Department of Electrical
and
Computer Systems Engineering

Technical Report
MECSE-20-2004

PHOTONIC TRANSMISSION MODELS FOR SYNCHROTRON-A
DEEP-ETCHED PHOTONIC FILTER STRUCTURES

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Abstract

Thin film multiplayer optical filters have been developed for several years. Periodic photonic structures with “brick-wall” like features can be fabricated with thick polymeric films and deep etching employing synchrotron radiation. These photonic devices need new design approaches.

This report describes a transmission representation of dielectric stratified media including the “optical” impedance and techniques for matching optical loads, hence transmission line analogue of optical waves propagation. The design of optical filters is outlined. The effects of the film thickness and refractive index on the equivalent optical impedance are given.

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1 Introduction

Electromagnetic waves play a major role in carrying the signals that contain the information. There are transverse electric and magnetic fields oscillating at very high optical frequencies of the order of 100 THz associated with the wave. It is desirable to increase transmission capacity in optical fiber communication by employing multi-carrier multiplexing in the optical domain, the wavelength division multiplexing (WDM). One of the most crucial photonic elements for DWDM systems is the optical filter.

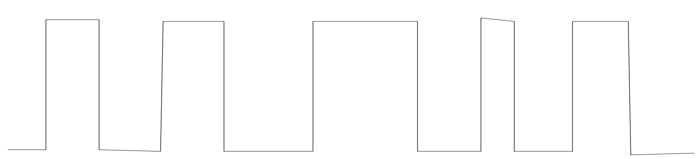
Optical filters with narrow pass-bands and high efficiency can now be precisely fabricated. Narrow band-pass optical filters play a major role in the elimination of noise in fiber amplifier, selecting certain wavelength channel. With narrow band-pass optical filter precise frequency discrimination will provide higher channel capacity. This offers tremendous improvements in the performance of a number of optical, infrared, and photonic systems. These filters can be designed to have center wavelength in the C-L- or S-bands of the 1550 nm window.

There are basically two common types of optical filters, dielectric multi-layer filter (DMF) and diffraction grating and fibre Bragg gratings (FBG)[1]. This document focuses on dielectric multi-layer filter (DMF), the theoretical foundation, and the design of a band-pass filters using dielectric multi-layers.

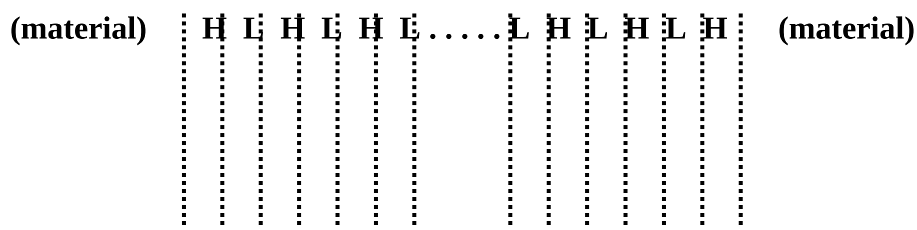
2 Fundamental of Optical Wave Propagation Properties

2.1 Structure of a DMF

Dielectric multi-layer (DMF) filters can be fabricated by cascading stratified dielectric layers of high (H) and low (L) RI materials, either isotropic or anisotropic. Materials applicable for fabrication using synchrotron radiation, polymeric materials are usually employed. Thus isotropic materials are considered in this work.



(a)



(b)

Figure 1 Structure of a DMF (a) refractive index distribution of high and low regions. (b) boundaries of stratified regions.

A typical frequency response of the filter is shown in Figure 2 which shows a center frequency, a bandwidth and a stop-band.

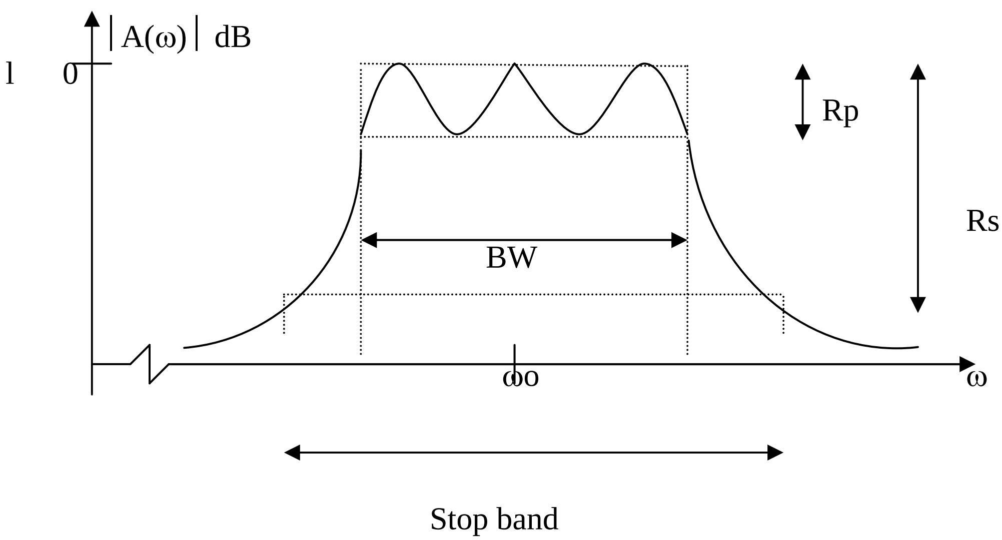


Figure 2 Frequency response of a band-pass filter

The layers are paired in high-low (HL) RI layers. The total optical thickness of these pairs is half wavelength thick of the center wavelength (frequency) and most of these pairs have high (H) and low (L) refractive index (RI) layers that are quarter-wavelength thick (optical thickness). There are also layers of variable optical thickness. Several HL pairs are stacked together to make up a reflector of higher reflection coefficient than a single pair or multiple-cavity resonant structure. This reflector is cascaded with another reflector of equal reflection coefficient (consisting of the same number of pairs) to form a generic ***resonator***. Hence, this type of resonator is symmetric. The frequency response of a resonator follows that of a band-pass filter as shown in Figure 1. A number of these resonators can be cascaded with a quarter wave coupling to form a BPF [1].

If the material is optically isotropic then this velocity is independent of the direction of travel. For homogeneous dielectric films ε, μ and the R [$n = (\varepsilon\mu)^{1/2}$] are constants. For non-magnetic dielectric material we have $\mu = 1$ and its intrinsic impedance $\eta = (\mu/\varepsilon)^{1/2}$, the RI relates to the intrinsic optical impedance by

$$\eta = 1/n \tag{1}$$

2.2 Optical thickness

It is common to describe an optical wave by its free space wavelength, which is its wavelength in vacuum, and the simple relationship between optical frequency f and free space wavelength λ_0 is:

$$c = f\lambda_0 \tag{2}$$

in the medium of a RI n the wavelength in such medium, the effective wavelength λ_{eff} is

$$\frac{c}{n} = f\lambda_{eff} \, . \tag{3}$$

Thus we have:

$$\frac{\lambda_0}{n} = \lambda_{eff} \tag{4}$$

Assuming that medium is not an optical waveguide then we could denote the physical thickness of a dielectric material by *h and* then *nh* is the ***effective optical thickness*** of the material. So nh/λ_0 gives the number of complete cycles of the optical electric field occurring in the medium. The effective *optical thickness* is a very important design factor of optical filter as it relates to the *quarter-wave* and *half-wave* section of the dielectric transmission lines. A dielectric layer with quarter wavelength optical thickness is analogous to a quarter wave section of a transmission line and so on.

2.3 Transmission and reflection

Optical wave propagation involves three kinds of phenomena: *transmission, reflection and diffraction.* The effect of diffraction can be ignored if the optical system has infinite aperture that can be assumed to be valid for the "brick-wall" gratings in thick polymeric films. This condition is usually meet because the operational free-space wavelength is about 1-2 μm. The most recent developments in thin-film deposition and in synchrotron-assisted radiation of deep etched polymeric materials permit the fabrication of surfaces with well-known transmission coefficients and with extremely low losses resulting from absorption and scattering.

Assuming a monochromatic uniform plane wave whose electric and magnetic fields are normal to the direction of propagation and of equal amplitude and phase in the transverse plane. When this plane wave falls normally on to a boundary between two homogeneous media of different optical properties, a transmitted wave proceeds into the second medium and a reflected wave is propagated back into the first medium as shown in *Figure 3*.

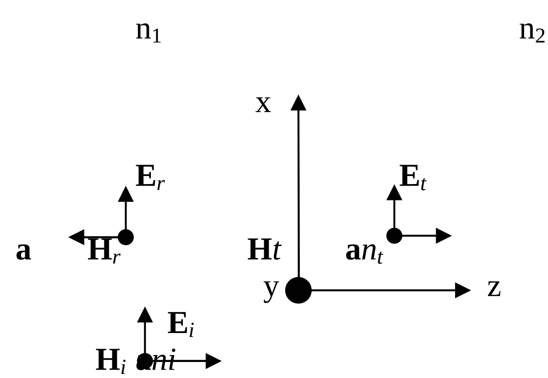


Figure 3 Plane wave incident normally on a plane boundary of two homogeneous media, incident, reflected and transmitted electric and magnetic field intensity phasors.

The existence of these two waves can be demonstrated from the boundary conditions. It is assumed that these waves are also plane waves.

2.4 Transmission line and dielectric layers

For the case that the wave is incident on multiple dielectric layers stratified between two materials of different constitutive parameters an analogy can be made between the dielectric layers and transmission lines. Each layer of the DMF is considered as a segment of transmission lines. The theory of wave characteristics on finite transmission lines is applied in DMF structures. The Helmholtz equations for transmission lines (both infinite and finite) are given as:

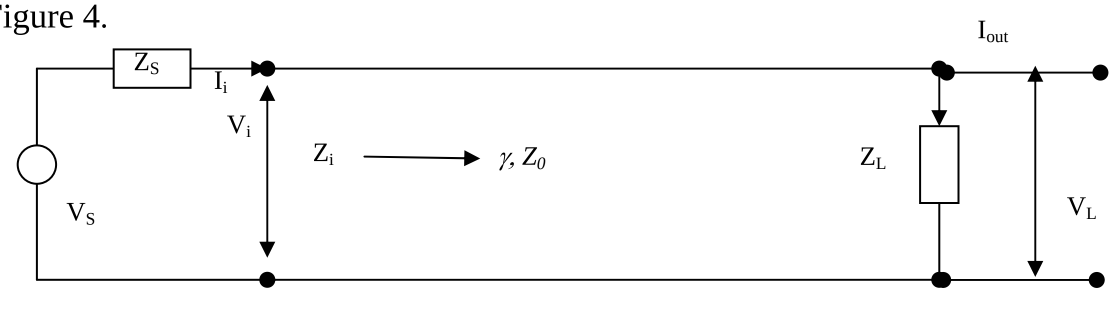
$$\begin{aligned} V(z) &= V_0^{+} e^{-\gamma z} + V_0^{-} e^{\gamma z} \\ I(z) &= I_0^{+} e^{-\gamma z} + I_0^{-} e^{\gamma z} \end{aligned} \tag{5}$$

where forward and backward traveling waves are used to indicate both the voltage and the current. $\gamma = \alpha + j\beta$ is the propagation constant whose real and imaginary parts, α *and* β are the attenuation constant (Np/m) and phase constant (rad/m) of the line, respectively.

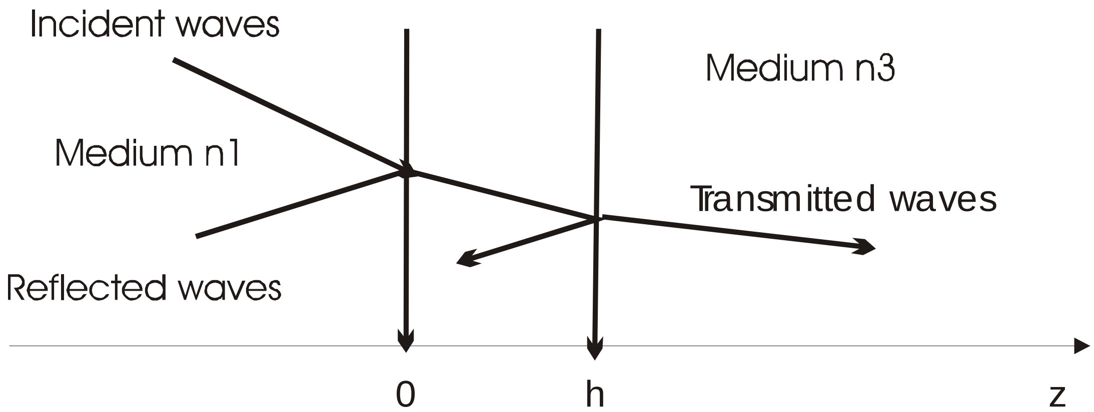
The characteristic impedance Z_0 of the transmission line relates the voltage and the current by:

$$\frac{V_0^{+}}{I_0^{+}} = -\frac{V_0^{-}}{I_0^{-}} = Z_0 \tag{6}$$

For optical filters, the multiple dielectric layers that are stratified in between two layers of different refractive indices, represent the transmission lines. *The intrinsic impedance of the first incident region is analogous to the internal source impedance*. *The intrinsic impedance of the final transmitting region is analogous to the load impedance*. The forward traveling wave is the incident wave that falls onto the DMF and the reverse traveling wave is the reflected wave. The transmitted wave travels into the third region represents the output voltage. The analogy between transmission lines and DMF is shown in Figure 4.



(a) Analogous transmission line model



(b) optical transmission model

Figure 4 Analogy between transmission line wave theory and stratified dielectric layers: *Dielectric layers stratified between two optical media; incident, reflected and transmitted waves.*

The optical filter is assumed to be lossless, hence the propagation constant is purely imaginary. Also, Z_0 represents the characteristic impedance of the lines, i.e. intrinsic impedance of the dielectric layers.

2.5 Total input optical impedance

Denote the amplitudes of the incident, reflected and transmitted waves by *A, R and T*, respectively which can be analogous to the voltage and current at any point along a transmission line in terms of the load current, I_L, load impedance, $Z_{L,}$ the propagation constant, γ, and the characteristic impedance, Z_0. This relationship is presented in the case of a dielectric layer in a matrix form

$$\begin{bmatrix} U(0) \\ V(0) \end{bmatrix} = \begin{bmatrix} \cos(k_o n_2 h/\cos\theta) & jp\sin(k_o n_2 h/\cos\theta) \\ jp\sin(k_o n_2 h/\cos\theta) & \cos(k_o n_2 h/\cos\theta) \end{bmatrix} \begin{bmatrix} U(h) \\ V(h) \end{bmatrix} \tag{7}$$

where $$p = \sqrt{\frac{\varepsilon}{\mu}}\cos\theta \tag{8}$$

$k_o = 2\pi/\lambda$ is the wave number in vacuum, n_2 is the RI of the dielectric layer, h = the physical thickness of the dielectric layer and θ denotes the angle of the normal to the wave with respect to the z-axis. If normal incidence is assumed then $\theta = 0$. We can replace all *p* with corresponding RIes, *n*.

