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SIMULINK Model for Optically Amplified Transmission
Systems: Part V: Linear and Nonlinear Fiber Propagation
Models

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MONASH
UNIVERSITY

**SIMULINK MODEL FOR OPTICALLY AMPLIFIED TRANSMISSION
SYSTEMS: PART V: LINEAR AND NONLINEAR FIBER PROPAGATION
MODELS**

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Abstract

We present the experimental and simulation results of an optical transmission system over SMF with and without dispersion compensation. The simulation is based on the Matlab SIMULINK R14. This report is a part of a series of SIMULINK simulator for optically amplified transmission systems (SOATS). The modelling of the fiber propagation of lightwaves under linear and nonlinear dispersion effects is the principal focus.

Both the split step Fourier technique and the low-pass filter transfer function are presented for the linear and nonlinear-induced effects on pulse shape. Eye diagrams obtained by simulation are compared with experimental results for the transmission systems employing advanced modulation formats such as NRZ, RZ, CSRZ, RZ-DPSK, RZ DQPSK. It is proven that the SIMULINK model is efficient for simulation of fiber communications system with minimum computation and development time.

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1 Introduction

Optical communications systems and networks have been installed throughout the world, from terrestrial to metropolitan to global information transport structures. The milestones include the invention of the low loss optical fibers in the 70s, optical fiber amplifier in the 80s and in line fiber Bragg grating (1990s). Optical communications remains a very diverse field, with ever increasing applications, e.g. long-haul communications, aircraft technology, underwater exploration, and high speed communications.

DWDM, known as Dense Wavelength Division Multiplexing is a multi-carrier fiber-optic transmission technique. Multiple-carrier optical signals are employed digitally modulated at bit-rate of 10 Gb/s up to 40 Gb/s in ultra-high speed and long haul transmission systems.

There is an ever increasing need, after a significant slowing down period, to upgrade the transmission capacity of multi-wavelength channels over ultra-long reach optical fiber distance. This could be in areas such as increased capacities, ultra-high speed, minimum transmission errors, and non-linear impairments in over long distances and maximising total number of channels over the C- L- and S- bands of the 1550 nm

window.

Modelling of an optically amplified transmission system is very important. The models must be simple and accurately representing the physical phenomena and corroborated with experimental systems. Although there are a number of modelling packages available in commercial environment (such as VPIsystems, OptiWave etc.) and in academic institutions[1]. They are either complicated and user-non-friendly or difficult to be correlated with experiments. Hence the need of a simple and graphical models that users can change and modify with minimum efforts to model their transmission systems.

This report is a part of a series of the simulator for optically amplified transmission systems (SOATS) for ultra-high speed ultra-long haul DWDM optical transmission systems[2]. The Simulink model presented in this report is principally focussed on the propagation models of lightwaves transmitting through fibers, transmission or dispersion compensating types. The regions of operation could be in either linear or nonlinear. These models are then integrated with other models developed for optical amplifiers[2] and other advanced modulation formats[3]. To the best of our knowledge, the Simulink simulator SOATS is successfully implemented for lightwaves propagation in optical fibers for the first time.

Matlab SIMULINK simulations results are verified with experimental results. Advanced modulation techniques are also developed in the simulator for demonstration of its effectiveness in long haul optically amplified communications systems. The principal contribution of this report is the development of a fiber propagation model capable of simulating the effects of linear dispersion, non-linear dispersion and optical power losses inherent to optical fibers.

This report is organised as follows: Section 2 gives an introduction of the propagation of modulated lightwaves over SMF and other types of fibers required for dispersion compensation and some essential photonic components are briefly

given. The modulation formats are introduced in Section 3 and integrated with the fiber propagation models in Section 4. Both linear and nonlinear models of the modulated lightwaves over fibers are described and the eye diagram in electrical domain are used to evaluate the BER and hence the performance of the transmission systems. Finally concluding remarks are made and suggestions for further studies are given.

2 Fibers and photonic components

Given that the line width of the laser source is very narrow that would allow transmission of ultra-high speed signals through optical fibers as expected from its minimum dispersion. However when it is directly modulated its spectrum is broadened and hence shortening the transmission distance, hence the need of external modulation technique. Mach Zehnder interferometric modulators are the most common photonic devices that are employed for modulating lightwaves via the electro-optic effects with minimum chirp effects on the carrier. Modern optically amplified transmission systems would require advanced modulation techniques to reduce the effective signal bandwidth and reducing the total optical power distributed to the carrier to minimize the nonlinear-induced dispersion.

Several modulation formats that can be considered, in this part and other parts including Non Return to Zero (NRZ), Return to Zero (RZ), Frequency Shift Keying (FSK), Phase Shift Keying (PSK), and Differential Quadrature Phase Shift Keying (DQPSK). Different formats are also reported in the other parts of the series [3].

Two types of modulators have emerged to be of higher popularity, the Mach-Zehnder Interferometric Modulator (MZIM) and the Electro-Absorption Modulator (EAM). The EAM is expected to be useful in specific modulation application such as in systems required its fast switching capabilities and linear characteristics. On the other hand the MZIM offers superior performance in amplitude and phase modulation. Its operating principles of constructive “ON” and destructive “OFF” are

depicted as shown in Figure 1:

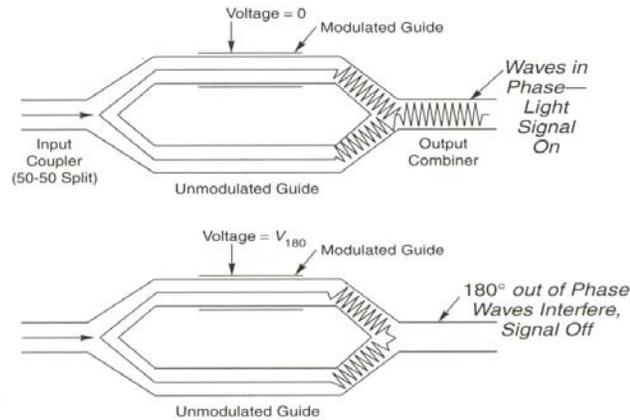


Figure 1: *MZIM at constructive and destructive interference operations [4].*

Should the voltage applied to the modulator be equivalent to the voltage for no phase shift, the modulator must be in its ON state as the interferometer would be producing constructive waves. If the voltage applied, this includes the DC biasing voltage and the modulating voltage, is equivalent to the phase shift voltage of V_{180} , the waves produced by the interferometer would be destructive waves with a phase shift of π rad/s. When combined with the optical waves passing through the unmodulated guide, this would negate all optical waves and produce near to no optical output. MZIM are prone to the effects of chirp which arises when there is a shift of refractive index (RI) in the cavity causing a shift in the resonant wavelength.

2.1 Optical Fibers

Commonly installed single mode optical fibers are mainly classified into standard non-zero dispersion shifted types (SSMF and NZ-DSF) for transmission spans. The guided mode is linearly polarized and composes of two orthogonally polarised modes propagating through a randomly deformed in its geometry and refractive index. The other fiber type is also used frequently for dispersion compensation, the DCM with its dispersion factor is in opposite sign of that of the transmission fibers. The significant difference between the fibers is their dispersion properties. The SSMF and NZ-DSF have, normally, a positive dispersion characteristic while the

DCM has a negative dispersion characteristic. The NZ-DSF fiber has zero dispersion at the wavelength just outside the transmission spectrum to avoid nonlinear four wave mixing effects in DWDM transmission. These properties can be attributed to the construction of the fibers.

