre and a planar slab waveguide are given in the Appendix. Their full and detailed derivations can be found in Refs. [15, 16]. Whilst $\{I_{f0}, I_{mnpl}\}$, $\{I_{fsmn}, I_{sfmn}\}$ are the additional fibre and slab coupling coefficients corresponding to the self- and cross-phase modulations, respectively, defined as

$$I_{f0} = \frac{k^2}{2\beta_{f0}} \int_{A\infty} n_2 F_0^4 dA,\ I_{mnpl} = \frac{k^2}{2\beta_s} \int_{A\infty} n_2 S_m S_n S_p S_l dA,\ I_{fsmn} = \frac{k^2}{2\beta_{f0}} \int_{A\infty} n_2 F_0^2 S_m S_n dA$$

$$\text{and} \qquad I_{sfmn} = \frac{k^2}{2\beta_s} \int_{A\infty} n_2 F_0^2 S_m S_n dA \tag{65}$$

where $k = 2\pi/\lambda$ is the free-space wave number, n_2 is the nonlinear coefficient as defined in (63), F_0 and $\{S_n\}$ are the fundamental fibre mode and the slab mode of transverse order n, respectively. Note that n_2 is generally a cross-sectional distribution of nonlinear coefficients of the cores and claddings, i.e. $n_2 = \{n_{2o}, n_{2s}, n_{2c}, n_{2f}\}$ and, therefore, it should remain as a part of the integrand as in (65), rather than be taken out of the integral[3]. In (64), the third and fourth terms in the right-hand side are the dominant additional nonlinear terms representing the self- and cross-phase modulation effects, respectively, whilst the nonlinear modulation on the coupling coefficients and nonlinear coupling at high intensity are assumed negligible[1, 3].

3.2 Coupling Coefficients

We consider, for simplicity, the case that *only the overlay of the slab* is a Kerr-like nonlinear medium. In order to obtain an analytical solution of the integrals of the above-defined nonlinear coupling coefficients, further approximations can be made. Firstly, from the definition (63) and (65), the cross-phase modulation terms can be ignored in comparison with the self-phase modulation terms, because the field-overlap terms of the evanescent fields between the two guides are generally smaller than the mode-power terms. That is, the fourth terms on the right-hand side of (64) are small compared to the third terms. Secondly, from the above defined nonlinear index distribution and the notation of (63), we have $n_2 = \{n_{2o}, 0, 0, 0\}$. Therefore, the integral of the first term of (65) is carried out in the cross-section area of the slab-overlay cladding only (i.e.$\int_{A\infty} \rightarrow \int_{A_o}$) and, because the integrand is then composed of very weak evanescent field (i.e. the tails of the field extending into the slab overlay) of the fibre mode, the integral I_{f0} can be neglected in comparison with the propagation constant β_{f0} and the self-coupling coefficient Q_{f00}, i.e. the first and second terms on the right-hand side of the first term of (65). The nonlinear self-modulation coefficient I_{mnpl} can therefore be obtained as:

(i) For the above nonlinear-index distribution, I_{mnpl} of (65) becomes

$$I_{mnpl} = \frac{k^2}{2} n_{2o} \int_{A_s} S_m S_n S_p S_l \, dA \tag{66}$$

(ii) For the slab overlay, the scalar mode field is given by

$$S_n\Big|_{A_o} = \frac{N_s k_s t}{2V_{so}} e^{-\gamma_o[x-(a+s+t)]} \cos(\sigma_n y) \tag{67}$$

(iii) Substituting (67) into (66), the exact solution of I_{mnpl} can be expressed as

$$I_{mnpl} = \frac{\alpha_o D k^2}{\gamma_o}\left(\frac{N_s k_s t}{4V_{so}}\right)^4 \{\delta_{(m+n-p-l),0} + \delta_{(m+n+p-l+1),0} + \delta_{(m+n-p+l+1),0} + \delta_{(m-n+p+l+1),0}$$

$$+\delta_{(m-n-p-l-1),0} + \delta_{(m-n+p-l),0} + \delta_{(m-n-p+l),0}\} \quad (68)$$