The incident and the reflected waves are incorporated into the input vectors by:

$$\begin{aligned} U(0) &= A + R, \\ V(0) &= p_1(A - R), \end{aligned} \tag{9}$$

and the transmitted wave with respect to the output vectors:

$$\left.\begin{aligned} U(h_1) &= T, \\ V(h_1) &= p_3 T, \end{aligned}\right\} \tag{10}$$

where $p_1 = \sqrt{\dfrac{\varepsilon_1}{\mu_1}}\cos\theta_1$ and $p_3 = \sqrt{\dfrac{\varepsilon_3}{\mu_3}}\cos\theta_3$. The above matrix relates the input and the output waves of a dielectric layer. This is the ***characteristic matrix***. Thus (9) can be rewritten as:

$$\left.\begin{aligned} U(0) &= A + R = (\cos\beta_2 h + j\frac{n_3}{n_2}\sin\beta_2 h)T, \\ V(0) &= n_1(A - R) = (jn_2\sin\beta_2 h + n_3\cos\beta_2 h)T, \end{aligned}\right\} \tag{11}$$

with $\beta_2 = k_0 n_2$ is the propagation constant of the traveling waves for non-guided medium In case that the transmission medium is n optical waveguide then $\beta_2 = k_0 n_{eff}$ with n_{eff} is the effective refractive index of the waves as seen in the propagation direction. The ratio *U(0)/V(0)* can be compared with the characteristic impedance of the transmission lines:

$$Z_0 = \frac{U(0)}{V(0)} = \frac{1}{n_2}\frac{n_2\cos\beta_2 h + jn_3\sin\beta_2 h}{n_3\cos\beta_2 h + jn_2\sin\beta_2 h} = \frac{1}{n_2}\frac{n_2 + jn_3\tan\beta_2 h}{n_3 + jn_2\tan\beta_2 h} \tag{12}$$

This represents the ***total input impedance*** of the dielectric layer.

Using the transmission line theory, a line is matched or maximum power transfer when the load impedance equates the characteristic impedance of the line. Here, when the total input impedance seen from the first medium layer of discontinuity matches with the load impedance then a maximum transfer of power from *a given voltage source to a load will occur*. Here, the first surface of discontinuity is the left-hand interface in Figure 4. In our case the load impedance is the reciprocal of the RI of the last medium. The maximum power is referred to the case when the lightwaves energy is completely transmitted and none reflected.

The voltage and the total input impedance are a function of frequency. For a certain frequency the total input impedance matches with the load impedance and we let this frequency as the center frequency of the optical filters. We can then normalize the total input impedance as

$$\frac{Z_O}{Z_L} = Z_O \times n_3 \tag{13}$$

Hence, we have to multiply $U(0)/V(0)$ by n_3. Therefore, we can write *the normalized total input impedance z*, in term of its real and imaginary parts as follows:

$$z = \frac{1}{n_2} \frac{n_2 n_3 + n_2 n_3 \tan^2 \beta_2 h}{n_3^2 + n_2^2 \tan^2 \beta_2 h} + j \frac{1}{n_2} \frac{(n_3^2 - n_2^2) \tan \beta_2 h}{n_3^2 + n_2^2 \tan^2 \beta_2 h} \tag{14}$$

NOTE: The real part is the resistive part and the imaginary part is the reactive part of the total input impedance.

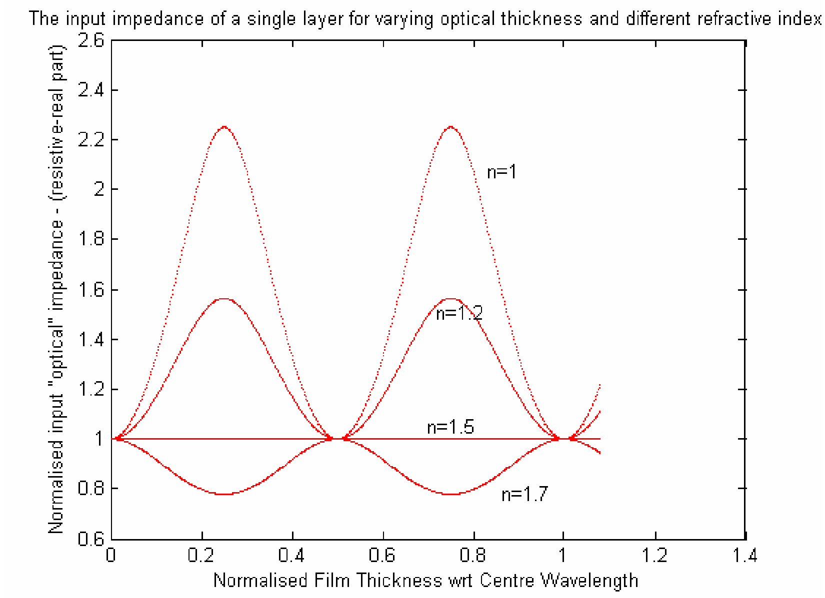
When n_2 and n_3 are invariant, the total input impedance is a function of βh, which in turn is a function of frequency, ω; $\tan\beta_2 h$ can be in the range $-\infty$ *to* $+\infty$. The reactive part of (14) which exhibits inductive or capacitive characteristic depending on n_2, n_3 *and* $\tan\beta_2 h$. We can plot the normalized input impedance of a single layer varying different parameters, and demonstrate their important features.

2.6 "Optical" impedance and optical thickness of a dielectric stratified layer

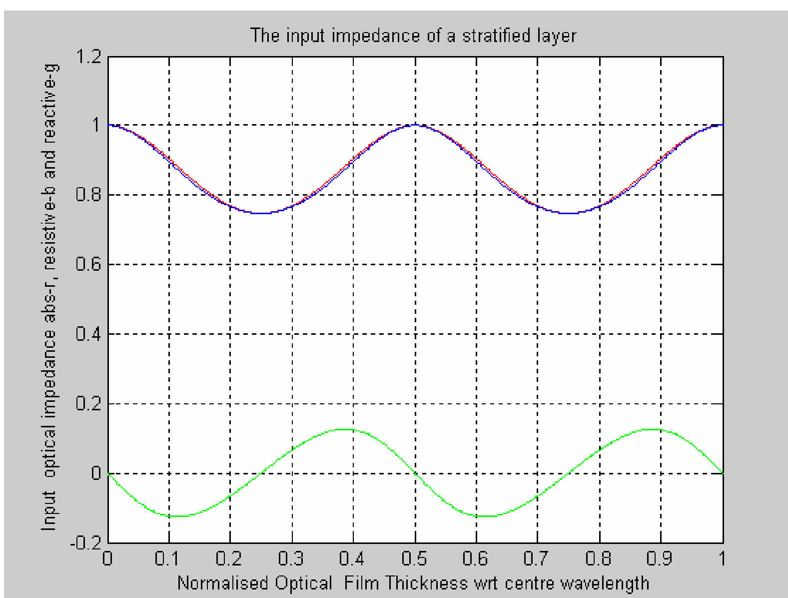
In this section we present the optical and characteristic "optical" impedance of a single dielectric layer sandwiched between two media of different RIes. These two media can be a substrate of glass and so on. Here, the two media are taken to be air/glass, the first media, RI n = 1 or 1.5, and glass the last or output medium with a RI n = 1.5.

A MATLAB program for simulation of a single layer is executed for different parameters, such as the optical thickness, RI and so on. The normalized total input impedance for varying optical thickness and different RI are plotted in ***Figure 5***.

Note that all curves are periodic with the optical thickness and the maxima and minima occur at *zero*, *0.25*, *0.5*, *0.75* and *1*. The optical thickness is normalized to the operating wavelength λ.. Hence, at odd multiples of quarter wavelength maxima or minima occur depending on the RI and at half-wavelength all the values are 1.



(a)



(b)

Figure 5 (a) variation of the optical impedance with the RI of the stratified layer (b) The magnitude(red), resistive(blue) and reactive(green) parts of the optical impedance with n= 1.7 stratified between input and output layers of refractive indices of n_1=1 (air) and n_3=1.5 (glass)

We recall from the transmission line theory that

"*a quarter-wave lossless line transforms the load impedance to the input terminals as its inverse multiplied by the square of the characteristic resistance*" [1]

and given as:

$$Z_i = \frac{R_o^2}{Z_L} \tag{15}$$

where Z_i is the input impedance, R_0 is the characteristic impedance/resistance, Z_L is the load impedance. Applying this to the dielectric layer case we have

$$Z_i = \frac{n_3}{n_2^2} \tag{16}$$

where $1/n_2$ is considered as the characteristic impedance of the transmission line, and $1/n_3$ is the intrinsic impedance of the third medium (output layer) which can be considered as the load impedance.

Again,

"a half wave lossless line transfer the load impedance to the input terminals without change" [3] :

$$Z_i = Z_L \tag{17}$$

From *Figure 5* we observe that at half-wave thickness the optical impedance take the value 1 (normalized by the load impedance).

RI	*Normalized total input impedance* *at odd multiples of quarter wave*	*at odd multiples of half wave*
1.0	2.25000000000000	1.000000
1.2	1.56250000000000	1.000000
1.4	1.14795918367347	1.000000
1.5	1.00000000000000	1.000000

1.7	0.77854671280277	1.000000
2.0	0.56250000000000	1.000000
3.0	0.25000000000000	1.000000

Table 1 Values of normalized optical impedance

The imaginary (reactive) and resistive parts of input optical impedance are also shown in *Figure 6*. It is observed that except for the case that the RI is 1.5 (no stratification) all others have their imaginary components and exhibit both capacitive and inductive reactance of the same amount within the range of a half-wavelength, that is for a normalized propagation constant β *varying from 0 to* π. We note the followings:

- For a RI less than n_3, the reactance exhibits inductive (variable) nature for the first quarter wavelength and capacitive (variable) nature for the second quarter wavelength and so on.
- It is in the opposite when RI is higher than n_3.
- When n_2=n_3 the imaginary term vanishes.

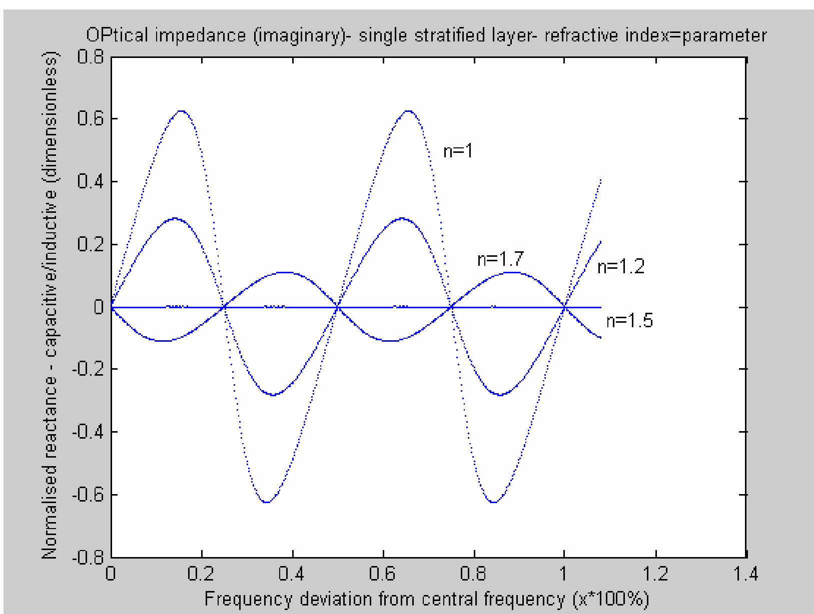


Figure 6 Reactive part of the input optical impedance with the RI of the stratified layer as a parameter.

Therefore, we can state that

"for an operating optical frequency the total input impedance of a dielectric layer act as a resistor in series with an inductor or a capacitor depending on the optical thickness of the layer"

We thus can represent the equivalent circuit of a dielectric layer as a coupling between a capacitor and an inductor as shown in ***Figure 7***:

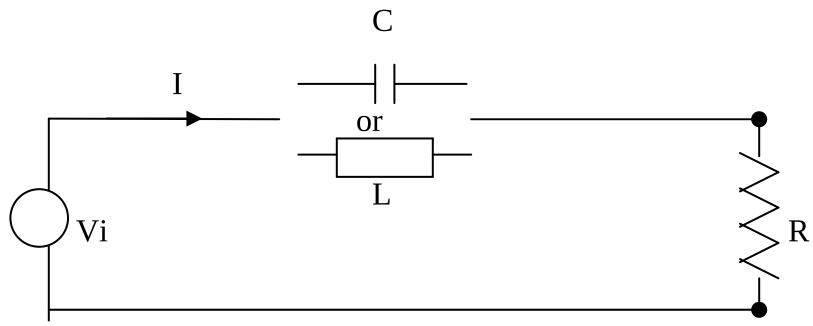


Figure 7 Equivalent series R L/C circuit representation of a dielectric layer.

The impedance is a variable of optical thickness. In our next chapter we will discuss about multiple numbers of layers and observe their important features.

2.7 Reflectivity, Transmittance/transmittivity and Attenuation

From (12) we can derive the reflection and transmission coefficient of the dielectric film given as:

.

$$r = \frac{R}{A} = \frac{(\cos\beta_2 h + j\dfrac{n_3}{n_2}\sin\beta_2 h)n_1 - (jn_2\sin\beta_2 h + n_3\cos\beta_2 h)}{(\cos\beta_2 h + j\dfrac{n_3}{n_2}\sin\beta_2 h)n_1 + (jn_2\sin\beta_2 h + n_3\cos\beta_2 h)},$$
$$t = \frac{T}{A} = \frac{2n_1}{(\cos\beta_2 h + j\dfrac{n_3}{n_2}\sin\beta_2 h)n_1 + (jn_2\sin\beta_2 h + n_3\cos\beta_2 h)} \qquad (18)$$

where r is defined as the ratio of the complex amplitudes of the reflected and incident "voltage" lightwaves, and, *t* is the ratio of the transmitted and the incident "voltage" lightwaves. In terms of *r and t,* the reflectivity or reflectance, R and transmissivity or transmittance, T can be defined as:

$$R = |r|^2$$

$$T = \frac{p_3}{p_1}|t| \qquad (19)$$

The phase of *r* is the phase change on reflection that happens at the first surface of discontinuity. The phase of *t* is the phase change on transmission that is referred to the plane boundary between the stratified medium and the last semi-infinite medium.