In normal optical transmission systems, the optical signal is transmitted over a SMF fiber with positive dispersion characteristics, this signal is then compensated for chromatic dispersion through a DCM fiber. Chromatic dispersion consists of material and waveguide dispersion. Material dispersion arises from the change in a material's refractive index with wavelength. The higher the refractive index, the slower light travels. Thus as a pulse spectrum containing a range of carrier and modulated sidebands passes through a material, it stretches out with the wavelength with lower refractive index going faster than those with higher indexes. The refractive index $n(\lambda)$ can be expressed in Sellmeier's dispersion formula as:

$$n(\lambda) = 1 + \sum_k \frac{G_k \lambda^2}{(\lambda^2 - \lambda_k^2)} \quad (1)$$

where G_k are Sellmeier's constants and k is integer and normally taken a range of $k=1-3$. The refractive indices are usually expressed using Sellmeier coefficients. These coefficients were utilized for designing fibers. Wave guide dispersion arises from the distribution and guiding of modulated lightwaves between core and cladding layers. Waveguide properties are a function of the wavelength. This indicates that varying the wavelength affects how lightwaves are guided in a SMF. The effect of wave guide dispersion can be approximated by assuming the material is independent of wavelength. This kind of dispersion depends strongly on Δ and V parameters. The wave guide dispersion factor can be defined as

$$D(\lambda) = D_{wg}(\lambda) = -\frac{n2\Delta}{C\lambda} V \frac{d^2(Vb)}{dV^2} \quad (2)$$

The main cause of chromatic dispersion is the change in wavelength of the transmission channel due to modulation with other frequencies. This causes a group of spectral components with minute differences in wavelength to travel within

the fiber. However, the SMF fiber propagates these signals at different wavelength at different speeds due to its positive dispersion characteristics. Should a pulse be transmitted, this effect would cause the pulse to spread over the transmission medium. Hence the need for dispersion compensation to equalise the pulse trains. It is achieved through a DCM which compensates this dispersion effect by slowing down and speeding the spectral components that are travelling faster or slower respectively through the SMF. The DCM inherits such properties due to its design and construction. It is constructed with multi-step index fiber of a number of cladding. The DCM is more prone to non-linear effects due to its small spot size. The spot size of a fiber determines the effective area of the fiber which guides light in the fiber. When the effective area is small, and the intensity of light is high, the concentration of light waves is forced through a smaller surface area, causing it to be prone to non-linear effects. If the spot size and hence effective area is small, the light waves are trapped closely to the core thus less susceptible to bending losses. Rayleigh scattering is another effect on optical fibers which causes losses in optical power. It is dependant on the difference in the reflective index between the materials utilized. Smaller differences would cause smaller losses due to this effect.

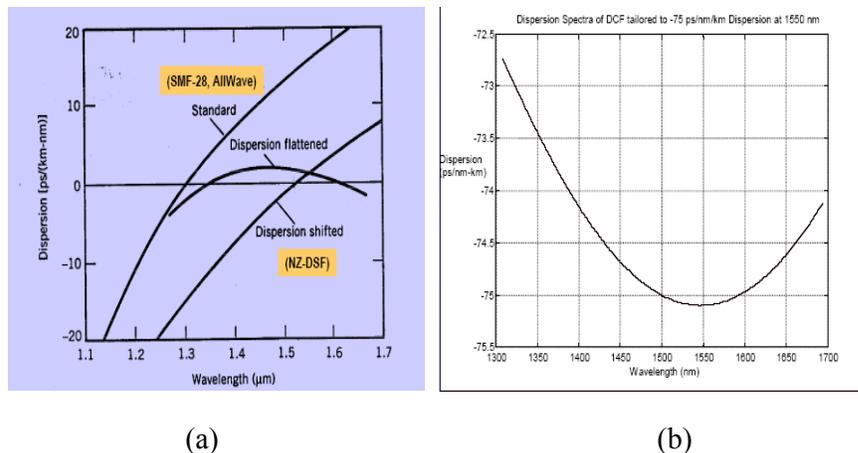


Figure 2: Dispersion properties of (a) SMF and DSF fiber[5] and (b) *Dispersion properties of DCM fiber*[5]

In a SMF light waves can exist as two orthogonal polarizations. In normal operations the signal in the SMF consists of both polarizations, these polarizations are not maintained within the fiber, allowing the light waves to couple from one polarization to the other randomly. In the SMF the ray path exhibits a different reflective index in each polarization, this is called birefringence. This causes a circular or elliptical polarization to form as the signal travels through the SMF. Dispersion from this effect is called PMD. Fibers have not been made truly circular, contributing to birefringence effects. Bending and twisting of fibers worsens the effects of PMD. A method of reducing PMD is using a polarization controller.

The ultra-wideband photo-detector utilized is usually a p-i-n type. In other words, it is a pn junction with a layer of intrinsic (non-doped) region inserted between the p-type and n-type materials forming a p-i-n junction. There is a potential barrier in the device which prevents majority carrier flow. This barrier is formed by the intrinsic region and its interfaces. For normal operations a reverse-bias voltage is applied across the junction to ensure that the intrinsic region is depleted of carriers. When a photon is absorbed, it gives up its energy to excite an electron from valence to conduction band. This forms an electron-hole pair, when the process repeats itself for many more photons, current flow will result in an external circuit. Therefore, the current flow is an indication of magnitude and frequency of the photons absorbed.

3 Modulation Formats

3.1 NRZ encoding

For low transmission rate (<2.5 Gb/s) communications systems, NRZ encoding is frequently utilized. In the NRZ code, binary 1s and 0s are depicted by two different light levels during bit duration. Hence, the presence of a high-light level in the bit duration would depict binary 1 and a low light level, binary 0. Although the NRZ codes are the most efficient in the utilization of system bandwidth, loss of timing

might occur in the presence of long strings of 1s and 0s. This causes a lack of level transition. Furthermore the total energy are low over the signal bands and the carrier energy is about 3 – 6 dB above the sidebands.

3.2 RZ Encoding

In RZ encoding, a binary 1 is represented by half period of optical pulse, present in the first half of the bit duration. Although the optical pulse is present in the first half of the bit duration, the light levels returns to 0 during the second half. Binary 0 is represented by the absence of an optical pulse during the entire bit duration. RZ encoding might require only half the bit duration for data transmission but it requires twice the bandwidth of NRZ encoding. However, in the presence of long strings of 0s, loss of timing could arise. RZ pulses have much greater tolerance to the effects of chromatic dispersion as compared to NRZ pulses. RZ pulses maintain practical residual tolerance at much higher launch powers thus allowing inexpensive unrepeated transmission over multiple segments of fiber. This format is commonly used in metropolitan optical communications systems.

3.3 Carrier Suppressed RZ

Carrier-suppressed return to zero, CS-RZ, combines the benefits of stable propagation that is displayed by RZ pulses with reduced optical spectral width to allow high spectral efficient long haul WDM transmission. The CS-RZ pulses in adjacent bit slots are transmitted with an optical phase difference, removing the optical carrier component in the optical spectrum and reducing spectral width by half. This allows closer WDM channel spacing and higher launched optical power thus increasing the possible spectral efficiency. It also avoids the effects of non-linear dispersion due to its lower optical power, allowing for more channels multiplexed in transmission.

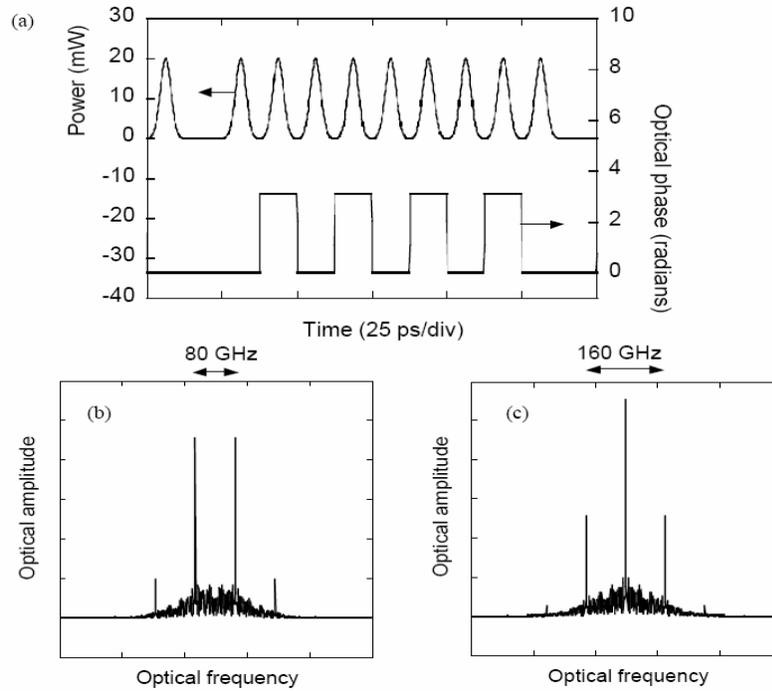


Figure 3 Intensity and phase waveforms of carrier-suppressed RZ signal. (b) Optical spectrum of an 80 Gbit/s CS-RZ signal. (c) Optical spectrum of an 80 Gbit/s conventional RZ signal[6].