3.3 Optical power tuning operations of the nonlinear asymmetric couplers

Numerical calculations have been carried out using practical parameters at a light wavelength λ = 1.55 μm, with fibre radius a = 2.5 μm, slab thickness t = 3 μm, refractive index of the fibre cladding n_c = 1.46, n_s = 1.4745, and the distance between the fibre and the slab s = 0.5 μm. The typical value of the self-focusing, nonlinear refractive index $n_{3,Io} = 10^{-9}$ m^2/W [6] is used and $n_{2o} = c\varepsilon_0 n_o^2 n_{3,Io}$ can be obtained, where n_o is the slab-overlay index of refraction, whilst c and ε_0 are the free-space light speed and dielectric constant, respectively.

Initially, the lightwave is launched (with unit power) into the fibre to excite its fundamental mode. Figure 2 shows that, when the core index of refraction of the fibre is greater than that of the slab, i.e. $n_f = 1.4817 > n_s = 1.4745$, (the fibre mode is out of phase with those of the slab transverse modes) the nonlinear effect is mainly displayed in the form of phase shift. The launched power largely remains in the fibre whilst a small amount couples back and forth between the fibre and the slab with an extended beat length (as a result of the nonlinear phase modulation of the guided modes).

When the fibre-core index of refraction is reduced to $n_f = 1.4756$ (greater than $n_s = 1.4745$, although the coupler is getting close to a phase match), Figure 3 demonstrates the interesting nonlinearity-assisted coupling which transfers up to a possible *100*% of the total power from the fibre to the slab. However, it appears that the exact amount of power transferred depends on the interaction (i.e. total coupling) length of the coupler due to the slow oscillation of the power between the two guides.

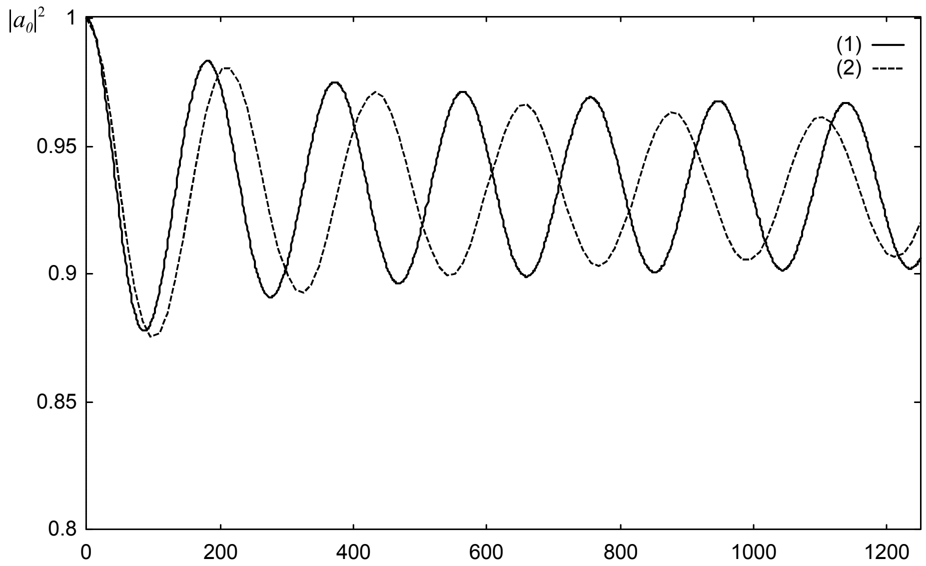

Figure 3. *Propagation of* $|a_0(z)|^2$. $n_f = 1.4817$, $n_s = 1.4745$, $n_c = 1.40$ *: (1) scalar CMT without nonlinearity; (2) scalar CMT with nonlinearity –this work.*