Therefore, the *power loss ratio or attenuation***,** *P_{LR} is obtained as:*

$$P_{LR} = \frac{Power\_available\_from\_source}{Power\_delivered\_to\_load} = \frac{P_{inc}}{P_{trans}} = \frac{1}{\frac{n_3}{n_1}|t|^2} \qquad (20)$$

Here, n_3 and n_1 are the reciprocals of the intrinsic impedance of the second last and the last or final media. The power loss ratio is important for identification of the filter response. *Figure 8* shows the attenuation of two typical filter responses of filters with equi-ripple and maximally flat responses:

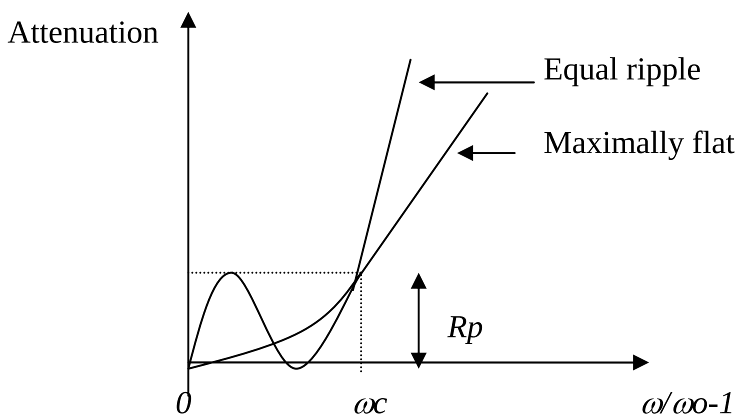


Figure 8 Maximally flat and equi-ripple filter responses; R_p is the ripple of the pass-band, ω_c = 3dB bandwidth, ω_o = center frequency.

2.8 Network equations and physical interpretation of the optical characteristic matrix

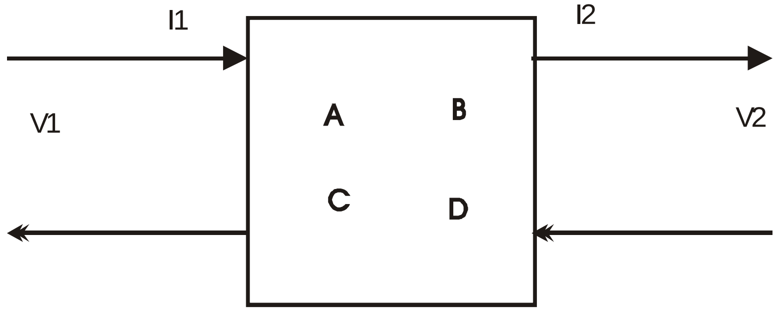


Figure 9 A four-terminal [ABCD] network

A four terminal network is shown in *Figure 9*. The relations between "voltages and currents" can be shown through three different network equations

$$\begin{aligned} V_1 &= AV_2 + BI_2 \\ I_1 &= CV_2 + DI_2 \end{aligned} \tag{21}$$

Note that the input vector is used as the resultant of the output vector and the transmission matrix. This form of cascading, normally termed as lattice structure, is advantageous when cascading several networks such as in the case of stacking several cascaded/stacked thin film layers of stratified dielectric regions. Alternatively the impedance and admittance can be used to represent the equivalence as:

- For impedance two port network:

$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned} \tag{22}$$

- For admittance two-port networks:

$$\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned} \tag{23}$$

Two port networks that give the same impedance from both terminals are the T and π-model are shown in *Figure 10*.

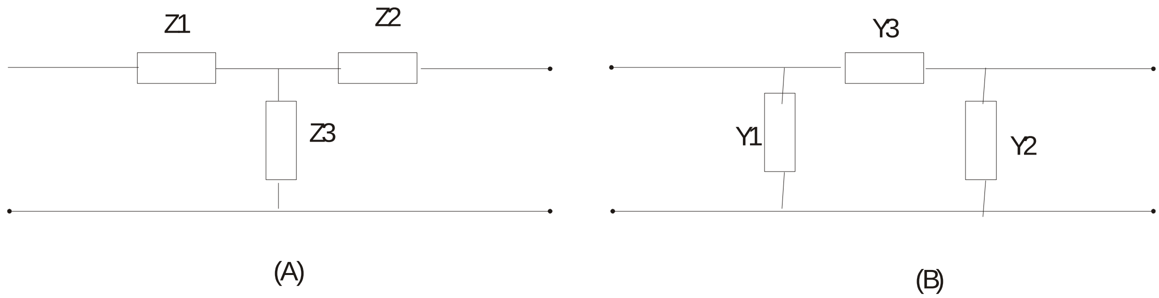


Figure 10 *Equivalent circuit representation for two-port networks. (a) T Z-equivalent. (b) π Y- equivalent*

We can thus equate above components in a circuit to the chain, impedance and admittance two-port networks the matrix components can be obtained (see Appendix 2) as

$$A = \frac{Z_{11}}{Z_{12}} = \frac{Y_{22}}{Y_{12}} \qquad B = \frac{1}{Y_{12}} \qquad C = \frac{1}{Z_{12}} \qquad D = \frac{Z_{22}}{Z_{12}} = \frac{Y_{11}}{Y_{12}} \tag{24}$$

$$\begin{aligned} Z_1 &= Z_{11} - Z_{12} \\ Z_2 &= Z_{22} - Z_{12} \\ Z_3 &= Z_{12} \end{aligned} \tag{25}$$

and
$$\begin{aligned} Y_1 &= Y_{11} - Y_{12} \\ Y_2 &= Y_{22} - Y_{12} \\ Y_3 &= Y_{12} \end{aligned} \tag{26}$$

Therefore the overall characteristic matrix of a resonator can be approximated at its center frequency as:

$$\begin{bmatrix} \dfrac{1}{j\pi x R^N\left(\left(n_a + n_b\right)\left(\sum_{k=N}^{1} r^k R^{N-k+1}\right) + nIR^N\right)} & \begin{matrix} j\pi x r^N\left(\left(\dfrac{1}{n_a} + \dfrac{1}{n_b}\right)\left(\sum_{k=N}^{1} r^k R^{N-k+1}\right) + \dfrac{1}{n} I r^N\right) \\ 1 \end{matrix} \end{bmatrix} \tag{27}$$

where $r = n_a/n_b$, $R = n_b/n_a$ depending upon the orientation; $I = 2, 4, \ldots$ the number of cavity layers; N is half the number of high-low (HL) pairs in a resonator. Therefore, using (24) we obtain:

$$Z_{11} = Z_{22} = \frac{A}{C} = \frac{-j}{\pi x R^N \left((n_a + n_b) \left(\sum_{k=N}^{1} r^k R^{N-k+1} \right) + nIR^N \right)} \qquad (28)$$

For two port Z-equivalent network, the T matrix components are:

$$Z_{12} = \frac{1}{C} = \frac{-j}{\pi x R^N \left((n_a + n_b) \left(\sum_{k=N}^{1} r^k R^{N-k+1} \right) + nIR^N \right)} \qquad (29)$$

$$\begin{aligned} Z_1 &= Z_{11} - Z_{12} = 0 \\ Z_2 &= Z_{22} - Z_{12} = 0 \\ Z_3 &= Z_{12} = \frac{1}{C} = \frac{-j}{\pi x R^N \left((n_a + n_b) \left(\sum_{k=N}^{1} r^k R^{N-k+1} \right) + nIR^N \right)} \end{aligned} \qquad (30)$$

- For optical frequencies lower than the center frequency ($\omega<\omega_0$) the above acts as an inductive element;
- for frequencies higher than center frequency ($\omega>\omega_0$) it acts a capacitive circuit and at center frequency it is infinity i.e. equivalent to an open "optical" circuit. The equivalent circuit is given *in Figure 11*.

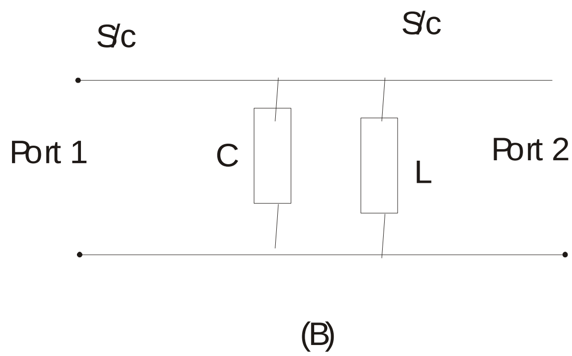


Figure 11 A Z- equivalent T-network of a dielectric layer resonator; at resonant (center) frequency the input impedance is infinite i.e. open circuit and voltage gain from port 1 to port 2 is unity.

Similarly for *Y*- equivalent π–network we have

$$Y_{11} = Y_{22} = \frac{D}{B} = \frac{-j}{\pi x r^N \left(\left(\frac{1}{n_a} + \frac{1}{n_b} \right) \left(\sum_{k=N}^{1} r^k R^{N-k+1} \right) + \frac{1}{n} I r^N \right)} \quad (31)$$

$$Y_{12} = \frac{1}{B} = \frac{-j}{\pi x r^N \left(\left(\frac{1}{n_a} + \frac{1}{n_b} \right) \left(\sum_{k=N}^{1} r^k R^{N-k+1} \right) + \frac{1}{n} I r^N \right)} \quad (32)$$

$$\begin{aligned} Y_1 &= Y_{11} - Y_{12} = 0 \\ Y_2 &= Y_{22} - Y_{12} = 0 \\ Y_3 &= Y_{12} = \frac{1}{B} = \frac{-j}{\pi x r^N \left(\left(\frac{1}{n_a} + \frac{1}{n_b} \right) \left(\sum_{k=N}^{1} r^k R^{N-k+1} \right) + \frac{1}{n} I r^N \right)} \end{aligned} \quad (33)$$

Therefore

- for frequencies less than center frequency the above acts as a capacitor;
- for frequencies higher than center frequency it acts an inductor and at center frequency it is infinity i.e. short circuit.

The equivalent π network is shown in *Figure 12*:

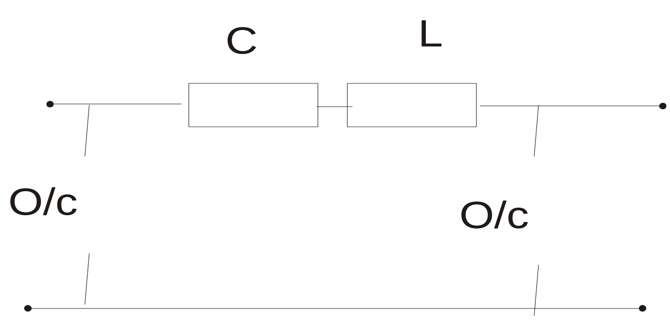


Figure 12 A -π equivalent circuit of a dielectric layer resonator; at resonant (center) frequency the input impedance is zero i.e. short circuit and voltage gain from port 1 to port 2 is unity.

Therefore, a dielectric multi-layer resonator has the characteristics of a parallel or series resonant circuit close to its center frequency. In the next section we show these properties by simulation of resonators. To apply above approximations the values of x must be very small for frequencies very close to the center frequency. Therefore, in our simulation of dielectric multilayer filters the frequency deviation is set to be very small with a maximum of about 10% of the center frequency.

3 Multiple layer optical resonators

Recalling that the total input "optical" impedance of a single dielectric layer can be resistive and either capacitive or inductive for different optical thickness and different RI. This section discusses the total input equivalent "optical" impedance of multiple dielectric layers. The design of a number of optical filters is described.

3.1 The characteristic matrix

In Section 2 the characteristic matrix is given for a single dielectric layer. Now let, $A = \cos\beta_2 h$, $B = \sin\beta_2 h/n_2$, $C = n_2 \sin\beta_2 h$, *and* $D = \cos\beta_2 h$. Hence if we denote U and V in terms of voltages and currents we have:

$$\begin{aligned} V_1 &= AV_2 + BI_2 \\ I_1 &= CV_2 + DI_2 \end{aligned} \tag{34}$$

It is noted that the input vector is presented as the resultant vector of the matrix and the output vector so as it is possible to construct the resultant matrix of cascaded layers with a product of chained matrices. The chain matrix is given by:

$$[M] = \begin{bmatrix} A & jB \\ jC & D \end{bmatrix} \tag{35}$$

whose determinant must obey the conservation of energy so that

$$|M| = AD - BC = \cos^2 \beta_2 h + \sin^2 \beta_2 h = 1 \tag{36}$$

For multiple dielectric layers the over resultant characteristic matrix is obtained by multiplying the characteristic matrices of each individual layer. The resulting matrix is thus a 2x2 matrix and can be presented as given in (35). Thence, the normalized total input impedance for multiple dielectric layers is given as:

$$\frac{V_1}{I_1} = n_3 \frac{AV_2 + jBI_2}{jCV_2 + DI_2} = \frac{A + jBn_3}{D + jC/n_3} = \frac{(AD + BC) + j(BDn_3 - AC/n_3)}{D^2 + (C/n_3)^2} \tag{37}$$

where $AD+BC = 1$ to satisfy the conservation of energy. For an operating center frequency we can select two layers of different materials of low (L) and high (H) RI. We can also select the optical thickness of each layer equal to a quarter wavelength corresponding to this center frequency. Multiple numbers of these layers can be cascaded alternatively.

At the center frequency the total input impedance as seen from the source end matches the load impedance, the intrinsic impedance of the last layer. Therefore the output power is the maximum at this frequency as compared to other frequencies, hence the band-pass optical frequency response.

The characteristic matrix of each layer at the center frequency can be expressed as:

$$[M(at\_center\_frequency)] = \begin{bmatrix} 0 & j/n \\ jn & 0 \end{bmatrix} \tag{38}$$

Then the resultant characteristic matrix of two cascaded layers (HL or LH) is obtained by multiplying the corresponding matrices [M] evaluated at the center frequency, thus we obtain:

$$[M_{HL}] = \begin{bmatrix} n_H/n_L & 0 \\ 0 & n_L/n_H \end{bmatrix} \tag{39}$$

3.2 Pairs of high H and low L RI layers

Consider two high and low RI dielectric layers can be stratified in cascade and sandwiched between two other media. Each of them has an optical equivalent thickness of a quarter wavelength, i.e.

$$n_L*h_{low} = n_H*h_{high} = 1/4\lambda_o. \tag{40}$$

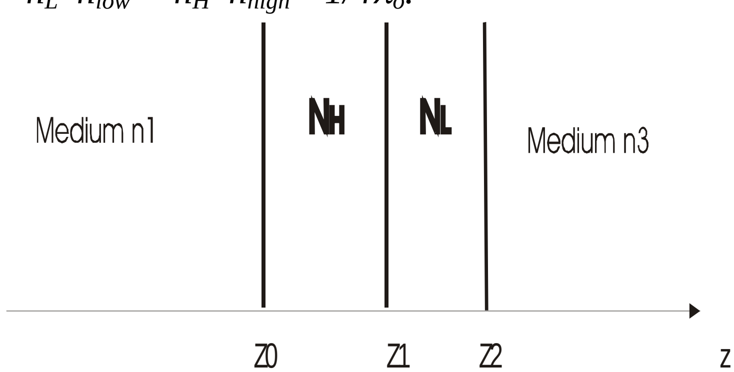


Figure 13 A pair of high and low RI layers; showing their RIes and physical thickness; each of optical thickness $\lambda_o/4$, direction of stratification is z, and the lightwaves propagating at normal incidence.