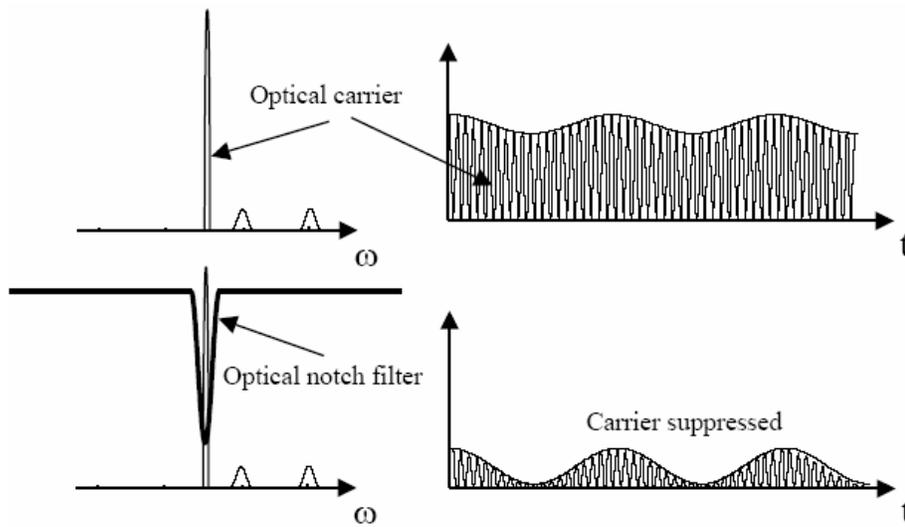


Figure 4 Illustration of optical carrier suppression [7]

3.4 Differential Phase Shift Keying (DPSK)

DPSK modulation is an encoding format which records changes in the binary stream.

The demodulator only determines changes in the incoming signal phase. At the modulator PSK signal is converted to a DPSK signal with two rules (see Figure. 2): a "1" in the PSK signal is denoted by no change in the DPSK; a "0" in the PSK signal is denoted by a change in the DPSK signal. This modulation format has less susceptance to dispersion effects due to a reduction on the harmonics involved to transmit the signal, hence a reduction in the variation of group of waves transmitted. However, it was noted that the cost of implementation and relative complexity of such a system has retarded its progress from becoming industry standard.

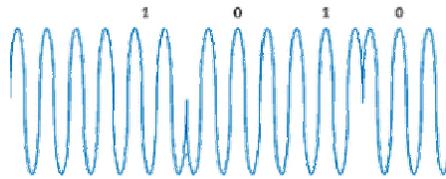


Figure 5 *Differential phase-shift keying*[8]

3.5 Differential Quadrature Phase Shift Keying (DQPSK)

DQPSK modulation encodes two parallel bit streams onto one of four possible optical phase states, thereby allowing each transmitted optical symbol to represent two bits. This property offers an advantage over the widely used NRZ modulation technique as it transmits only one bit per optical symbol. Therefore, it is less efficient with the use of optical bandwidth, allowing fewer channels to be packed in a limited bandwidth. DQPSK due to the same properties (Reduction in Harmonics) as the DPSK modulation format offers improved tolerance to chromatic dispersion and polarisation mode dispersion (PMD) over NRZ modulation format, together with increased spectral efficiency, while requiring the same OSNR for a given bit rate.

4 Photonic Transmission Model

4.1 Fiber Propagation Model

The propagation of these modulated lightwave channels are considered in this section. Fiber linear and nonlinear induced dispersion effects exist over the entire length of

an optical fiber. Thus differing lengths of fiber incur variable dispersion effects. Furthermore, fiber effects are not a result of one singular parameter, rather results stem from numerous parameters. As such, any attempt to model fiber propagation must take into account both the length of fiber, and the various effects that occur. Further complicating the issue of modeling is the nature of fiber effects. Some components occurring during propagation are linear in nature, others non-linear. Naturally a perfect model taking all factors into account is complicated to the point of impracticality, especially given the time frame of this project and the computer systems running this simulator. Analysis of various research papers and articles led to the adoption of the split step model [9, 10]. This method aims to consider the major effects of fiber propagation in a way that is, comparatively simpler than more precise models. The Non-Linear Schrodinger Equation (NSE) is regarded as the propagation equation of an optical pulse in its single mode. The NSE is represented by

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + \frac{j}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial t^3} = j\gamma |A|^2 A - \frac{\alpha}{2} A \quad (3)$$

The β matrices correspond to the various dispersion components of the fiber. The β_1 term representing the PMD fiber effects, and the chromatic dispersion represented by the β_1 and β_2 terms. Losses over the length of fiber are considered through the attenuation α parameter, and fiber non-linearities are represented by the γ term. This can be rewritten in a normalized form by defining

The above equations are used to represent the amplitude of a propagating signal in both its x and y directions. Rewriting (4) shows this propagation equation

$$\frac{\partial A}{\partial z} + \left\{ \beta_1 \frac{\partial}{\partial t} - \frac{j}{2} \beta_2 \frac{\partial^2}{\partial t^2} - \frac{1}{6} \beta_3 \frac{\partial^3}{\partial t^3} \right\} A + \alpha A = -j \frac{\gamma}{3} \left\{ \begin{array}{l} \left(3|A_x|^2 + 2|A_y|^2 \right) A_x \\ \left(3|A_y|^2 + 2|A_x|^2 \right) A_y \end{array} \right\} \quad (4)$$

The split step model attempts to model various fiber effects over the length of fiber. The model requires that the fiber be split into a number of small sections, each

usually represented as δz . Over each length of delta z the various fiber effects are assumed to act independently of one another. Each split step, delta z, is assumed to consist of a number of digital filters, each of which corresponds to a different fiber effect. This is illustrated in Figure 6. A fiber propagation model is developed to simulate the optical fiber propagation effects. This propagation model is developed using Matlab SIMULINK. A mathematical modelling of these effects was utilized to develop the model. The split-step Fourier model [10] is applied for the fiber and the transmission distance is divided it into a number of split-steps.

4.1 Split-step Model

The different optical fiber effects include chromatic dispersion, polarization model dispersion and other nonlinearities which affect optical propagation over the fiber. These different optical fiber effects were modelled over a length of the optical fiber by splitting the optical fiber into numerous small sections in the split-step model. The diagram below depicts the split-step model. Each section is denoted by δz . Each of these sections of δz acts independently from one another with regards to the different fiber effects. Each δz can be modelled by a number of digital filters in Matlab. These digital filters would be modelled accordingly to match the different effects of the optical fiber and used for numerical simulations.

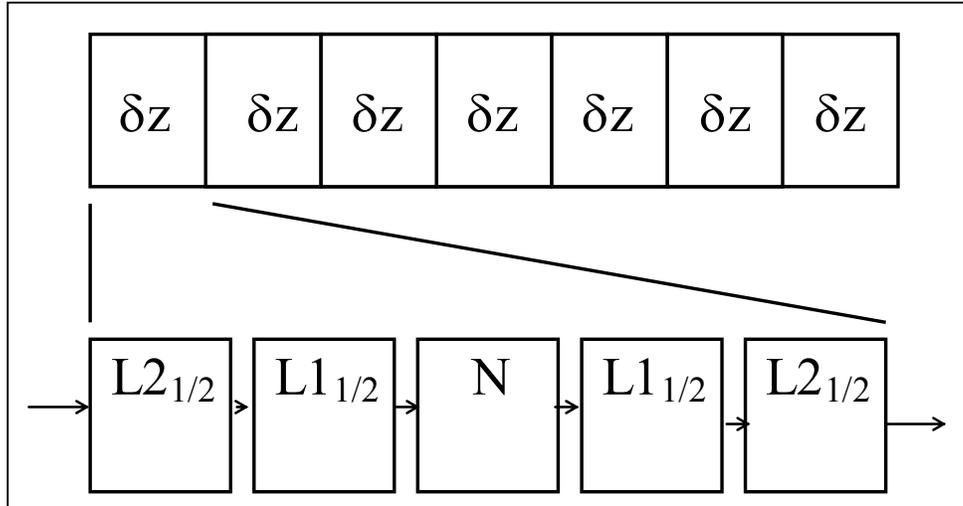


Figure 6: Split step model- δz split step can be 500 km or shorter dependent on the accuracy required.