When the core refractive indices become equal, i.e. $n_f = n_s = 1.4745$, the coupler is almost exactly phase-matched (i.e. $\beta_{f0} \cong \beta_s$) and Figures 5 and 6 further indicate the nonlinearity-assisted coupling, with a nearly complete power transfer in this case. The above phenomenon of optically-enhanced coupling may be explained by the phase-detuning effect[1] due to the nonlinear phase modulation. The phase modulation is optically induced, initially starting in the fibre (the launching guide) and moves into the slab as the light is gradually coupled from the fibre to the slab. Here the slab self-phase modulation (67) is the contributing factor to the enhanced, phase-matched coupling, which drives the coupler to the state of a full power transfer (in analogy to the crossed state of a two-mode coupler).

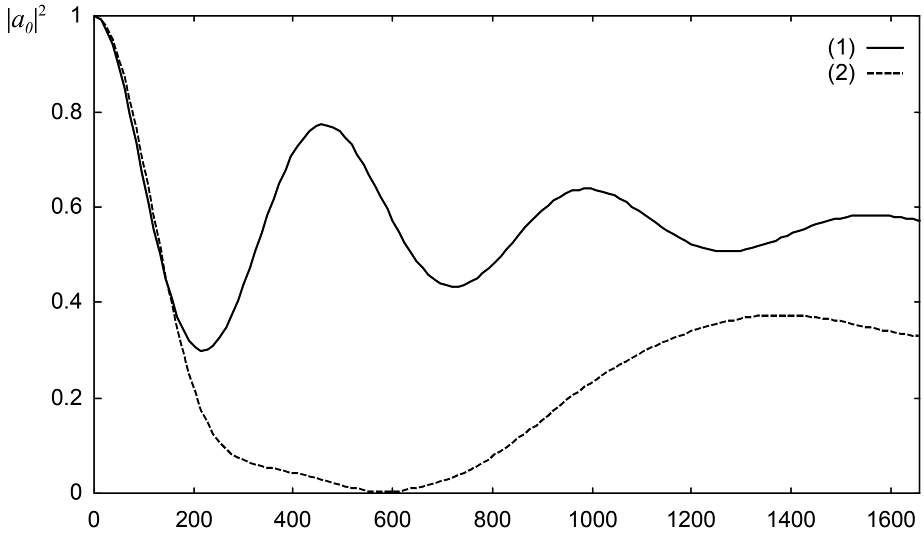

Figure 4. *Propagation of* $|a_0(z)|^2$. $n_f = 1.4756$, $n_s = 1.4745$, $n_c = 1.40$ *: (1) scalar CMT without nonlinearity; (2) scalar CMT with nonlinearity – this work.*

Finally, when the fibre-core refractive index becomes smaller than that of the slab, i.e. $n_f = 1.4709 < n_s = 1.4745$, the fibre mode is phase-matched with one of the slab transverse modes (i.e. at the absence of the nonlinear effect). As a result, the nonlinearity does not seem to significantly assist the coupling in terms of the power transfer, if at all. Instead, it alters the power-beating properties, such as the beat length and the amplitudes of the power oscillation, through the nonlinear phase self-modulation.