Using the *quarter wave lossless line property,*, the total input impedance seen at Z_1 plane is:

$$Z_i = n_2 / n_L^2 \tag{41}$$

Then again, at Z_0 plane it is:

$$Z_i = \frac{n_L^2}{n_H^2} \times \frac{1}{n_2} \tag{42}$$

Hence the normalized total input "optical" impedance of the pair is given by:

$$Z_{i\_normalized} = \left(\frac{n_L}{n_H} \right)^2 \tag{43}$$

This can also be achieved by using the cascaded chain matrix of (39). A MATLAB program for simulating a HL/LH pair by using their characteristic matrix is given in Appendix 1. To observe the frequency response, the frequency is varied from the center frequency as % frequency deviation. Its total input impedance is plotted as a function of frequency. The same procedure can be carried out for different HL/LH pairs. The results are tabulated in ***Table 2***.

No.	Low RI	High RI	Normalized total input impedance at the center frequency	
			HL configuration	LH configuration
1st	1.46	2.30	0.40294896030246	2.48170388440608
2nd	1.20	1.70	0.49826989619377	2.00694444444444
3rd	1.35	1.38	0.95699432892250	1.04493827160494

Table 2 Normalized total input impedance for different high-low pair configurations.

Table 2confirms the normalized total input impedance at the center frequency is the square of the ratio of two RIes depending on the orders of which they are placed. It is noted that the total input impedance for HL is the reciprocal for LH, as expected

Instead of a pair of HL/LH layers if we cascade a multiple numbers of these pairs we expect the normalized total input impedance seen at the source end at center frequency to be:

$$
\begin{aligned}
Z_i &= \left(\frac{n_L}{n_H}\right)^{2N} \quad for\_(HL)^N\_configuration \\
Z_i &= \left(\frac{n_H}{n_L}\right)^{2N} \quad for\_(LH)^N\_configuration
\end{aligned}
\tag{44}
$$

where N is the number of pairs. ***Table 3*** shows some the normalized input impedance at the evolution of the center frequency with respect to the number of cascaded resonators. In this case a 20 pairs of resonators are used.

No. of resonator pairs	Normalized total input impedance at center frequency HL configuration high n=2.30; low n=1.46
1	0.40294896030246
2	0.16236786460883
3	0.06542596223066
4	0.02636332345763
8	6.950248237317212e-004

Table 3 Normalized total input impedance for multiple number of high-low pairs, HL configuration.

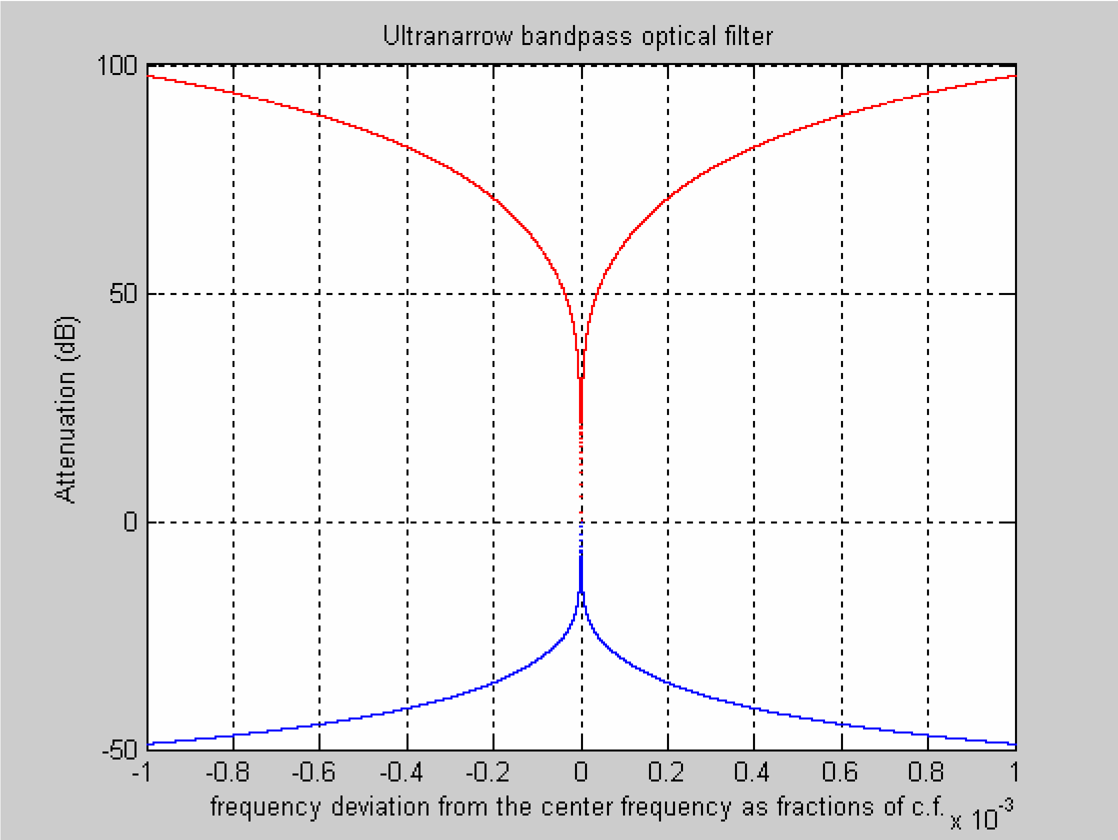


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Figure 14 Spectral response of an optical filter with 20 cascaded resonators of HL and LH layers (see Appendix for MATLAB program.)

4 Multiple-layer optical resonators

From Section 2 if we cascade a multiple number of HL pairs with the same number of LH pairs having identical properties, then at the center frequency the normalized total input impedance is:

$$\left(\frac{n_L}{n_H}\right)^{2N} \times \left(\frac{n_H}{n_L}\right)^{2N} = 1 \tag{45}$$

where N is the number of HL or LH pairs. The basic structure of a resonator is given as

N(HL)*2mL(or 2mH)*N(LH).

Here, N = 1, 2,… is the number of HL pairs; m = 1, 2,… number of $\lambda_0/4$ single layers, of either low or high RI. That means the structure is symmetric and has even multiples of single layers in the middle and high and low pairs at both ends. The reason for even multiple of single layer is in the middle is to satisfy the half-wave line theory. The characteristic matrix of a resonator at center frequency, with the above configuration is given as

$$\begin{bmatrix} (-\frac{n_L}{n_H})^N & 0 \\ 0 & (-\frac{n_H}{n_L})^N \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} (-\frac{n_H}{n_L})^N & 0 \\ 0 & (-\frac{n_L}{n_H})^N \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad (46)$$

Therefore, normalized total input impedance of a resonator according to (46) is thus unity. Hence, the total input impedance matches the load impedance at the center frequency. So, there would no reflection occurs at the center frequency. Thence a maximum transfer of power from a given "voltage" optical source to an optical load will occur. We present the total input impedance of the resonators of HL pairs used previously ***Figure 14***. High RI n = 2.30 and low RI n = 1.46. From ***Figure 15*** and ***Figure 16*** it is clear that at center frequency the normalized total input impedance is unity for all the resonators. It resembles the input impedance of a parallel resonator. ***Figure 17*** shows the attenuation, transmission and input optical impedance of such staked structure.

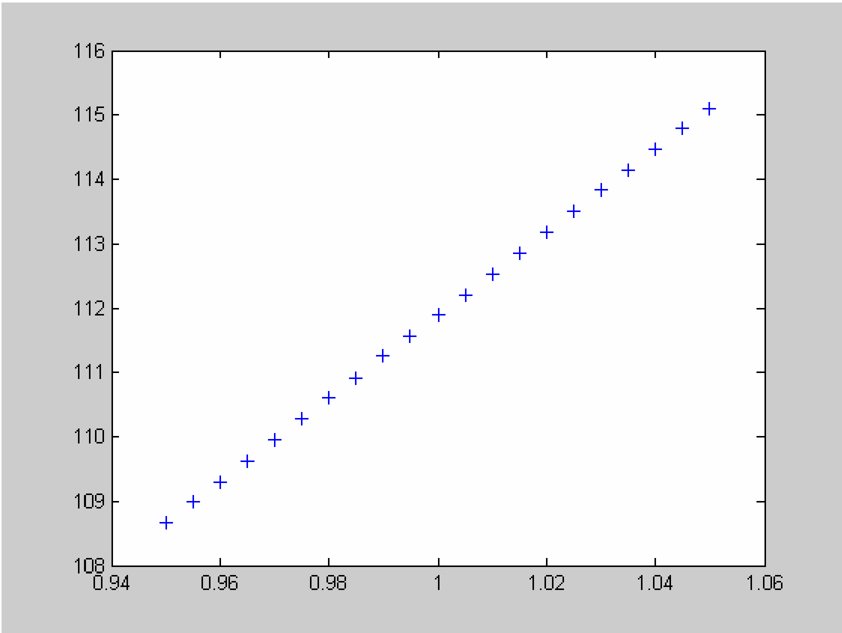


Figure 15 Loaded Q-factor as a function of the change of layer thickness

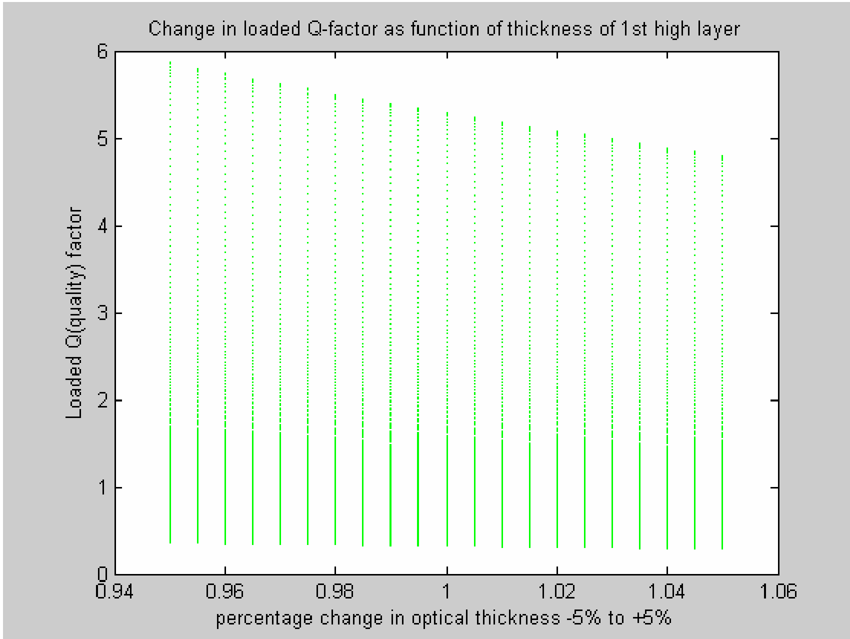


Figure 16 Change of the total input impedance as a function of change of the thickness of first input layer

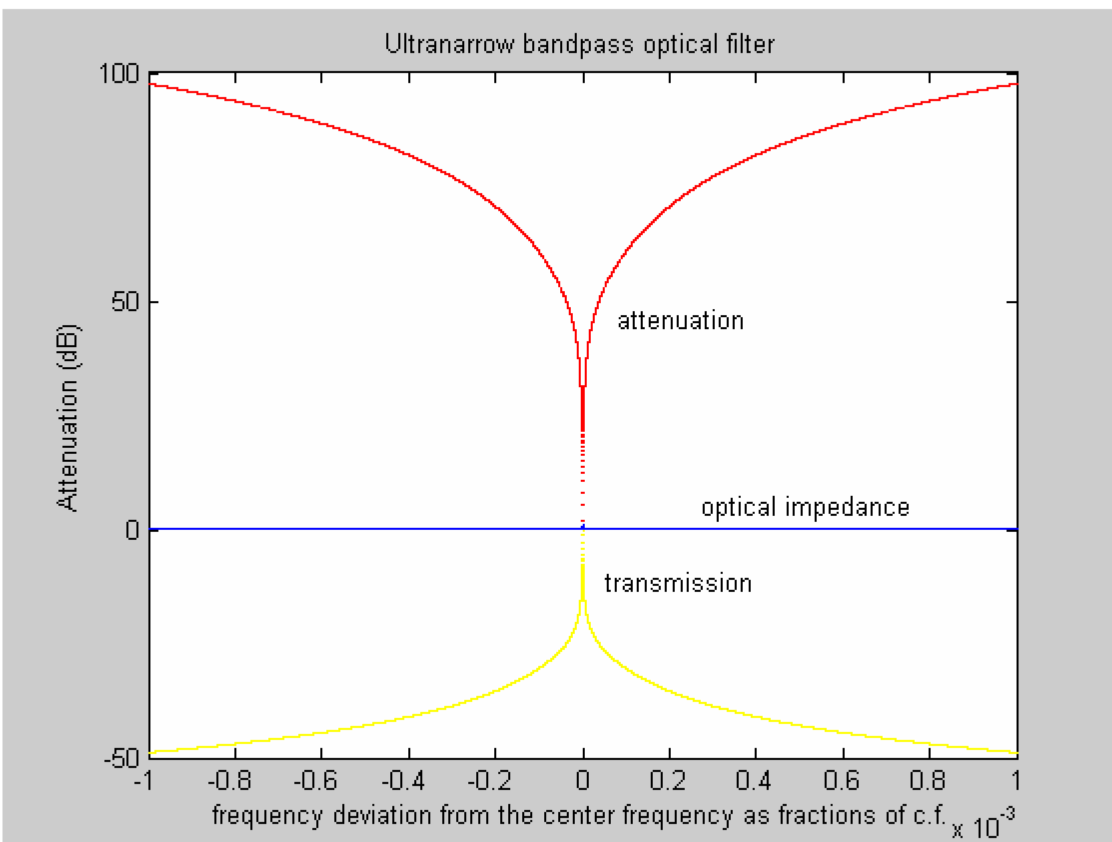


Figure 17 attenuation, transmission and optical input impedance of N of LH resonators.