4.2 Design of the Digital Filter

In the schematic diagram shown in Figure 7, the pulse sequence passes into the purely linear dispersive component of the fiber first. These are denoted by digital filters $L_1^{1/0}$ and $L_1^{1/2}$. The presence of chromatic dispersion and PMD in both halves of the split-step are described by subscript $1/2$ in the operators of $\underline{L}_1^{1/2}$ and $\underline{L}_2^{1/2}$. The pulse would go through non-linearity before going through a purely dispersive region of the fiber.

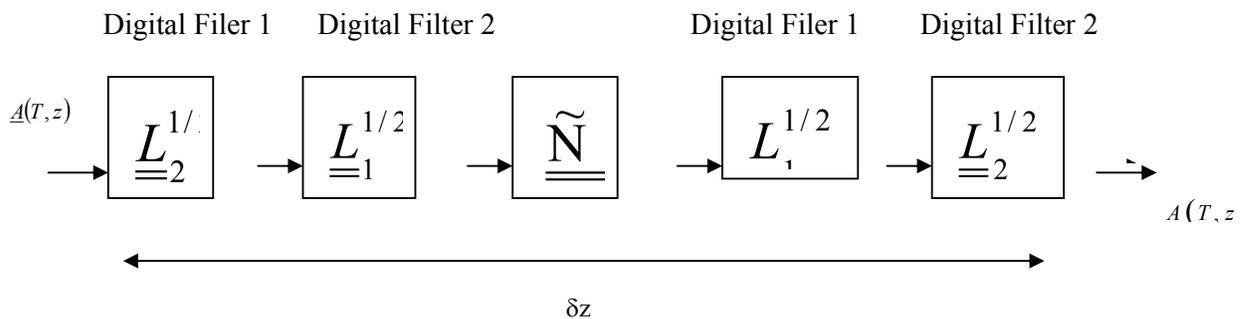


Figure 7: A split step δz modelled by cascading digital filters and a nonlinear operator

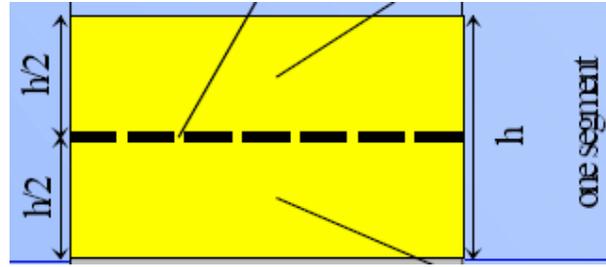


Figure 8: A transmission distance issplit into several split step.

A pulse propagating through an optical fiber can be modeled by multiple iterations of the split step. Each split step can be considered as a solution to the Nonlinear Schrödinger Equation (NSE) which is the propagation equation of an optical fiber in single mode. This solution is given by

$$A(T, z + \Delta z) \approx L_2^{1/2} L_1^{1/2} \exp\left\{\int_z^{z+h} N(z') dz'\right\} L_1^{1/2} L_2^{1/2} A(T, z) \quad (5)$$

Digital filter 1, $L_2^{1/2}$, is models the chromatic dispersion effects of the fiber propagation. This equation is represented by[3]:

$$L_2^{1/2} = \begin{bmatrix} H_d(z) & 0 \\ 0 & H_d(z) \end{bmatrix} \quad \text{with} \quad H_d(z) = H_{lin}(z) H_{par}(z) \quad (6)$$

$H_d(z)$ is represented as a combination of the linear $H_{lin}(z)$ and parabolic filters $H_{par}(z)$.

With reference to the equation above $H_d(z)$ is represented by the following equation:

$$H_{lin}(z) = uv \frac{z - j\frac{1}{u}}{z - ju} \frac{z - \frac{1}{v}}{z - v} \quad (7)$$

Where u and v are real numbers, the poles and zero pair of the filter are matched to function like an all pass filter that causes phase distortion. As it is operating within Nyquist range, the filter can be defined as unit amplitude and the equation redefined as:

$$1 = 4 \frac{u^2(1-u^2)(1-v)^3}{v(1+u^2)^3(1+v)} \quad (8)$$

The resulting filter group delay is given by:

$$\tau_{H_{lin}} = \tau_0 + 2T_c^2 \frac{u(1-u^2)}{(1+u^2)^2} \Omega \quad (9)$$

A constant pure delay denoted by τ_0 is common to most digital filters. It does not impact chromatic dispersion and can be negated and ignored. The linear group delay is represented by the other term in the equation. The function of u was plotted using Matlab to determine the range of values that can be taken.

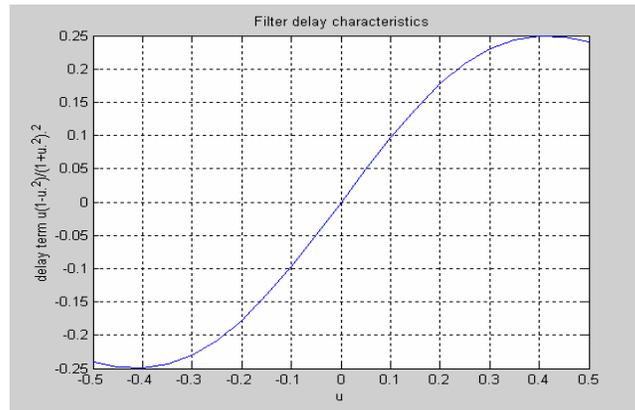


Figure 9: *Function of u*

From Figure 9, the value of u is limited to the range ± 0.4 where the curve is almost linear. Taking the most linear region of the curve, the value of u was found to be about ± 0.282 . Substitution of this value yielded a linear delay of $+0.4455 T_c^2 \Omega$. It is only slightly below the previously calculated maximum delay value of $\pm 0.5 T_c^2 \Omega$ when we used the value of ± 0.4 . The v is calculated by substitution u into equation and it gives a v value of 0.13355. Substitution into the previous filter equation resulted in a transfer function representative of the filter that would represent the linear component of chromatic dispersion:

$$H_{lin}(z) = \frac{(-0.038 - j0.135)z + 0.282}{(1 + j0.282)z - 0.135} \quad (10)$$

The other parabolic dispersion filter can be represented by the following transfer function:

$$H_{par}(z) = v \frac{z - \frac{1}{v}}{z - v} \quad (11)$$

A real number, represented by v , characterizes the filters group delay, which is given by:

$$\tau_{Hpar} = \tau_0 + T_c^3 \frac{v(1+v)}{(1-v)^3} \quad (12)$$

To determine a maximum value for v , $\frac{v(1+v)}{(1-v)}$ is plotted. A value of 1/8 was obtained for v and a maximum delay of $(T_c^3/8)\Omega^2$. The value of v was later substituted into the equation to give the transfer function for the parabolic component of chromatic dispersion. The total chromatic dispersion could be represented by cascading the two filter designs using SIMULINK.