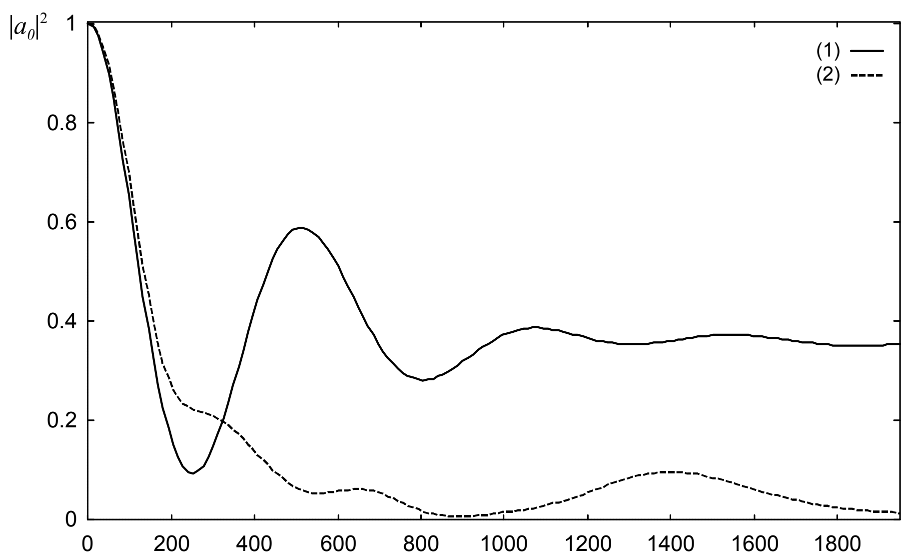

Figure 5. *Propagation of* $|a_0(z)|^2$. $n_f = 1.4745$, $n_s = 1.4745$, $n_c = 1.40$ *: (1) scalar CMT without nonlinearity; (2) scalar CMT with nonlinearity – this work.*

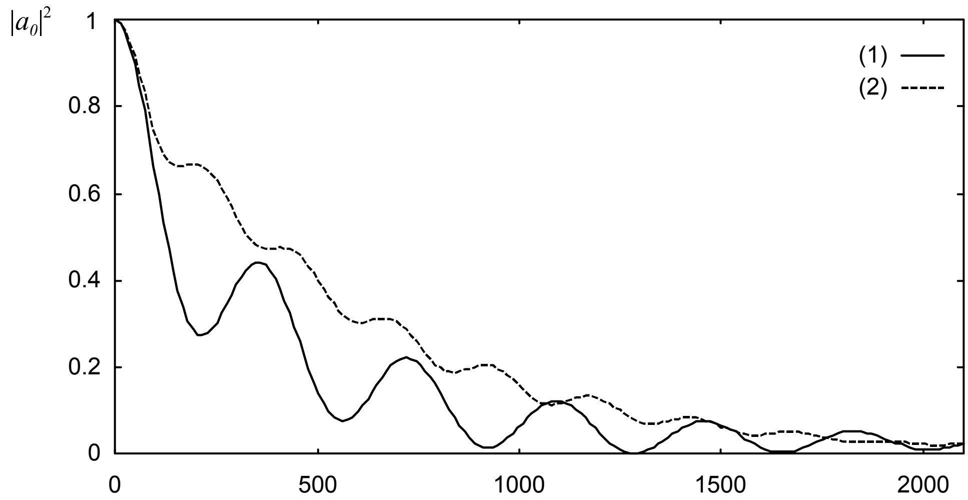

Figure 6. *Propagation of* $|a_0(z)|^2$. $n_f = 1.4709$, $n_s = 1.4745$, $n_c = 1.40$ *: (1) scalar CMT without nonlinearity (2) scalar CMT with nonlinearity – this work.*

4 Concluding remarks

A unified theoretical analysis has been developed and described for nonlinear asymmetric optical couplers utilising the scalar CMT whilst treating the effect of Kerr-like nonlinearity as a perturbation on the index-profile of the whole coupler structure. The most general cases (i.e. including the nonlinear guided-modes and power-nonorthogonality) are considered and full, power-conserved and nonorthogonal coupled-mode equations in the power parameters are proposed and analytically solved. Without losing simplicity (e.g. the format of the simplified analysis), the new formulations and solutions generalise the previous results[1,3,6] with all the coupling coefficients and constants updated to include the mode-nonorthogonality (i.e. the filed-overlap effects) ignored previously.