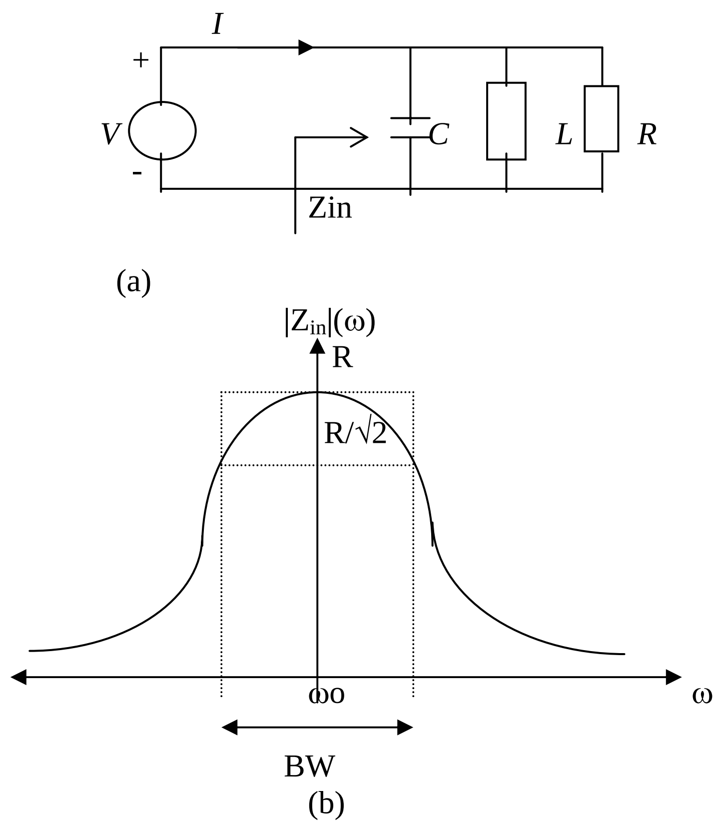
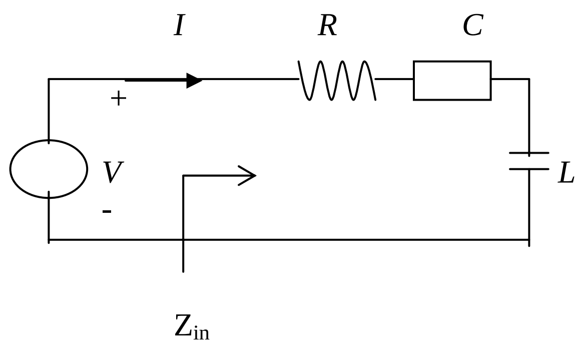


Figure 18 Circuit representation of dielectric multi-layer resonators; (a) A parallel RLC circuit, (b) The input impedance as a function of frequency, ωo = center frequency

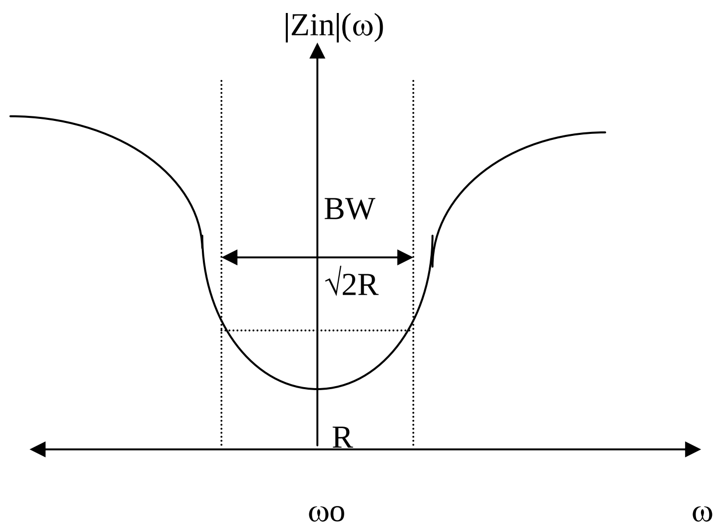
A parallel resonance circuit has an input impedance of:

$$\frac{1}{Z_{in}} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \qquad (47)$$

Therefore, the reciprocal of the input impedance is equal to the input impedance of a series circuit with the capacitor and the inductor values exchanged. For the case of dielectric layers inverting of total input impedance can be achieved by inserting a single layer of quarter wave optical thickness between the first medium and the resonator.



(a)



(b)

Figure 19 Circuit representation of dielectric layer resonators (a) A series RLC circuit, (b) The input impedance as a function of frequency, ω_o is the center frequency

4.1 Quality factor

Another important factor of a resonator is its quality factor Q defined as:

$$Q = \omega \frac{\text{average energy stored}}{\text{energy loss/second}} \tag{48}$$

$$\text{or } Q = \frac{1}{BW} \qquad Q = \frac{\omega_0}{2\Delta\omega} \tag{49}$$

Here, $\Delta\omega$ is the difference between the center frequency and the frequency such that *sqrt(* $|$*total input impedance* $|$*) = sqrt(R)/2,* then the average real power delivered to the circuit is one-half that is delivered at resonance. The Q-factor is an important factor in the design of optical filters as a measure of bandwidth of a resonator. In the following table we present the values of Q-factors calculated using MATLAB program of Appendix.

No. of pairs	Quality factor	Error
3	61.56856118614775	0.29546251086132
4	1.566789888712407e+002	0.43016667253121
8	6.035475540675679e+003	0.47256017136146

Table 4 Q-factor for resonators N(HL)N(LH) configuration

Table 4 shows clearly that the quality factor increases with the number of HL pairs. To avoid large error in calculating Q-factor method of false position was used. To keep the error values small for higher values of Q-factors we needed to reduce the interval between x-values. Hence to keep computation time, a reasonable x-axis range is imposed.

4.2 Sensitivity analysis of a resonator

In this section we examine the impact of the changes of different parameters of each layer on the optical transfer function properties of the resonators so as to determine which layer has the most influential on the resonator's performance thus its sensitivity.

For all resonators the following parameters were used.

- High RI, n_H = 2.30 (TiO_2); Low RI, n_L = 1.46 (SiO_2);
- RI of glass n_G = 1.5; RI of air n_A = 1

4.2.1 Q-factor vs. RI

The resonator configuration is : **(air)4(HL)*2L*4(LH)(glass)**

Due to symmetry the resonator is divided into two halves: LHS and RHS. The layers are numbered in increasing order from left to right e.g. 1st high layer, 1st low layer and so on until 5th low layer. Symmetry is retained in the numbering process.

Firstly the RI of each layer was varied from –5% to +5%. To keep the optical thickness constant the physical thickness of the corresponding layer needs to be changed accordingly. The results are presented in four different sets, high layer first half, high layer second half, low layer first half, low layer second half. We observe significant amount (maximum approx. 26.41) of change in Q-factor due to small change in the RI of a single layer. Obviously, layers on the right-hand side have much more effect on the resonator Q-factor than the layers on the left hand-side. And this is true for both high and low RI layers. For few layers, especially low layers first half the change in Q-factor is not a constant decreasing or increasing. Therefore, it will be hard to determine the Q-factor for a given RI of these layers.

Another set of similar results is presented in ***Figure 18***. Slight difference in the resonator configuration is introduced:

(air)4(HL)*2H*4(LH)(glass)

Two high layers instead of low layers inserted in the middle of the resonator. The results are very similar to the previous case.

4.2.2 Q-factor vs. optical thickness

Another important parameter is the optical thickness of the layers. Optical thickness of all the layers were chosen to be the same and quarter wave thick. We will vary the physical thickness of each layer one at a time, keeping the RI fixed, which will in turn change the optical thickness. We want to observe how does little change (-5% to +5%) in the optical thickness of each layer affect the Q-factor of the resonator.

The resonator configuration: **(air)4(HL)*2H*4(LH)(glass)**

Again, we group our results in the way we did for the first set of results. The results are can be . The Q-factor is not a continuous function of the optical thickness. It has continuous sudden drops. This might contribute to some difficulty to build a resonator of a given Q-factor that is very close to the edge of those lines. Also, our observation tells us, we can achieve a very large range of Q-factor by changing the optical thickness of each layer very slightly. This range increases as we step towards the middle of the resonator. In other words, the resonator is much more sensitive to the layers that are close to the middle of the resonator. Just a minute change in optical thickness can result into six fold or more change in the Q-factor. This might be useful in building a resonator with few layers and very high Q-factor with the risk of building the particular layer to the exact thickness. But, if we want to build a resonator of Q-factor that is not far from original Q-factor and can be achieved for a reasonable thickness then we should choose a layer that is close to the ends of the resonator. The original Q-factor for the above resonator is, 1.450756365179007e+002 the estimated error in the Q-factor is, 0.14825162851018. Now, let us try to build a resonator having Q-factor of 130.

From graph 8a, Q-factor of 130 might be achieved within the range of 102% to 104% optical thicknesses. Firstly we set the error in Q-factor to around 0.2 depending on Δx, the difference between discrete frequencies. This error can be further reduced by choosing smaller Δx. After several times running the MATLAB program and each time reducing the interval to find a more accurate Q-factor shown in ***Table 5***:

Q-factor	Error in Q-factor	O.T. (normalized by quarter wavelength)
1.300000348803267e+002	0.13003495009670	1.0301279000
1.299999869918581e+002	0.12998696584312	1.0301280000

Table 5. The Q-factor closest to 130.

The Q-factors can be approximately written as 130.0±0.1. The plot of Q-factor and optical thickness is almost a straight line. Therefore, we apply the method of false position to get a value for the optical thickness using the two values given above. This gives O.T. ≈ 1.03012797 with an error in O.T. of -7.3e-008

Hence we can construct resonators of any given Q-factor. One very important factor to know is that when any of the layers' optical thickness is changed the center frequency also changes. Here also we notice that layers closer to the center of the resonator has much effect on the shift in center frequency than the layers that are closer to the ends. Center frequency shift shows linear relationship with the optical thickness.

In the next section multiple resonance optical circuits are combined in cascade to form novel filter structures.

4.3 Some HL/ LH optical filter structures

Referring to ***Figure 13***, the RIes are chosen to be 1.46 (SiO_2) and 2.30 (TiO_2). Frequencies are varied 10% lower to 10% higher from the center frequency. The input impedance is separated in its real and imaginary components. Two different configurations are considered, HL and LH. In the following we discuss the results.

The significant features are, the graphs are symmetrical at the center frequency and at the center frequency the input impedance only has real component and no imaginary component, i.e. it is resistive. At all other frequencies the input impedance is complex, having both real and imaginary components.

Now, we change the configuration to LH and compare its result to the case for HL configuration. The significant differences are, a maximum occurs at the center frequency of low-high configuration but there is a minimum for high-low configuration. Also, the imaginary parts are the opposite of each other. Meaning, for the case of low-high configuration for frequencies lower than the center frequency the input impedance is resistive and capacitive and for higher frequencies the input impedance acts like resistive and inductive. The case is opposite for high-low configuration.

Therefore, for any given frequency apart from the center frequency the pair acts like a resistor in series with a capacitor or an inductor very much the same as the single layer in the previous section.

We can now return to the characteristic matrix representation of the layers. We rewrite the matrix in a slightly different way, as given below:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \tag{50}$$

The input impedance can thus be given as

$$Z_{in} = \frac{V_1}{I_1} = \frac{AV_2 + jBI_2}{DI_2 + jCV_2} \tag{51}$$

Similarly the output impedance can be defined as $Z_{out} = V_2/I_2$. thus we can write

$$Z_{in} = \frac{V_1}{I_1} = \frac{AV_2 + jB\,{}^{V_2}\!/\!_{Z_{out}}}{DI_2 + jCI_2 Z_{out}} = Z_{out}\,\frac{A + j\,{}^{B}\!/\!_{Z_{out}}}{D + jCZ_{out}} \tag{52}$$

alternatively we have,

$$\frac{Z_{in}}{Z_{out}} = \frac{A + j\,{}^{B}\!/\!_{Z_{out}}}{D + jCZ_{out}} \tag{53}$$

Multiplying both numerator and denominator by the complex conjugate of the denominator and use the unity property of the transmission matrix AD+CB=1 we have the real part contributes as a resistance:

$$r_{in} = \frac{R_{in}}{Z_{out}} = \frac{1}{D^2 + C^2 Z_{out}^2} \tag{54}$$

and the normalized reactance:

$$x_{in} = \frac{X_{in}}{Z_{out}} = \frac{j\, {BD}\big/(Z_{out} - ACZ_{out})}{D^2 + C^2 Z_{out}^2} \tag{55}$$

The reactance can be split into two parts: (i) Positive (or inductive) reactance: $jBD/Z_{out}/(D^2+ C^2{}^\wedge Z^2{}_{out})$ and (ii) Negative (or capacitive) reactance: $-jACZ_{out}/(D^2+ C^2Z^2{}_{out})$. These inductive and capacitive reactances are dependent on the operating frequencies.

Finally, we can represent the dielectric layers with an equivalent circuit at a particular optical frequency as shown below:

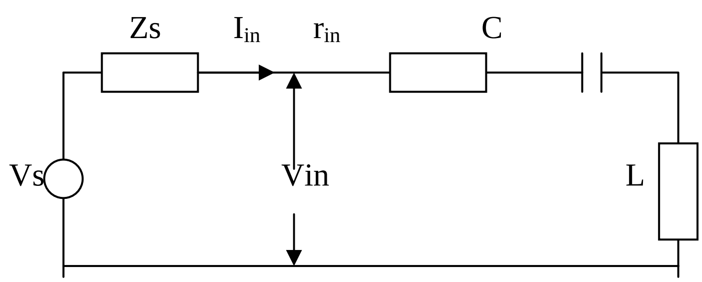


Figure 20 An equivalent circuit representation of single or multiple dielectric layers, showing normalized input resistance r_{in}, capacitance C and inductance L.

4.4 The Resonators

Based on the model developed in the previous sections a number of basic structures are designed herewith in this section. The basic structure of a resonator can be

$$\mathbf{(HL)^N*2mL(or\ 2mH)*(LH)^N.}$$

with $N = 1, 2,\ldots$ number of high-low pairs and $m = 1, 2,\ldots$ number of quarter wavelength long single layers **L or H**. The structure is thus symmetric and has even multiples of single layers in the middle region and identical high and low pairs on both sides. The reason for even multiple of single layer is in the middle is to use the half-wave transformation. The characteristic matrix of a resonator at center frequency of the cascaded resonantor is given as:

$$[(-n_L/n_H)^N\ 0\ ;0\ (-n_H/n_L)^N][-1\ 0\ ;0\ -1][(-n_H/n_L)^N\ 0;0\ (-n_L/n_H)^N] = [-1\ 0\ ;0\ -1]$$

Therefore, $Z_{in} = V_o/I_o = (-1*V_1)/(-1*I_1) = Z_L$. Hence, the input impedance at the center of thepassband is equal to the load impedance, so no reflection occurs at due to impedance matching.

The quality factor can be related to the input impedance close to the resonant frequency as:

$$\mathrm{Z_{in}} \approx R + j2QR\frac{\Delta\omega}{\omega_0} \tag{56}$$

When the frequency is such that $/Zin/^2 = 2R^2$, then the average real power delivered to the circuit is one-half that is delivered at resonance. Then,

$$\left| R + jQR\frac{\Delta\omega}{\omega_0} \right|^2 = 2R^2 \tag{57}$$

with $Q = 1/BW$ as given in (49).

4.5 The thickness of the layers

4.5.1 HL or HL configurations:

The same pair as was used previously is used again. All reactance are drawn in positive scale regardless of either capacitive or inductive type. Again we notice that at center frequency all the reactance values are zero and also the input normalized resistance is maximum/minimum at this point. Comparing this result with previously obtained result we notice, the resistance part is very much the same; the only difference is in normalization.

4.5.2 HL – 2H configuration:

Now it is possible to cascade these two pairs to construct a symmetric configuration, H-L-2H structure. We plot here only the resistive part. At the center frequency the graph has the value of 1. This is a normalized case. Therefore, at center frequency the source sees the input impedance to be a unity resistance normalized by the output resistance.

Another configuration of high-low 4-high also gave unity for the normalized input resistance. Therefore, it can be concluded that for symmetrical configuration the input impedance is unity at the center frequency.