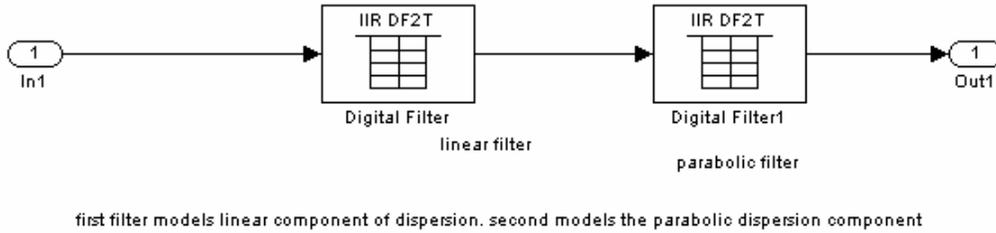


Figure 10: *Digital filters in SIMULINK*

$L_1^{1/2}$, another digital filter was used to model the PMD effects. It would in turn pass into the non-linear operator, designed to model any non-linearity or attenuation through the fiber. This is represented by the following equation:

$$L_1^{1/2} = \begin{bmatrix} H_p(z) & 0 \\ 0 & 1 \end{bmatrix} \quad (13)$$

The digital filter resulting in the PMD is represented by $H_p(z)$. It affects one mode

of propagation and is caused by the fiber birefringence. By propagating the optical signal through an all pass filter, the effects of PMD is modelled. An all pass filter generating the required effects is given by the following formula:

$$H_p(z) = z^{-k} \quad (14)$$

Where k is a real number representing the number of time samples, the total delay given by the filter can be given by the following equation:

$$\tau_p = kTc \quad (15)$$

Tc represents the sampling rate of the input signal and determines the number of sample occurring over the course of an optical pulse by substituting the value of k in the equation. This gives a filter function that can be implemented using the digital filter design block.

$$H_{PMD}(z) = z^{-1} \quad (16)$$

2.4.1 The Non-Linear Operator

N , the non-linear parameter, represents a number of non-linear effects such as Brillouin, Rayleigh and Raman scattering. This parameter acts as a multiplier value dependant on the injected laser power which is defined by the following equation:

$$N^2 = \gamma P_0 L_D \quad (17)$$

The non-linear parameter, γ , represent non-linearity that are present in a length of optical fiber. This parameter is defined by:

$$\gamma = \frac{2\pi n_2}{\lambda A_{eff}} \quad (18)$$

n_2 depicts the non-linear index coefficient. For a single mode silica fiber, it is usually $10^{-20} \text{m}^2/\text{W}$. λ represents the operating wavelength while A_{eff} the effective area of fiber. A_{eff} is defined by the following equation:

$$A_{eff} = \pi r_0^2 \quad (19)$$

r_0 depicts the fiber spot size. This is defined on the SMFC-28 data sheet as 4.1 μm . This gives an effective area of $5.281 \times 10^{-11} \text{m}^2$.

D , the dispersion value, can be calculated using:

$$D = \frac{-2\pi c}{\lambda^2} \beta_2 \quad (20)$$

The value can be obtained from the SMF data sheet as 17/ps/nm.km. The values were converted to s/m^2 by multiplying the values by 10^{-6} to ensure accuracy. Substituting and the values obtained the value of -2.167×10^{-26} for β_2 . The sign in this value is critical as it represents the direction where pulse dispersion occurs. By The value of γ can be estimated to be $7.677 \times 10^{-4} \text{m}^{-1} \text{W}^{-1}$. The dispersion length, L_D , is derived from the following equation[10]:

$$L_D = \frac{T_0^2}{|\beta_2|} \quad (21)$$

T_0 represents the width of the data pulse in seconds. For a 10ps pulse, the dispersion length was 4614.675m. The threshold power, depicted by P_{TH} , was determined using the following formula:

$$P_{TH} = \frac{|\beta_2|}{\gamma T_0^2} \quad (22)$$

The value of the threshold power obtained was 282.3mW. Any laser power greater than 28.2mW (282.3/10) would require non-linearity effects to be taken into consideration. Power greater than the threshold power would cause N to be defined using the following formula:

$$N = \sqrt{\gamma P_0 L_D} \quad (23)$$

Power levels less than the threshold would define N as 1. This would cause non-linear effects to be negligible. Non-linear effects are minimal at low power levels when it is in single channel systems. However, in multi-channel system,

with high power, propagation would be greatly affected by non-linearity.

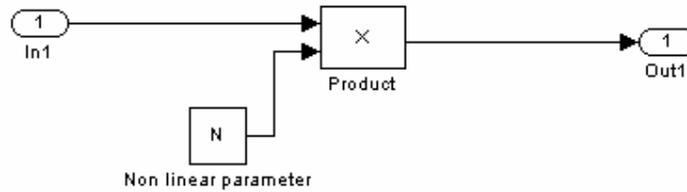


Figure 11: *The SIMULINK non-linear effects model*

To determine the pulse peak power from the input signal, a Fast Fourier Transform, FFT, was utilized in the model. This will allow the value of N to be computed and multiplied by the input signal in FFT form. The Zero pad block changes the dimensions of the input matrix from M_i -by- N_i to M_o -by- N_o by zero-padding or truncating along the columns and rows before input into the FFT block. The FFT block then computes the FFT of each channel for manipulation by the digital filters in frequency domain. The IFFT block after the digital filters converts output back into its original format.

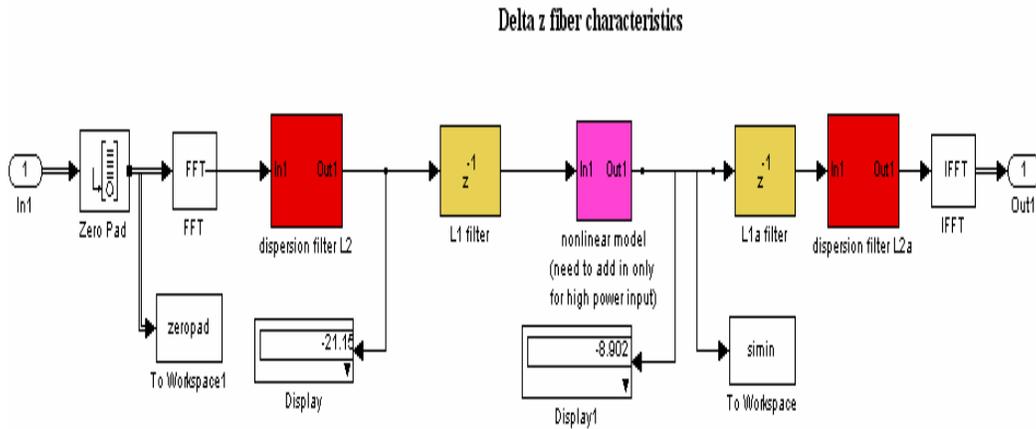


Figure 12: *Fiber propagation model*

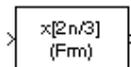


Figure 13: *FIR rate conversion block*

The FIR Rate Conversion block resample the discrete-time input to a period K/L times the input sample period, where the integer K is specified by the Decimation factor parameter and the integer L is specified by the Interpolation factor parameter. In short, the main purpose of the FIR is use as a pulse-shaping filter that shapes the rectangular eye diagram from the propagation into the eye diagram that can be observed from the scope.

Within the FIR, FIR filter coefficient of $rcosine(1,8,0.5,3)$ is used. An explanation of how to arrive at the FIR filter coefficients is provided by

$$rcosine(Fd,Fs,type_flag,r,delay)$$

Sampling frequency for digital input signal, $Fd = 1$. Sampling frequency for filter, $Fs = 8$ The ratio Fs/Fd must be a positive integer greater than 1. Default roll-off factor, $r = 0.5$. Group delay, $delay = 3/Fd = 3$

4.3 SMF Transfer Function Model

It has been found that the split step model allows only the change of dispersion parameters by cascading or removing the digital filter blocks, hence limiting the simulations to SMF as the change dispersion parameters causes the variation in length, dispersion, frequency or wavelength to dictate a cumbersome fluctuation in the number of digital filters utilized. Thus the SMF Transfer Function Model is proposed in this work. The Single Mode Fiber Transfer Function Model (SMFTF) allows the change of dispersion parameters and effectively model various fiber types of SMF, DSF and DCF. A non-linear fiber model is also developed allowing the change in fiber type by a simple change in A_{eff} and attenuation characteristics.