This generalised analysis would find immediate application in the design and analysis of optical nonlinear guided two-mode coupler system, especially for the cases of weak guidance and weak to moderate nonlinear couplings. A simplified, scalar coupled-mode analysis is presented to demonstrate the essential nonlinear effects on the optical composite fibre-slab guided-wave couplers. A simple, numerical example is used to demonstrate some interesting nonlinear coupling features that are quite similar to those of the grating-assisted coupling in a fibre-slab coupler which would be described in another article. When the parameters of the coupler are such that there is no phase synchronism between the fibre and the slab modes, the nonlinear phase-modulation comes into play and it assists coupling, resulting in up to a total power transfer from the fibre to the slab. However, when there is a phase synchronism between the two sets of guided modes, the effects of the Kerr-like nonlinearity seems greatly reduced and displayed as some minor changes to the small power beating on the power-decay curve.

Based on the above nonlinearity-assisted coupling feature we can propose an all-optical, in-line fibre to slab coupler device that is easy to fabricate, considering the matured polished-fibre or D-fibre technique. This composite fibre-slab guided-wave system may find immediate applications in the areas of in-line fibre couplers for optical switching and sensing technology. It may be designed to achieve the desired level of power transfer from the fibre (in-line) to the slab waveguide through an optimised selection of the materials and launched power.

5 References

[1] Jensen, S. M., “The nonlinear coherent coupler” *IEEE J. Quantum Electron., vol.* QE-18, pp.1580-1583, 1982.

[2] Thylen, L., Wright, E. M., Stegeman, G. I., Seaton, C. T. and Moloney, J. V., “Beam propagation method analysis of a nonlinear directional coupler exhibiting a Kerr nonlinearity” *Opt. Lett*. 11, QE-24, pp.739-741, 1986.

[3] Chen, Y., “Solution to full coupled mode equations for nonlinear coupled systems”, IEEE J. Quantum Elect., vol. 25, No. 10, pp. 2149-2153, 1989.

[4] Cada, M. and Begin, J. D., "An analysis of a planar directional coupler with a lossless Kerr-like coupling medium" *IEEE J. Quantum Electron*. Vol. 26, pp. 361-371, 1990.

[5] see Chuang, S. L. "A coupled-mode formulation by reciprocity and a variational principle" *J. Lightwave Technol*. LT-5, pp.5-15, 1987 and Chuang, S. L., "Application of the strongly coupled-mode theory to integrated optical devices" *IEEE J. Quantum Electron*. QE-23, pp.499-509, 1987.

[6] Meng, X. J. and Okamoto, N. (1991) "Improved Coupled-Mode Theory for Nonlinear Directional Couplers" *IEEE J. Quantum Electron*. Vol.27, pp.1175-81, 1991.

[7] Daino, B., Gregori, G. and Wabnitz, S., "Stability analysis of nonlinear coherent coupler", *J. Appl. Phys*. **58** (12), pp. 4513-4514, 1985.

[8] Pham, A.T. and Binh, L.N., "Nonlinear optical directional coupler used as optical amplifier with twin biasing beams", *Int. J. Optoelectron*. 5, pp. 381-386, 1990.

[9] Pham, A.T. and Binh, L.N., "All-optical modulation and switching using a nonlinear-optical directional coupler", *J. Opt. Soc. Am.* B, vol. 8, pp. 1914-1931, 1991.

[10] Snyder, A. W., Mitchell, D. J., Poladian, L., Rowland, D. R. and Chen, Y., "Physics of nonlinear fiber couplers", *J. Opt. Soc. Am*. B., vol. 8, pp. 2102-2118, 1991.

[11] Shen, Y. R. "The principles of nonlinear optics", Wiley, New York, 1984.

[12] Sammut, R. A. and Pask, C., "Gaussian and equivalent-step-index approximations for nonlinear waveguides" *J. Opt. Soc. Am*. B, vol. 8, No.2, pp. 395-402, 1991.

[13] Abramowitz, M. and Stegan, I. A., "Handbook of mathematical functions" New York: Dover, 1965.

[14] Ankiewicz, A., "Novel Effects in Non-linear Coupling" *Opt. Quant. Elect., vol. 20*, pp. 329-337, 1988.