4.6 Variable index/thickness pair

We would like to see the results of another configuration where we have a pair of dielectric layer, high and low RI, but their optical thicknesses are not equal as was the case before. The configuration is, a high RI layer is in between two low RI layers. The high layer is an independent layer, meaning the optical thicknesses of other two low layers are dependent upon the optical thickness of the high layer. The total optical thickness is still λo/2. Below the configuration is shown:

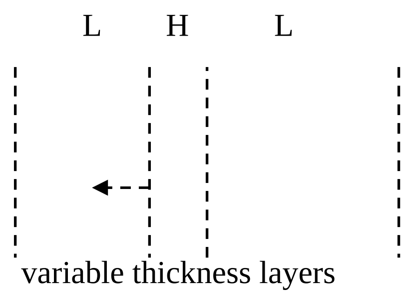


Figure 21 **Fig. 3.3 Variable thickness pair low-high-low configuration.**

In the above diagram the high layer is placed in the middle of two low layers. The optical thickness of the high layer varies from zero to λo/4. The first high layer varies as 0.5×optical thickness of high layer and the last low layer varies as 1.5×optical thickness of the high layer. After running the MATLAB program we achieved the following results (refer to variable pair graphs):

We notice that normalized input resistance is no longer unity at the center frequency and also not the maximum or the minimum. The reactance part also is not zero at the center frequency and the zero for capacitance does not occur at the same point as the zero for inductance.

5 Filter design using cascaded resonators

We have discussed about the normalized wave impedance of resonators, their Q-factor, and how the Q-factor can be altered by changing the constitutive parameters of individual layer. In this section resonators are coupled in cascade so as to design a band-pass filter

5.1 N identical resonators and coupled resonators

Multiple number of identical resonators are stratified together. Graph 11 shows the results of different number of resonators. The bandwidth becomes narrower as the number of resonator increases. Therefore, the Q-factor also increases. In the table below, we present the values of the Q-factors:

No. of resonators of the filters	The Q-factor
1	2.503043868491580e+002
2	5.050658195478696e+002
3	7.651586758484709e+002
4	1.030645180132406e+003

Table 6 N identical resonators: Resonator configuration 4(HL)(HH)4(LH); low n=1.46, high n=2.30.

When multiple resonators are coupled the center frequency of all the resonators should be matched for maximum power transfer at this frequency. Hence, all resonators should be have the same optical thickness.

Now, if the resonators are coupled together with a quarter wavelength thick layer inserted between the resonators, a band-pass filter would be formed. We have shown that a resonator has wave impedance similar to the input impedance of a parallel resonator, and this can be inverted to give the input impedance of a series resonator by inserting a single layer at the beginning of the resonator. Therefore, when we couple two or more resonators together with a single layer in between we have the equivalence of parallel and series resonators connected together. Below we show the circuit equivalence of it:

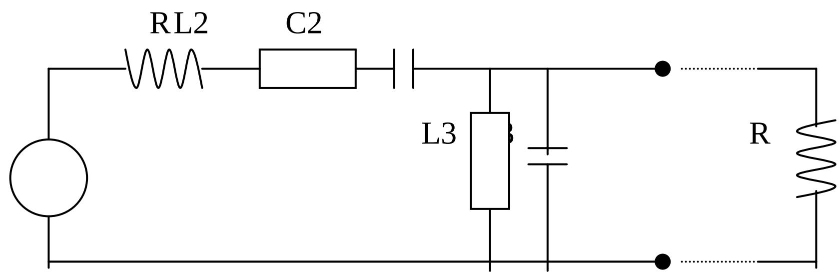


Figure 22 Equivalent circuit of coupled resonators with quarter wave coupling; structure of a band-pass filter.

In designing filters four characteristics of the filter are important: the center frequency, and the 3-dB bandwidth, hence the Q-factor, the roll-off between the pass-band and stop-band, and the ripple of the passband. Q-factor of a resonator has been introduced and investigated. The power loss or attenuation in between the pass-band and the stop-band for filters, such as maximally flat, equal ripple, etc. and the order of the filter can be obtained from charts provided in the reference [4]. For the case of filters of dielectric layers we can plot the attenuation curves for us to build our own band-pass filters of given specifications. Once we know the order of the band-pass filter and know the Q-factor of each resonator then we can build each resonator to meet the required Q-factor Here, we present the 3-dB bandwidth, the stop-band frequency as a fraction of the center frequency.

5.2 Coupling two identical resonators

For the case of dielectric resonators coupling is done with a quarter-wave dielectric layer. The optical thickness of that layer is thus kept constant so that quarter-wave, half-wave line theories can be applied to it. Therefore, the RI of the layer can be varied to observe the changes in passband ripples, 3-dB bandwidth and Power loss ratio. A resonator structure can be chosen as :

Resonator Configuration:	*4(HL)4(LH)*
High RI:	*2.30*
Low RI:	*1.46*
Q-factor:	*1.56*

*T*hese resonators are coupled with a single layer inserted between them. Referring to graph 12, the RI of the coupling layer was chosen to be 1.5, i.e. of a glass. The 3-dB bandwidth is close to 1% of the center frequency, which gives an overall Q-factor of approximately 110. This value is less than the Q-factor of individual resonators. There are no ripples in the pass-band for this given set of resonators. We have approximately 13.75 dB attenuation or power loss at frequency deviation of 1%.

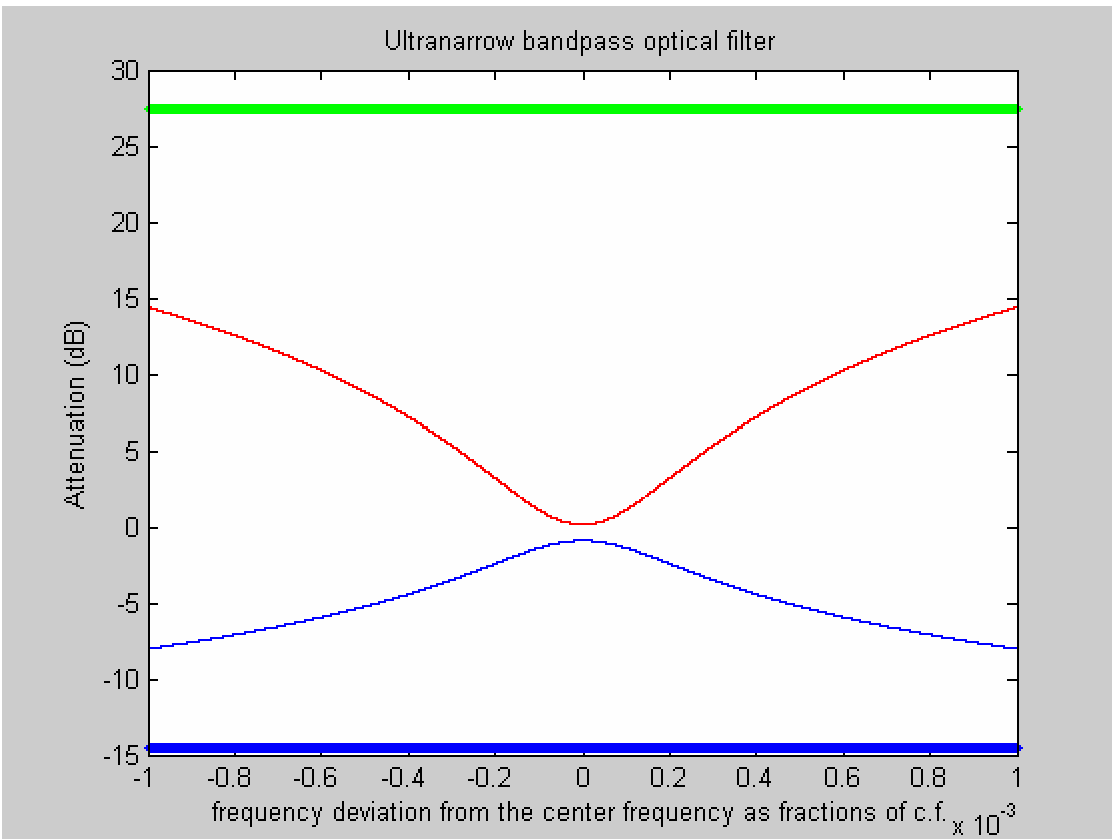


*Figure 23 The transmission (blue) and attenuation (red) factor of a 4(HL)*4(LH) resonator optical filter*

For other RI we observe ripple in the pass-band (refer to ***Figure 23***). The ripple and 3 dB BW are tabulated in ***Table 7***

RI of the coupling layer	Size of ripple in the pass-band in dB	3-dB bandwidth as a fraction of c.f.	Attenuation at 1% frequency deviation
1.35	0.04812139531757	0.00821785622150	14.76906956267890
1.38	0.03015936932202	0.00834876296174	14.55963623819277

1.5	0	0.00905636441678	13.75683792555771
2.3	0.77044510265037	0.01343129397549	9.34021437722554
2.45	1.00596387836308	0.01414918918033	8.62581888443643
2.8	1.59223760350057	0.01572944003438	7.04310338435548

Table 7 *Two identical coupled resonators with varying RI of the coupling layer; size of ripple in the pass-band, 3-dB bandwidth and power loss at 1% frequency deviation.*

From above table we notice, by increasing the RI of the coupling layer we can increase the size of the ripple in the pass-band. Also, the 3-dB bandwidth increases with increasing RI, and the rate of power loss in dB at frequencies away from the center frequency decreases.

The ripple in the pass-band can be explained in the following way, the single layer inserted between the resonators acts as a quarter-wave transformer and inverts the load impedance (here, reciprocal of the RI of the last medium). Thus, causes the normalized wave impedance not to be unity at the center frequency instead we have the values given by the quarter-wave line equation (1.14).

Hence, the power loss at center frequency is not zero. It is only zero when the RI of the coupling layer is also the same as the RI of the final medium. This is what we observe in the table 3.2 above. Again, referring to graph 13, the RI that are relatively low, i.e. 1.35 (cryolite) and 1.38 (magnesium fluoride) are used as antireflection film, that is a film which reduces the reflectivity of a surface. Therefore, they increase the transmissivity, and hence lower power loss at the center frequency. On the other hand, a material of sufficiently high RI will cause the reflectivity of the surface to increase a lot. That is why, the power loss is higher for these materials. The case of titanium dioxide (n = 2.45), zinc sulphide (n = 2.3) and stibnite (n = 2.8), but for case of stibnite 8 percent of the incident light is absorbed by the film [3]. In the ripple versus RI of the coupling layer can be estimated and that to keep the ripple lower than 0.5 dB the coupling layer of RI 1.7 or lower should be used.

In designing filters we have to choose the RI of the coupling layer such that the ripple size is small, such as 0.5 dB and the 3-dB bandwidth meets the specification or the given Q-factor and the minimum attenuation is met at the specified frequency deviation.

5.3 Cascading more than two resonators

More than two resonators can be cascaded to form sharp roll of optical filters. The RI of the coupling layer is the only constitutive parameter that is varied in order to see changes in the ripples, 3-dB bandwidth and power loss. In the following we present our results, but before that we want to specify the important features of the coupled resonators that will be used in all cases:

High RI:	*2.30*
Low RI:	4(HL) 4(HL)
Resonator configuration:	1.46
Q-factor:	*1.558233538672287e+002*
Coupled resonators' configuration:	*R-C-R-C …*
3-dB bandwidth:	*1% passband of the center frequency*
stop-band:	*1% frequency deviation from c.f*
	R = resonator and C = quarter-wave coupling layer

5.3.1 BPF passband ripple

Among the two types of filters, namely maximally flat and equal ripple, ripple exists in the pass-band of the second type as the name suggests. Below, we present a diagram of the ripple in side the pass-band of a filter response.

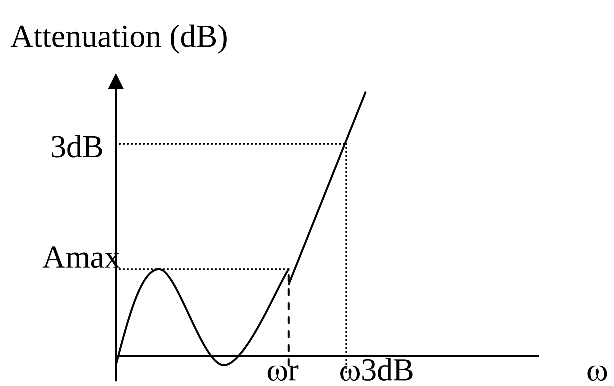


Figure 24 Ripple in the pass-band; ripple size is given as Amax, also ripple frequency and 3dB frequency is shown.

The ripples in the pass-band of dielectric filters can be controlled. As the number of resonator increases the number of ripples increases. To keep the 3-dB bandwidth within the range of 1% of the center frequency the RI of the coupling layer must be changed for each case. An important feature to notice is that the ripples are not of equal size. The ripples close to the center frequency are smaller in size than the ones away from the center frequency. Now, we wish to vary to certain degree (95% to 105%) the RI of the quarter-wave coupling layers between every two resonators one at a time and present the change in the size of ripples in our next set of results.

The RI of the coupling layer is chosen to be $n = 1.55$ in order to keep the 3-dB bandwidth within the range of 1% of the center frequency. For three coupled resonators we have two coupling layers and two ripples. The ripples are of the same size. Changing the RI of either of the coupling layers gave the same results. Therefore, we notice symmetry is observed.

Each case had only two equal ripples. For every case the results were exhibiting symmetry. In the last of this series of graphs we present the case of 11 resonators. Where there are 8 ripples. The highest ripple by changing each coupling layer one at a time. An important observation is the output is more sensitive to layers closer to the center of the structure, e.g. 5^{th} & 6^{th} layers for the case of 11 resonators. Another observation is that the size of the ripples increases as the number of resonator increases, e.g. for the case of 3 or 4 or 5 resonators.

Therefore, if identical resonators are chosen to keep low ripple and also keep the 3-dB bandwidth within 1% of the center frequency, then we could choose three resonators structures.

5.3.2 RI of the coupling layer & 3-dB bandwidth

3-dB bandwidth is defined as the bandwidth of a filter. Range of frequencies within this bandwidth are transmitted with little loss in power, nearly zero at the center frequency. Here, we are going to present the change in the 3-dB bandwidth for change in the RI of the coupling layers. Again we keep the RI to a value that will give close to 1% bandwidth and vary it slightly.

5.3.3 Power loss (attenuation) in dB between pass-band and stop-band

Here, again we have presented the change in power loss or attenuation in dB at 1% frequency deviation as a function of slight change in the RI of the coupling layer. For the most familiar types of filters, that are maximally flat and equal-ripple, the attenuation versus normalized frequency curves as provided in Ref.[4]. We have used the same resonator with configuration as given earlier, with quarter-wave coupling layers. The RI of the coupling layer was changed so that, the 3-dB bandwidth for different number of resonators were the same (1% of the center frequency). Now, a coupled resonator can be constructed with minimum of 20 dB power loss in the stop-band. The stop-band is at a frequency deviation of 1% from the center frequency, i.e. twice the pass-band. To meet the specifications we can choose three resonators' case from the graph, with the coupling layer RI, n = 1.55. The curve shows clearly that the attenuation is slightly more (21.984 dB) than 20 dB. If we require 40 dB loss within that range of frequency deviation then we need to couple five resonators. Five resonators with coupling layer RI, n = 1.35 (RI of cryolite) 44.850 dB loss of power are obtained at a frequency deviation of 1% from the center frequency.