4.4 Derivation of SMFTF Model:

From the time shifting property of the Fourier transform we have

$$\begin{aligned}
x(t_c) &\leftrightarrow x(f_c) \\
x(t_c - \Delta t) &\rightarrow e^{-j\Delta T \omega} X(f_c) \\
&\rightarrow XF [x(t_c - \Delta t)] \\
&\rightarrow e^{-j\Delta T (2\pi f_c)} X(f_c) \\
&\rightarrow e^{-j2\pi\Delta T f_c} X(f_c)
\end{aligned} \tag{24}$$

where $\Delta T = \frac{\Delta\phi}{\Delta\omega}$ is the change of phase with respect to the frequency variation. The group velocity is dependent on the propagation constant B which is dependant on the respective reflective index for the wavelength of concern. The group velocity is also dependant on frequency of the light waves.

$$Vg\alpha \frac{1}{\Delta\beta} \rightarrow Vg\alpha\Delta\omega \rightarrow \therefore Vg = \frac{\Delta\omega}{\Delta\beta} \tag{25}$$

$$\begin{aligned}
\frac{1}{Vg} &= \frac{\partial\beta}{\partial\omega} \\
\Delta T &= \frac{\partial T}{\partial\omega} = \frac{\text{dis tan ce}}{Vg} \Delta\omega = L \frac{\partial^2\beta}{\partial\omega^2} \Delta\omega \quad \text{where } T = \frac{\text{Dis tan ce}}{\text{Velocity}} = \Delta T = \frac{LD\lambda^2}{c} \Delta f \\
&= \frac{LD\lambda}{f} \Delta f
\end{aligned}$$

$$\begin{aligned}
\therefore X(t_c - \Delta T) &\leftrightarrow X(f_c) = e^{-j\Delta T (2\pi)(f - f_c)} \\
&= e^{-\frac{j2\pi LD\lambda}{f_c} (f - f_c)^2} = e^{-j\alpha f^2}
\end{aligned} \tag{26}$$

The SMFTF linear model can thus be expressed as

$$H(f) = e^{-j\phi(f)} = e^{-j\alpha f^2} = e^{-j \left[\alpha B^2 \left(\frac{f}{B} \right)^2 \right]} \tag{27}$$

where $\alpha = \pi D(\lambda) \frac{\lambda^2}{c} L$, Length of fiber, $L = 80000\text{m}$, the wavelength, $\lambda = 1550\text{nm}$, the speed of light, $c = 299.792458 \times 10^6$, frequency, $f = 1.93414489 \times 10^{14}$, the dispersion, $D = 17\text{ps/nm.km (SMF)} = 3\text{ps/nm.km (DSF)}$

$$\begin{aligned} \therefore e^{-j\alpha f^2} &= e^{-j\pi D \frac{\lambda^2}{c} L f^2} \\ &= e^{-j\pi D \frac{\lambda^2}{f\lambda} L f^2} \end{aligned} \quad (28)$$

$$= e^{-j\pi D \lambda L f} \quad (29)$$

Where D represents the dispersion at the operating wavelength while λ represents the operating wavelength. L represents the length of the fiber while f represents the frequency of the optical carrier and its sidebands. The SMFTF Non Linear Model can be approximated as

$$H(f) = e^{-j\phi_{NL}} \quad (30)$$

where $\phi_{NL} = \gamma P_{in} L_{eff}$ (31)

$$\gamma = \frac{2\pi\bar{n}_2}{\lambda A_{eff}} \quad (32)$$

$$L_{eff} = \frac{[1 - \exp(-\alpha L)]}{\alpha} \quad (33)$$

$$P_{th} \approx \frac{16\alpha(\pi w^2)}{g_R} \quad (34)$$

where

Raman gain coefficient, $g_R = 1 \times 10^{-13} \text{ m/W}$

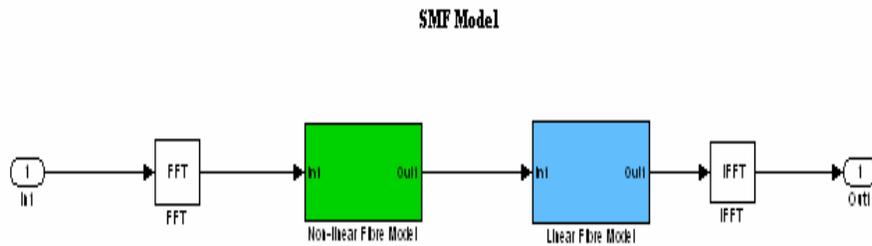


Figure 14 *SMF propagation model consisting non-linear fiber model and Linear*

SMFTF Model

The Non-Linear Fiber Model (NLFM) is positioned before the SMFTF linear fiber model due to its dependency on input power. The SMFTF attenuates the input power according to the optical power loss characteristics of the fiber modeled. If the order of the two models is swapped the equivalent model would not be accurate.

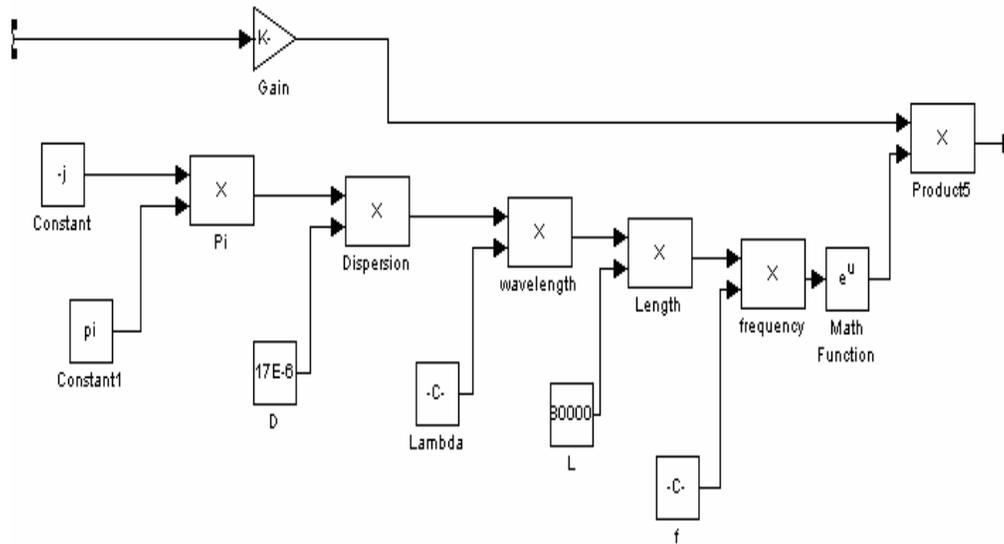


Figure 15 *The Simulink Linear Fiber Propagation Model*

The fiber can be modeled by the SMFTF shown in Figure 15. The fiber dispersion characteristics can be easily accessed and changed according to the optical Fiber modeled. It can be obtained through calculations with the equations provided on the manufacturers' specifications using the Sellmeier's constants. The dispersion shown in the Figure 15 is calculated for SMF-28 at 1550nm. L represents the length of the fiber, an 80 km span for the simulation example. The gain block represents the optical power loss, which can be calculated from loss per km of fiber multiplied by the length of the fiber. The loss in dB was converted into gain and input into the gain block.

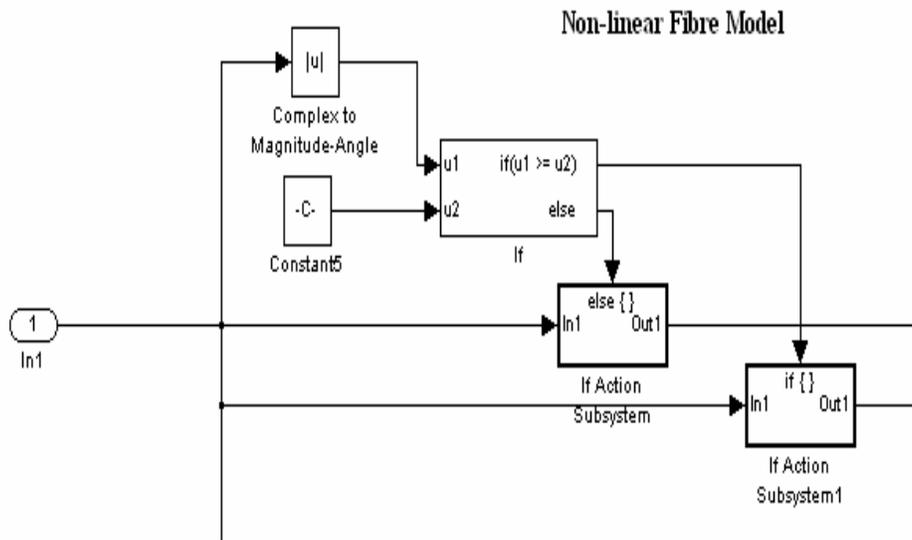


Figure 16 *Non-linear Fiber Propagation Model Logical Unit*

The logical unit of the NLFM shown in Figure 16 detects the input power level and converts it into magnitude for comparison with predetermined optical power level where non-linear dispersion effects start to occur. This optical power level can be calculated in the initialisation file. If the optical power level is below the predetermined level, non-linear dispersion effects are deemed negligible hence the signal is passed from input to output without interference. However, when the input power level is above the predetermined level, the non-linear dispersion effects are significant and the non-linear dispersion induced are included. Figure 17 illustrates the NLFM. The constants γ and L_{eff} (effective length) are given in the initialisation file. Hence, the power input was connected as part of the exponential function.