[15] Marcuse, D., "Investigation of coupling between a fiber and an infinite slab", IEEE J. Lightw. Tech., vol. 7, pp. 122-130, 1989.

[16] Zheng S., G.P. Simon and Binh, L.N., "Light coupling and propagation in composite optical-fiber slab waveguides", IEEE J. Lightw. Tech., vol. 13, pp. 244-251, 1995.

[17] Snyder A. and Chen Y., "Nonlinear fibre couplers: switches and polarisation splitters", Opt. Lett., vol. 14, pp. 517-519.

[18] Marcuse, D., "Directional couplers made of non-identical asymmetric slab: Part I: synchronous couplers", IEEE J. Lightw. Tech., vol. LT-5, No. 1, pp. 113-118, 1987.

[19] Marcuse, D., Personal communications in relation to the paper of D. Marcuse [19].

6 Appendix: CMEs of The Fibre-Slab Linear Coupling System

The transverse electric field in the coupled fibre and slab waveguides can be well represented by the superposition of the guided-modes of the fibre and the slab, i.e. $\{F_0, S_n\}$

$$E(x,y,z) = a_0(z)F_0(x,y) + \sum_n b_n(z)S_n(x,y) \qquad \text{(A1)}$$

where $\{a_0(z), b_n(z)\}$ ($n = 0, 1, 2, \cdots$) is a set of z-dependent expansion (excitation) coefficients (modal amplitudes), whilst the summation is carried out over the set of discretised transverse slab modes of the planar waveguide.

In the co-ordinate system shown in Figure 2, the dominant LP_{01} mode (the transverse electric field) of the isolated fibre is given by

$$F_0 = N_f \begin{cases} \dfrac{J_0(k_f r)}{J_0(k_f a)} & \quad r \le a \\ & for \\ \dfrac{K_0(\gamma_f r)}{K_0(\gamma_f a)} & \quad r > a \end{cases} \qquad \text{(A2)}$$

where J_0 and J_1 are the Bessel functions of the first kind, K_0 is the modified Bessel functions of the second kind and $N_f = \dfrac{\gamma_f J_0(k_f a)}{\sqrt{\pi} V_f J_1(k_f a)}$ is the normalisation constant of the fibre mode.

The parameter a denotes the fibre radius, $k_f^2 = n_f^2 k^2 - \beta_{f0}^2$ and $\gamma_f^2 = \beta_{f0}^2 - n_c^2 k^2$ are constants in which $k = 2\pi/\lambda$ (λ is the free-space light wavelength), β_{f0} is the propagation constant obtained from the LP_{01} mode dispersion equation[5] and n_f and n_c are the refractive indices of the fibre core and cladding respectively. $V_f = ka(n_f^2 - n_c^2)^{1/2}$ is a dimensionless waveguide parameter related to k_f and γ_f via

$$V_f^2 = a^2(k_f^2 + \gamma_f^2) \qquad \text{(A3)}$$

In the same co-ordinate system (i.e. Figure 2), the nth guided transverse slab mode is obtained in the form

$S_n(x,y) =$

$$N_s\cos(\sigma_n y)\begin{cases} V_{so}\exp[\gamma_c(x-h)]/V_{sc} & x < a+s \\ \{\cos[k_s(x-h-t)]-(\gamma_o/k_s)\sin[k_s(x-h-t)]\} \quad \mathit{for} & a+s \le x \le a+s+t \\ \exp[-\gamma_o(x-h-t)] & x > a+s+t \end{cases}$$