5.4 Case study of a DMF

In this section we illustrate the design example of a particular dielectric multilayer filter as described in Ref.[1]. The filter design is based upon the transmission line theory described in Section 2. Three resonators, two of which are identical, are coupled to give an overall symmetric structure. Their approach to design the filter is summarized in the following:

5.4.1 Filter specification

The filter was designed to meet the following characteristics:

3-dB bandwidth:

stop-band: at a frequency deviation of 1%
stop-band attenuation: 15 dB
ripple in the pass-band:less than 0.5 dB.

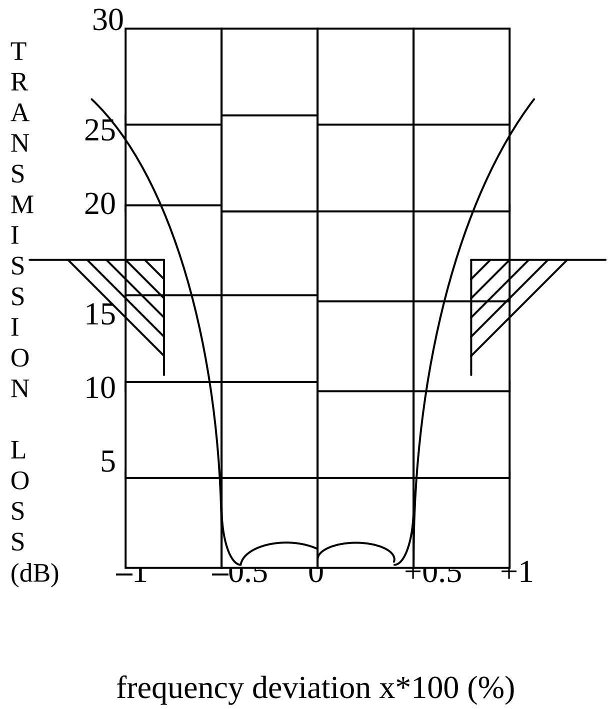


Figure 25 Fig 3.3 Spectral response of the BPF

5.4.2 Filter design

Select the filter type: a Chebyshev type with 0.5 dB ripple in passband is selected as the BPF.

Loaded Q: the loaded Q is given as the reciprocal of the 3-dB bandwidth. Therefore, Q = 100 for the overall filter.

Stop-band attenuation: 15 dB or more is at a frequency deviation of 1% from the center frequency. From the attenuation versus normalized frequency for equal-ripple filter prototypes curves in the reference [2] for 0.5 dB ripple level the order of the filter is obtained to be 3. Consequently, three stages of resonators are necessary.

Element values: from the table of element values for equal-ripple low-pass filter prototypes in the reference [4] the following element values are obtained:

1.5963 1.0967 1.5963

These values are either for capacitor or inductor, and they alternate as in the ladder network realization. Individual loaded Q: the loaded Q for each resonator is given as, Q1 = Q3 = 93.2 and Q2 = 64.0.

Material selection: SiO_2 (n=1.46) and TiO_2 (n=2.30) are selected as low RI and high RI materials respectively.

Thickness of the layers: the thickness of each layer was obtained from charts that relate the thickness and the loaded Q for a resonator for given RIes. This filter contained variable optical thickness layers and their thickness is also obtained from the chart.

Thus the final filter that was obtained is given below:

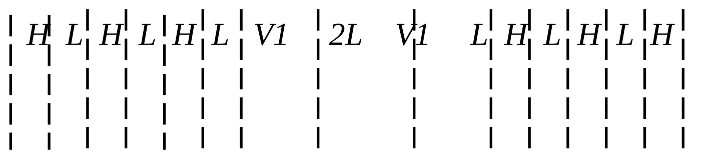


Figure 26 Configuration of the first or the third resonator, H = high RI layer, thickness = 1, L = low RI layer, thickness = 1 and V1 = variable layers, LHL, thickness 0.227, 0.514, 1.277 respectively; thickness normalized by π/2.

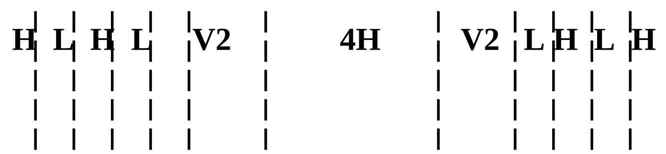


Figure 27 Configuration of the second resonator, H = high RI layer, thickness = 1, L = low RI layer, thickness = 1 and V2 = variable layers, LHL, thickness 0.111, 0.756, 1.111 **respectively; thickness normalized by π/2.**

Above three resonators are joined with a single quarter wave low RI layer inserted in between each resonator.

5.4.3 Filter evaluation:

Above BPF is simulated in a MATLAB program. We will observe the simulated response of the above filter and discuss important features.

Refer to graph 21, that shows the spectral response of the BPF. The 3-dB bandwidth is 0.998113% i.e. very close to 1% but it does not meet other specifications, such as it is not symmetrical to the center frequency. The size of the ripple in the pass-band is huge compared to 0.5 dB. The attenuation at –1% frequency deviation is much less than 15 dB and attenuation at +0.5% frequency deviation is 22.9320 dB which is too far from what we expect (refer to fig. 3.3).

Now, to fix these discrepancies using our previous findings we want to make some changes to the parameters of the filter and do a sensitivity analysis to obtain a filter that meets the specifications more accurately. Here, we present the changes in steps.

Shift in center frequency and symmetry: Previously in Section 2 we notice that a change in optical thickness of any layer causes shift in the center frequency. The above design contains variable layers placed close to the middle region of the filter where the filter is more sensitive. This shift in the center frequency can be fixed if the variable layer are replaced with (i) HL pairs or (ii) two L layers for $1^{st}/3^{rd}$ resonator and HL pairs for the 2^{nd} resonator. After such modifications we have the following two configurations:

(i) 4(HL)2(L)4(LH)–C–3(HL)4(H)3(LH)–C–4(HL)2(L)4(LH)

(ii) 3(HL)4(L)3(LH)–C–3(HL)4(H)3(LH)–C–3(HL)4(L)3(LH)

Where, N(HL) or N(LH) are N layers of HL/LH pairs. C is coupling layer with RI n=1.35 so as to decrease the reflectivity. Furthermore N(H) or N(L) are used with N is the number of quarter-wave single H or L RI layers. Their frequency responses are given in graphs 22 (a) and (b) respectively.

Passband Ripple: From previous findings we know the quarter wave coupling layers play a major role in the size of the ripple. Therefore, we change the RI of the coupling layer to give ripple less than 0.5 dB. The result is plotted in graph 22 (c). From this result we see that if we choose the coupling layer to be the low RI (n=1.46) layer as is used in the filter then the ripple size will be slightly more than 0.5 dB. Therefore, choosing a lower RI we can expect the ripple to be smaller. Hence, the RI was chosen to be, n=1.35.

3-dB bandwidth: Above two filters both have ripple in the pass-band less than 0.5 dB and attenuation at frequency deviation of 1% is more than 15 dB, but the second filter response is closer to the given specifications in terms of the 3-dB bandwidth. The second filter's specifications are listed below:

3-dB bandwidth: 1.007737%

stop-band attenuation: 20.318 dB at 1% frequency deviation.

ripple in the pass-band: 0.042 dB

Therefore, this configuration is selected for further analysis.

5.4.4 Sensitivity

The aim of this sensitivity analysis is to find which layer gives the 3-dB bandwidth closest to 1% of the center frequency. We will slightly vary the RI of every layer, from 95% to 105%, and observe any changes to the 3-dB bandwidth of the filter. Since, the filter is symmetrical, and the response is the same for each half of the filter, we only need to do calculations on half of the filter.

H and L layers are numbered as shown:

Resonator 1 **Resonator 2**

1 2 3 4 5 6 7 8 9 10 1 2 3 4 5

HL HL HL L L L L LH LH LH–C–HL HL HL H H

Here, only half of the filter is shown. Each layer gives closest to 1% bandwidth at a particular RI and are tabulated in ***Table 8***.

Layers		RI	3-dB bandwidth
1st resonator	H	2.2310	0.999903
	L	1.5038	0.999900
	H	2.2356	0.999894
	L	1.50088	1.000064
	H	2.2425	1.000023
	L	1.49358	1.000071
	L	1.43080	1.000123
	L	1.43664	1.000071
	L	1.4819	1.000152
	L	1.44102	0.999896
	L	1.47752	0.999818
	L	1.44394	0.999806
	H	2.3253	1.000066
	L	1.4454	0.999768
	H	2.323	0.999821

	L	1.4454	0.999936
	H	2.323	0.999898
2nd	H	2.323	0.999815
	L	1.4454	0.999749
r	H	2.3253	1.000023
e	L	1.44248	1.000237
s	H	2.3299	0.999994
	L	1.43664	1.000133
	H	2.3552	0.999911

Table 8 3-dB bandwidth versus RI of each layer.

If we plot the 3-dB bandwidth values against the number of layers the closest is achieved by the 3rd high-ref. layer in the second resonator for a RI of 2.3299. Now, a change toan be obtained. We note that the 3-dB bandwidth is very close to 1%. At the center frequency there is very low transmission loss of 0.000725 dB. The ripples are well below 0.5 dB and the stop-band attenuation is more than 15 dB. The filter configuration is as

3(HL)4(L)3(LH)–C–2(HL)(VL)4(H)3(LH)–C–3(HL)4(L)3(LH)

where V is the variable layer with RI n = 2.3299. There could be still other solutions to the above filter specification. Other layers also gives close to 1% bandwidth, we can choose a different layer and change its RI. Also other parameters such as the number of HL pairs in each resonator, optical thickness of the layers could be varied to give a different solution. The RI of the layers of the optical filter may be changed due to some environmental conditions. In the next section the stability of an optical filter is investigated.

5.5 Other DMFs and environmental effects

There are other dielectric multilayer filters which could form ultra narrow bandwidth optical filters, e.g. 0.3 nm passband at the center wavelength of 1500 nm corresponding to a center frequency of about 200 THz. One of these configurations is given as:

9(HL)H(2L)H9(LH)

It is important that the filter characteristics remain stable during environmental variations. The materials and deposited film density affect the environmental stability of the dielectric multilayer filters significantly. Therefore, changes can be done to the parameters so that the filter is stable. This changes are change in material, such as using Ta_2O_5 as the high RI layer, n=2.08, rather than usual TiO_2, n=2.30. Ta_2O_5 is a physically and chemically a stable material. Also, the deposition density affects the stability. For example, the lower RI layer is SiO_2, which is low in density, but in humid condition it can absorb water and increase in density. This will cause the RI to increase. Therefore, the density is increased by the O_2 ion-assisted technique and the stability of the filter is improved due to increase in density. Ta_2O_5 adjusts with the ion-assisted deposition technique better than TiO_2.

Because of these improvements it was possible to realize very narrow band-pass filter which is stable during environmental variations and aging. Such filter is used in elimination of noise in fiber amplifiers and wavelength selection in high-density wavelength division multiplexing [5]. The most important feature of this filter is high accuracy in selecting the required wavelength. Graph 26 displays the spectral response of this filter.

6 Some OBPFs

The input impedance of a dielectric multilayer resonator circuit close to its center frequency is shown to exhibit the characteristics of the input impedance of a parallel resonator. A single quarter-wave layer acts as an impedance inverter and if added before a resonator it transforms the input impedance of a parallel resonator into the input impedance of a series resonator. The Q-factor was shown to increase as the number of high-low pairs in the resonator increased.

The Q-factor of a given resonator is subject to vary a large range for changes in the RI of the layers on the second half of the resonator, with the first medium being air and last being glass. If we want to build an optical filter of a given Q-factor by changing the optical thickness, then layers that are further from the center of the resonator should be chosen to give a stable result. Change in optical thickness will result in shift in center frequency. This shift will also be small if a layer further from the center is chosen. Therefore, to build a desired resonator small shift in center frequency and slight asymmetry in the response can be accepted.

A number of resonators joined side by side without any coupling layer inserted in between will give a response with higher Q-factor but no ripples in the pass-band. Number of coupled resonators should not be many so those ripples are few in numbers. The RI of the coupling layer showed to have effects the size of ripples, on the 3-dB bandwidth and little effect on the stop-band attenuation. If we increase the number of coupled resonators then we need reduce the RI of the coupling layer to have small ripples. From the attenuation curves we can decide on how many resonators of a given configuration will give the desired attenuation in the stop-band.

In fabricating a DMF we have to keep in mind that the RI of the coupling layer should be low enough to keep the ripples small. Change in optical thickness will give an asymmetric response and shift in center frequency. There are more than one solution of DMF for a given filter specification. A filter can be made stable during environmental variation by increasing the density of the layers by O_2 ion-assisted deposition. Since the stability of the filter can be improved it is possible to construct the ultra narrow band-pass filter. The output response of the band-pass filter is discussed in the next section.

6.1.1 Sensitivity

This filter can consist of series and parallel resonators. Each resonator is a pair of high-low layers of equal and variable optical thickness. Further there is a single layer of low RI inserted between these two resonators to form a pi-equivalent photonic circuit with an interchange between parallel resonance and series resonance as shown in Figure 28.

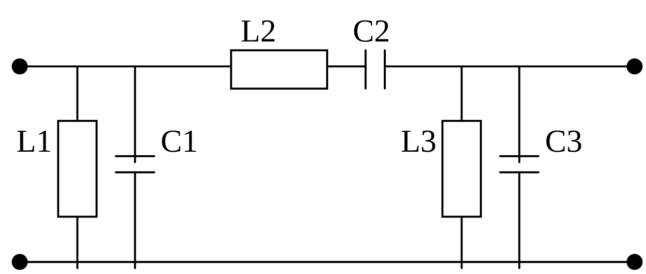


Figure 28 Equivalent circuit of a band pass filter.

The following is an example of designing of the optical filter given by the Japanese filter [2] design.

Filter Specifications: A band pass filter with following specifications, *Optical 3-dB Bandwidth: 1%;*

Rejection: more than 15dB at frequency deviation of 1%;

Ripple: less than 0.5 dB in the pass band.

A Chebyshev type filter is selected as the BPF. We divide the filter into three parts.

6.1.2 Symmetric multi-layer optical filter

From the filter design the number of fixed reflecting pairs is ***three*** on each side due to symmetry reason. Variable pairs are in between the fixed layers and there are two low RI layers are in the very middle section. We plot of the input resistance at different stages of the resonator.

(i) First stage: the variable layers are removed and plot the input resistance. The graph is symmetrical at the center frequency and its value is normalized to one.

(ii) Second stage: Only one of the variable layers is added to one side of the resonator. This increases the filter size by $\lambda_o/2$. The maximum input resistance does not occur at the center frequency and less than unity. When the variable layer is removed and another variable layer is added symmetrically to the other side of the filter struture, the maximum input resistance is changed but occurs at the same frequency. What symmetrical variable layers mean is shown *Figure 29*.