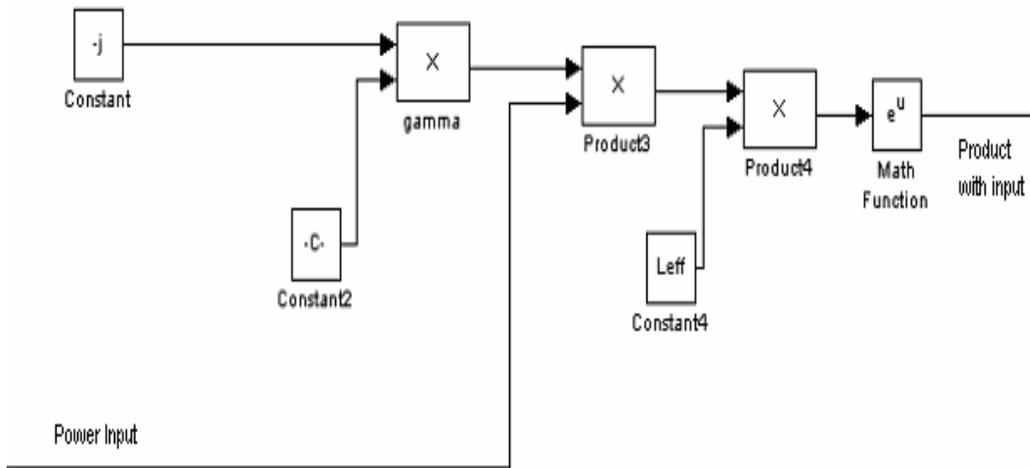


Figure 17 Non-Linear Fiber Propagation Model Dispersion unit

5 SIMULINK simulator for transmission without dispersion compensation

The Simulation model is constructed to compare the simulated and experimental results. The block diagram of the simulation model utilised for this stage is shown in Figure 18.

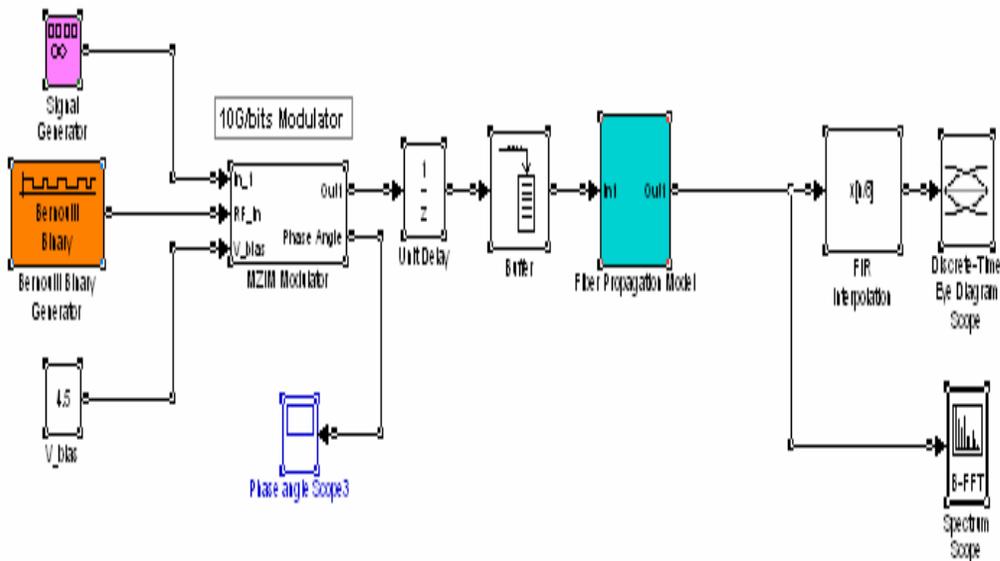


Figure 18: Block diagram of optical transmission system w/o dispersion

compensation.

A unit delay was inserted before the fiber model to discretize the signals without affecting its properties. While the buffer inserted after the unit delay to convert the signal into frame mode. The blocks are necessary as the SMFTF operating in discrete mode and frame inputs. This fiber propagation is an extension of an existing developed model using the split step model as described in a previous section. However, several deficiencies have been found with this model. These deficiencies have been addressed in the SMF propagation model. The MZIM is modeled with a direct look up table to simulate the effects and output of the MZIM characteristics shown in Figure 18.

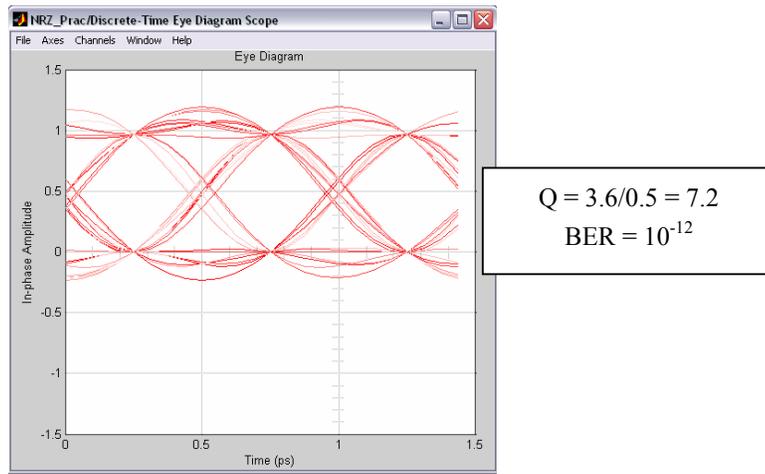


Figure 19: *Electrical Eye diagram obtained from simulations at 2.0Gbps without fiber*

The simulated waveforms from Figure 19 and Figure 20 and the Q factors, hence the BER are very close to the error free values.

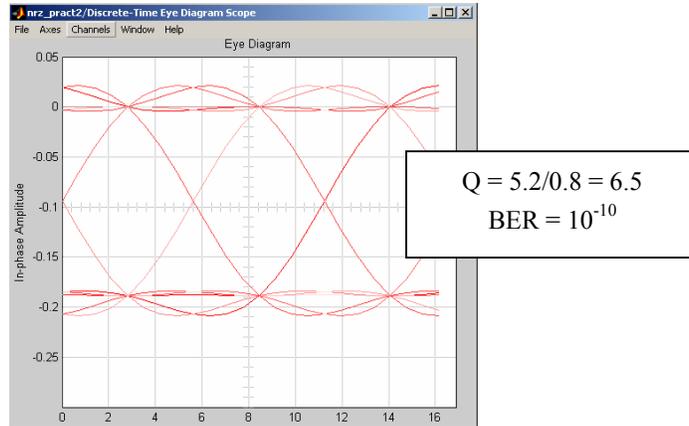


Figure 20: Eye diagram obtained from simulations with fiber propagation effects.