(A4)

where $n = 0, 1, 2, \cdots$, $k = 2\pi/\lambda$, $k_s^2=n_s^2k^2-\beta_s^2$, $\gamma_c^2=\beta_s^2-n_c^2k^2$, $\gamma_o^2=\beta_s^2-n_o^2k^2$, $V_{sc}=kt(n_s^2-n_c^2)^{1/2}/2$, $V_{so}=kt(n_s^2-n_o^2)^{1/2}/2$, $N_s=[\frac{2\gamma_c\gamma_o}{(\gamma_o+\gamma_c+\gamma_c\gamma_o t)D}]^{\frac{1}{2}}$ is the normalisation constant and σ_n is defined from $\beta_{sn}^2=\beta_s^2-\sigma_n^2$ for n= 0,1,2…., i.e. the transverse propagation constant of the lightwaves travelling along the z-direction. The parameter t denotes the slab thickness, s the minimum distance between the surface of the fibre core and that of the polished flat (see Figure 1(b)), β_s is the propagation constant of the slab mode, n_s and n_o are the refractive indices of the slab and the overlay cladding, respectively. V_{sc} and V_{so} are dimensionless waveguide parameters related to k_s, γ_c and γ_o via

$$V_{sc}^2 = t^2(k_s^2+\gamma_c^2)/4 \qquad \text{and} \qquad V_{so}^2 = t^2(k_s^2+\gamma_o^2)/4 \tag{A5}$$

The propagation constant β_s of the slab guided mode of the mth order can be obtained from the well known dispersion equation of the asymmetric planar slab waveguide

$$k_s t-\tan^{-1}(\gamma_c/k_s)-\tan^{-1}(\gamma_o/k_s) = m\pi \qquad (m = 0, 1, 2, \cdots) \tag{A6}$$

The CME are obtained when the field expansion (A1) is substituted into the wave equation of the total field, and then the resultant equation is multiplied successively with F_0 and S_n and integrated over the transverse (x, y) plane whilst making use of the wave equations and orthogonal relations of the fibre and slab modes. Using the scalar approximations, i.e. assuming a slowly varying envelope of the modal amplitudes and weakly coupled fibre and slab modes, both the second-order derivatives and the field overlap integrals may be neglected[15] and the following first-order coupled-mode equations are derived

$$a_0' = -j(\beta_{f00}+Q_{f00})a_0 - j\sum_n K_{f0n}\, b_n \qquad \text{(A7.a)}$$

$$b_m' = -j\sum_n (\delta_{mn}\, \beta_{sn}+Q_{smn})b_n - jK_{sm0}a_0 \qquad \text{(A7.b)}$$

where $a_0' = \dfrac{da_0}{dz}$, $b_m' = \dfrac{db_m}{dz}$, $m, n = 0, 1, 2, \cdots$, δ_{mn} is the Kronecker delta function, $\{Q_{f00}, Q_{smn}\}$ and $\{K_{f0n}, K_{sm0}\}$ are the self- and cross-coupling coefficients, respectively. They are defined as follows

$$Q_{f00} = \frac{k^2}{2\beta_{f0}}\int_{A_\infty}[n^2(x,y) - n_f^2(x,y)]F_0F_0dA \qquad \text{(A8.a)}$$

$$Q_{smn} = \frac{k^2}{2\beta_{sm}}\int_{A_\infty}[n^2(x,y) - n_s^2(x,y)]S_mS_ndA \qquad \text{(A8.b)}$$

$$K_{f0n} = \frac{k^2}{2\beta_{f0}}\int_{A_\infty}[n^2(x,y) - n_s^2(x,y)]F_0S_ndA \qquad \text{(A.9a)}$$

$$K_{sm0} = \frac{k^2}{2\beta_{sm}}\int_{A_\infty}[n^2(x,y) - n_f^2(x,y)]S_mF_0dA \qquad \text{(A9b)}$$

where $m, n = 0, 1, 2, \cdots$, $dA = dxdy$ and A_∞ indicates integration over the infinite cross-section, i.e. the entire transverse (x, y) plane, of the coupler system. From the refractive index profiles the coupling coefficients defined above can be categorised in two ways. $\{Q_{f00}, Q_{smn}\}$ are self coupling coefficients that represent the coupling among the fibre or slab modes due to the presence of the other, $\{K_{f0n}, K_{sm0}\}$ are cross coupling coefficients that couple the slab modes to the fibre mode or vice versa.