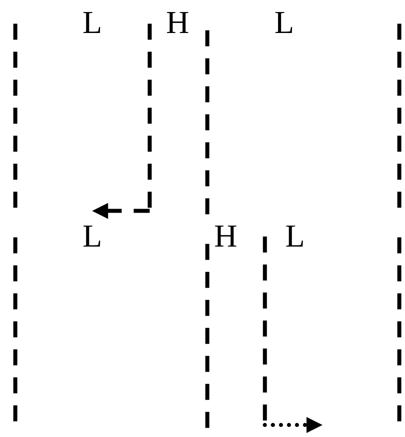


Figure 29 Symmetrical variable pair low-high-low configuration.

(iii) Third stage: Variable reflection layers are added to both sides. The maximum value for input resistance is unity. However the maximum does not occur at the center frequency.

The schematic diagram of the structure of the resonator is given in *Figure 30*.

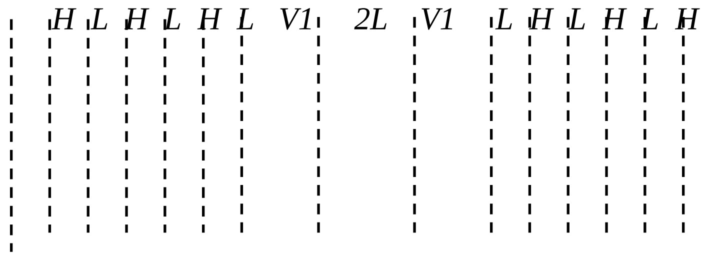


Figure 30 Configuration of the first resonator, H = high RI layer, L = low RI layer and V1 = variable layer for the first resonator.

6.1.3 Symmetric and variable edge multilayer optical filters

The second resonator consists of four fixed reflection pairs, two variable pairs whose quantitative values are different from the variable layer discussed tion 5.1.2, and four high layer of quarter wavelength optical thickness are placed in the middle.

Again we take the resonator through all three stages. The results were very much the same as was obtained earlier. They are given as shown in ***Figure 31***.

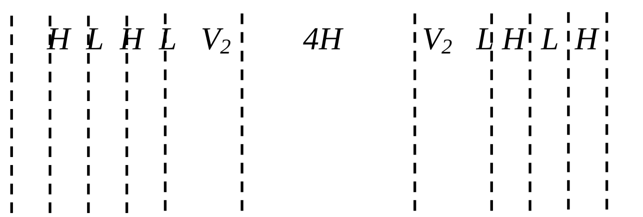


Figure 31 Configuration of the second resonator, H = high RI layer, L = low RI layer and V_2 = variable layer for the second resonator.

6.1.4 Low index transformer multilayer optical filters

These resonators follow the structure of the symmetric fixed multilayer resonator with a single layer of low RI alternately placed between all three resonators. After running the designed filter in MATLAB program we achieved the following response of the filter.

6.1.5 Traveling waves through the optical filter?

We develop a tracing program for monitoring the traveling wave through the filter of the filters in Ref.[2]. We let light waves of different wavelengths be the input of the simulated filter and received different plots of the magnitude and the phase of the traveling wave. The high, low and variable RI layers were represented by different symbols. The derived expressions of the optical transmittance and reflectance are used.

6.1.6 Traveling of the center-frequency wave

The filter was designed based on the wavelength of the centre passband or the Bragg wavelength i.e. each layer of the fixed HL pair has an optical thickness of a quarter wavelength. The magnitude of the light wave changes in a periodic fashion. Within the first half of the first resonator the magnitude becomes very small. It recovers back to nearly half the initial magnitude (initial magnitude is unity) as it travels through the rest of the resonator. Then, it enters the second resonator. Similar pattern is observed in the second resonator. Finally, in the third resonator the light wave gains back its magnitude towards unity (refer to the plot of center wave).

A closer observation of the magnitude plot indicates that the change in magnitude occurs in steps and each step involves a pair of H and L layers. The magnitude of the transmitted wave decreases for the first half of the resonators and increases for the second half of the resonators.

Now some changes of the structure of the filter can be made. All the layers except the H and L fixed reflection pairs are removed and the magnitude of the traveling wave at centre frequency is monitored as it passes trough the layers. It is observed that for the first half of each resonator the magnitude decreases and for the second half the magnitude increases of the same magnitude.

An observable feature is that the high and low layers are paired together and they get paired only in the direction of transmission leaving out any single high layer by itself

6.1.7 Off-band Traveling Wave

Waves of different wavelength deviations are traveled through the filter layers. The wavelength deviation increases the transmitted light wave attenuates rapidly.

7 Concluding remarks

Having outlined the analytical technique for analysis of multilayer thin film, the most important issue at this stage is the synthesis of optical filters with specific optical properties.

This can be implemented through the following stages:

- Employing several commonly available filter design techniques – both analog and digital – e.g. MATLAB filter design tool box
- Find the impulse responses of the desired filter and its corresponding frequency domain
- Convert the poles and zeroes into number of cascaded resonators
- For each resonator transfer the number of layers into equivalent refractive index profile using digital thin file technique
- Finalize the layer thickness, its height and periodicity.

8 References:

Ramo, S., Whinery J.R. and T. Van Duzer *"Fields and waves in communications electronics"*,J. Wiley pp.351-352, Tokyo, 1965

Kajikawa,M.; Kataoka,I.; Lightwave Technology, IEEE Journal of , Vol.15 , No.9, Sept.1997 , pp.1720 – 1727.

W.H. Southwell , " *Coating design using very thin high- and low- index layers*", Applied optic, vol. 24., no.4, pp457, 1985.

8.1 Program "OPTICAL IMPEDANCE"

```
% This program plots the absolute magnitude, the resistive and reactive parts of the total optical impedance of a single stratified
% layer (of a nominated RI n bounded by air (RI n1=1) and glass (RI n3=1.47)
% The Optical thickness is normalised wrt the optical wavelength.
Format long;
clear;
n1 = 1;        % RI of air
n3 = 1.47;     %    "    "  "  glass
n = input('RI of the layer n = ');  % RI of the layer
N = 1;
for x = 0:0.002:1.0,     % ratio of optical thickness and wavelength
    k = 2*pi*x;
    M = [cos(k) j*sin(k)/n; j*sin(k)*n cos(k)];
    % Characteristic matrix of the layer
    z(N) = (M(1,1)+M(1,2)*n3)/(M(2,1)/n3+M(2,2));% normalised input impedance
    plot(x,abs(z(N)),'r.');
    plot(x,real(z(N)),'b-');
    plot(x,imag(z(N)),'g-.');
    hold on;
    N = N + 1;
    end
grid;
```

```
title('The input impedance of a stratified layer');
ylabel('Input  optical impedance abs-r, resistive-b and reactive-g');
%ylabel('normalised reactance - capacitive/inductive (dimensionless)');
%xlabel('Frequency deviation as factor of central frequency (x*100%)');
xlabel('Normalised Optical  Film Thickness wrt centre wavelength');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program is used for sensitivity analysis of a resonator. It plots
% the Q-factor of resonator as the RI of a layer is varied
format long;
clear;

n1 = 1;         % RI of air
nl = 1.46;      % RI of the low ref. layer
nh = 2.30;      % RI of the high ref. layer
n3 = 1.5;       % RI of glass (base material)
%c = input('c= ');
%b = input('b= ');
%c = 1.026404;

for c = 0.95:0.005:1.05,

N = 1;
for x = -0.02:0.00008:0.02,    % ratio of frequency/wavelength deviation and c. f.
```

```
    k = (1+x)*pi/2;
    MH = [cos(k) j*sin(k)/nh; j*sin(k)*nh cos(k)];
    ML = [cos(k) j*sin(k)/nl; j*sin(k)*nl cos(k)];
    C = [cos(k) j*sin(k)/c; j*sin(k)*c cos(k)];    % the variable layer
    M =
C*ML*MH*ML*MH*ML*MH*ML*MH*MH*ML*MH*ML*MH*ML*MH*ML*MH;
    Zin = (M(1,1)+M(1,2)*n3)/(M(2,1)/n3+M(2,2));
    % normalised input impedance
    Z(N) = abs(Zin);
    N = N + 1;
end

N = 1;
    while (abs(0-20*log10(Z(N)))) > 3
            N = N + 1;
    end;
Z1 = 10*log10(Z(N-1));
Z2 = 10*log10(Z(N));
N = N + (Z1+3)/(Z1-Z2);
Ner = abs((Z1+3)/(Z1-Z2));
x = abs(-0.005+(N-1)*0.00001);
xer = 0.00001*Ner;
Q = 1/(2*x);
Qer = abs(xer/(2*x^2));
```

```
figure(1)

plot (c,Q,'b+');

hold on;

figure(2)

plot(c,Z,'g');

hold on;

end

grid;

%xlabel('frequency deviation (x*100%)');

xlabel('percentage change in optical thickness -5% to +5%');

%ylabel('normalized wave impedance (dB)');

ylabel('Loaded Q(quality) factor');

%title('Normalized wave impedance for different resonators, N(HL)N(LH) configuration
- high n=2.30 and low n=1.46');

title('Change in loaded Q-factor as function of thickness of 1st high layer');

*********************************************************************
%       optical filters with stacked layer filters M1 and M2

%

%

format long;

clear;

n1 = 1.5;         % RI of glass

nl1 = 1.46;       % RI of the low 1

nh1 = 2.08;       % RI of the high 1

nl2 = 1.46;       % RI of the low 2
```

```
nh2 = 1.70;     % RI of the high 2
n3 = 1.5;       % RI of glass
c = 1.35;       % RI of the coupling layer
N = 1;
for x = -0.001:0.0000001:0.001,     % ratio of wavelength deviation and center wavelength
        k = (1+x)*pi/2;
         % ABCD matrix of the H and L RI layers MH ML
        MH1 = [cos(k) j*sin(k)/nh1; j*sin(k)*nh1 cos(k)];
        ML1 = [cos(k) j*sin(k)/nl1; j*sin(k)*nl1 cos(k)];
        C = [cos(k) j*sin(k)/c; j*sin(k)*c cos(k)];
        MH2 = [cos(k) j*sin(k)/nh2; j*sin(k)*nh2 cos(k)];
        ML2 = [cos(k) j*sin(k)/nl2; j*sin(k)*nl2 cos(k)];
  % layer resultant matrix of 20 pairs of 2x2x20 layers with HL alternate stacking
  M1 = (MH1*ML1)^20*(ML1*MH1)^20;
  M2 = (MH2*ML2)^20*(ML2*MH2)^20;
  M = M1*C*M2;
  t = 2*n1/((M(1,1)+M(1,2)*n3)*n1+(M(2,1)+M(2,2)*n3));       % transmissivity
        P(N) = n1/(n3*(abs(t))^2);     % power loss ratio
        plot(x,10*log10(t),'b');
        plot (x,10*log10(P(N)),'r');
        hold on;
        N = N + 1;
end
```

```
grid;
%-------------------------------------3-dB
% Calculating the 3dB bandwidth of the optical filter
N = 1;
        while 10*log10(P(N)) > 3
                N = N + 1;
        end;
P1 = 10*log10(P(N-1));      % calculating error for determining the 3dB bandwidth
P2 = 10*log10(P(N));        %
N = N - (P1-3)/(P1-P2);
Ner = abs((P1-3)/(P1-P2));
x1 = 2*abs(-0.01+(N-1)*0.00001)
xer = 0.00001*Ner;
%-----------------------------------------
grid on;
title('Ultranarrow bandpass optical filter');
ylabel('Attenuation (dB)');
xlabel('frequency deviation from the center frequency as fractions of c.f.');
##############################################################################
```

APPENDIX 2 TWO PORT NETWORKS AND EQUIVALENT CIRCUITS:

Conversion relationships for two-port parameters

	Impedance Z		Admittance Y		Hybrid H		Reverse Hybrid g		Transmission T		Reverse transmission t	
Z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$-\frac{g_{12}}{g_{11}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$	$\frac{d}{c}$	$\frac{1}{c}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta_g}{g_{11}}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{\Delta_t}{c}$	$\frac{a}{c}$
Y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{\Delta_g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$	$\frac{a}{b}$	$-\frac{1}{b}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$	$-\frac{1}{B}$	$\frac{A}{B}$	$-\frac{\Delta_t}{b}$	$\frac{d}{b}$
H	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{g_{22}}{\Delta_g}$	$-\frac{g_{12}}{\Delta_g}$	$\frac{B}{D}$	$\frac{\Delta_T}{D}$	$\frac{b}{a}$	$\frac{1}{a}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{g_{21}}{\Delta_g}$	$\frac{g_{11}}{\Delta_g}$	$-\frac{1}{D}$	$\frac{C}{D}$	$\frac{\Delta_t}{a}$	$\frac{c}{a}$
g	$\frac{1}{z_{11}}$	$-\frac{z_{12}}{z_{11}}$	$\frac{\Delta_y}{y_{22}}$	$\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta_h}$	$-\frac{h_{12}}{\Delta_h}$	g_{11}	g_{12}	$\frac{C}{A}$	$-\frac{\Delta_T}{A}$	$\frac{c}{d}$	$-\frac{1}{d}$
	$\frac{z_{21}}{z_{11}}$	$\frac{\Delta_z}{z_{11}}$	$-\frac{y_{21}}{y_{22}}$	$\frac{1}{y_{22}}$	$-\frac{h_{21}}{\Delta_h}$	$\frac{h_{11}}{\Delta_h}$	g_{21}	g_{22}	$\frac{1}{A}$	$\frac{B}{A}$	$\frac{\Delta_t}{d}$	$-\frac{b}{d}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$	$\frac{g_{22}}{g_{21}}$	A	B	$\frac{d}{\Delta_t}$	$\frac{b}{\Delta_t}$
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$	$\frac{\Delta_g}{g_{21}}$	C	D	$\frac{c}{\Delta_t}$	$\frac{a}{\Delta_t}$
t	$\frac{z_{22}}{z_{12}}$	$\frac{\Delta_z}{z_{12}}$	$-\frac{y_{11}}{y_{12}}$	$-\frac{1}{y_{12}}$	$\frac{1}{h_{12}}$	$\frac{h_{11}}{h_{12}}$	$-\frac{\Delta_g}{g_{12}}$	$-\frac{g_{22}}{g_{12}}$	$\frac{D}{\Delta_T}$	$\frac{B}{\Delta_T}$	a	b
	$\frac{1}{z_{12}}$	$\frac{z_{11}}{z_{12}}$	$-\frac{\Delta_y}{y_{12}}$	$-\frac{y_{22}}{y_{12}}$	$\frac{h_{22}}{h_{12}}$	$\frac{\Delta_h}{h_{12}}$	$-\frac{g_{11}}{g_{12}}$	$-\frac{1}{g_{12}}$	$\frac{C}{\Delta_T}$	$\frac{A}{\Delta_T}$	c	d

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$, $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$, $\Delta_T = AD - BC$

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$, $\Delta_g = g_{11}g_{22} - g_{12}g_{21}$, $\Delta_t = ad - bc$