The above integrals are all solved exactly except Q_{f00}, for which a large-argument asymptotic approximation of the modified Bessel function K_0 has to be used. The closed-form expressions of the coupling coefficients are given below

$$Q_{f00} = \frac{\pi^2 N_f^2 V_{sc}^2}{\sqrt{2}\beta_{f0}\gamma_f^2 t^2 \mathrm{K}_0^{\,2}(\gamma_f a)}[\mathrm{erf}(\sqrt{2\gamma_f(a+s+t)}) - \mathrm{erf}(\sqrt{2\gamma_f(a+s)}) + A_s \mathrm{erfc}(\sqrt{2\gamma_f(a+s+t)})] \tag{A.10a}$$

$$Q_{smn} = \frac{\pi N_s^2 k_s^2 V_f^2 t^2 \mathrm{e}^{-2\gamma_c(a+s)}}{8\beta_{sm}V_{sc}^2}[\frac{\mathrm{I}_1(\sqrt{4\gamma_c^2 - (\sigma_m+\sigma_n)^2}\,a)}{\sqrt{4\gamma_c^2 - (\sigma_m+\sigma_n)^2}\,a} + \frac{\mathrm{I}_1(\sqrt{4\gamma_c^2 - (\sigma_m-\sigma_n)^2}\,a)}{\sqrt{4\gamma_c^2 - (\sigma_m-\sigma_n)^2}\,a}] \tag{A.10b}$$

$$K_{f0n} = \frac{\pi N_f N_s V_f^2 k_s t \mathrm{e}^{-\gamma_c(a+s)}}{2\beta_{f0} a V_{sc} \mathrm{J}_0(k_f a)(k_f^2 + \gamma_c^2 - \sigma_n^2)}[\sqrt{\gamma_c^2 - \sigma_n^2}\,\mathrm{J}_0(k_f a)\mathrm{I}_1(\sqrt{\gamma_c^2 - \sigma_n^2}\,a) + k_f \mathrm{J}_1(k_f a)\mathrm{I}_0(\sqrt{\gamma_c^2 - \sigma_n^2}\,a)] \tag{A.11a}$$

$$K_{sn0} = \frac{\pi N_f N_s V_{sc} k_s \mathrm{e}^{-\sqrt{\gamma_f^2+\sigma_n^2}(a+s)}}{\beta_{sn}\mathrm{K}_0(\gamma_f a) t \sqrt{\gamma_f^2 + \sigma_n^2}} \left\{ \frac{\sqrt{\gamma_f^2 + \sigma_n^2} + \gamma_c - B_s(\sqrt{\gamma_f^2 + \sigma_n^2} - \gamma_o)\mathrm{e}^{-\sqrt{\gamma_f^2+\sigma_n^2}\,t}}{k_s^2 + \gamma_f^2 + \sigma_n^2} + \frac{A_s B_s \mathrm{e}^{-\sqrt{\gamma_f^2+\sigma_n^2}\,t}}{\gamma_o + \sqrt{\gamma_f^2 + \sigma_n^2}} \right\} \tag{A.11b}$$

where $V_f = ka\sqrt{n_f^2 - n_c^2}$, $V_{sc} = \frac{kt}{2}\sqrt{n_s^2 - n_c^2}$, $V_{so} = \frac{kt}{2}\sqrt{n_s^2 - n_o^2}$, $A_s = \frac{n_o^2 - n_c^2}{n_s^2 - n_c^2}$, $B_s = \frac{V_{sc}}{V_{so}}$,

$B_s = \frac{V_{sc}}{V_{so}}$, erf($x$) is the error function, erfc(x) = 1 - erf(x), I_0 and I_1 are the modified Bessel functions of the first kind. In the above expressions, the new parameters A_s and B_s are introduced to quantify the geometric asymmetry of index profile of the slab waveguide. For the case involving a symmetric slab waveguide, we have $A_s = 0$ and $B_s = 1$.