6 Simulator for dispersion compensating fibers

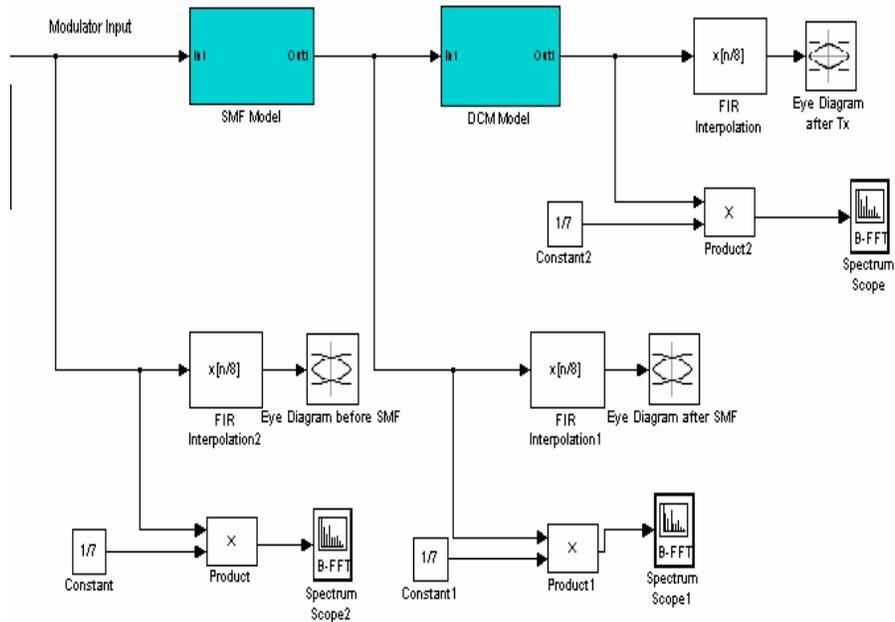


Figure 21 Generic system model for integration with different modulation formats and dispersion compensation

The above generic SMF and DCF fiber propagation models are integrated with NRZ, RZ, CS-RZ modulation formats.

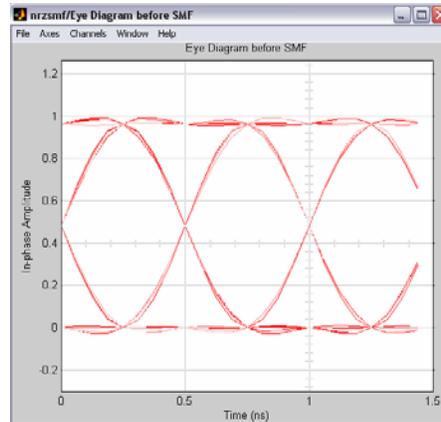


Figure 22 Eye diagram for NRZ before propagation 2Gb/s BER = 10^{-12}

The BER and eyediagram in electrical domain (after the PD) obtained for the NRZ modulation format before fiber transmission can be corroborated with experimental results achieved in Figure 23,

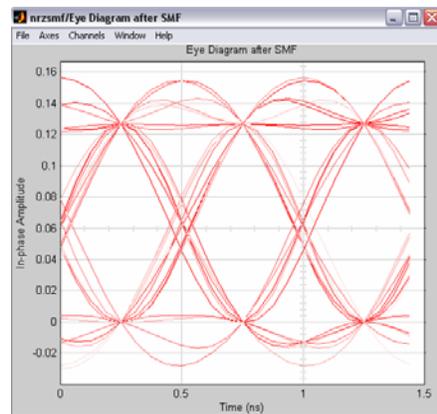


Figure 23 Eye diagram for NRZ after propagation through SMF 2Gb/s BER = 10^{-5}

Significant pulse broadening can be observed from the eye diagram after propagation through SMF, the BER is also compromised as expected due to linear dispersion characteristics of the fiber. The optical power from the eye diagram is attenuated according to the loss characteristics of the SSMF.

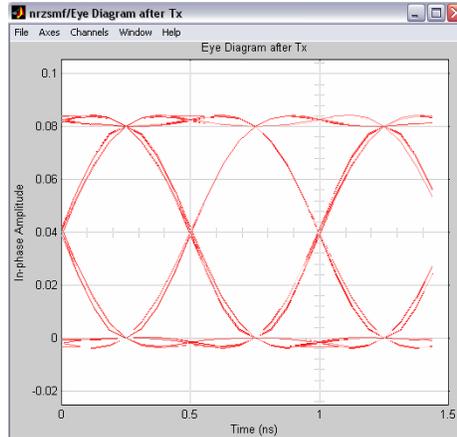


Figure 24 Eye diagram for NRZ after propagation through DCM 2Gb/s $BER = 10^{-12}$.

The effect of pulse broadening is effectively compensated by the DCF module as the BER improved to 10^{-12} , which is considered reasonable for a typical optical transmission system. Further optical power attenuation can be observed from the diagram in accordance to the power loss characteristics of 16 km of DCF.

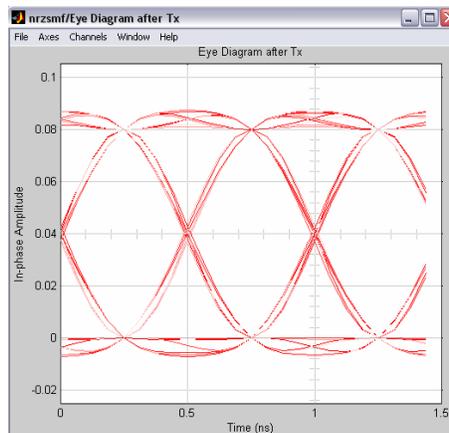


Figure 25 Input optical power to SMF = P_{th} $BER = 10^{-12}$.

The above eye diagram is generated with the input optical power set at P_{th} , this is to observe the effects of the non-linear model. It must be noted that the SMF model attenuates the input power. Hence non-linear effects are not dominating in the DCF model. Significant pulse broadening can be observed when comparing with Figure

25. However, the pulse broadening is not sufficiently high to reduce the eye opening and hence the BER.

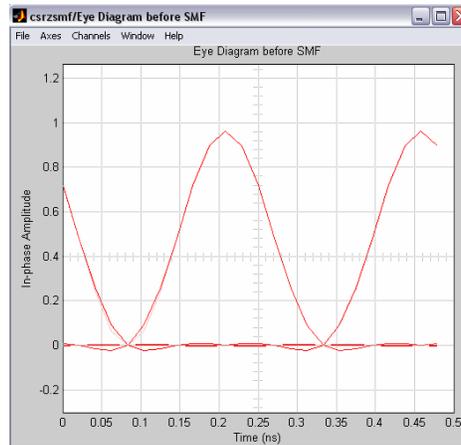


Figure 26 Eye diagram for CSRZ before propagation 2Gb/s BER = 10^{-12} .

The pulse observed in Figure 26 is near ideal with very little distortion. The shape of the pulse concurs with pulse shape obtained for CSRZ experiments in several published articles.

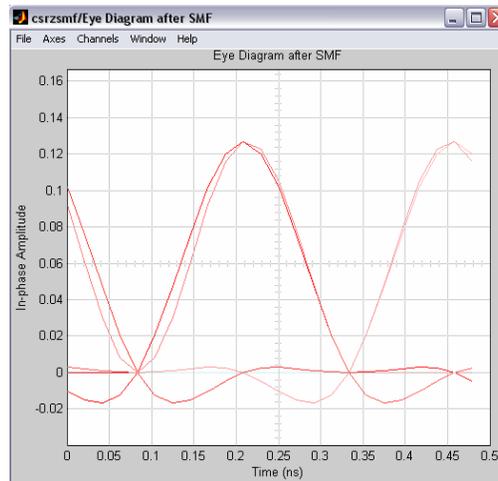


Figure 27 Eye diagram for CSRZ after propagation through SMF- BER = 10^{-7}

Comparing Figure 27 with Figure 28 a significant amount of pulse broadening can be observed. Consequentially, the BER is higher. Attenuation can be observed from the diagrams indicating the optical power loss characteristics of the fiber.

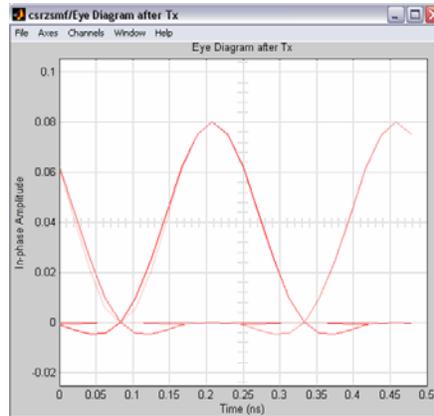


Figure 28 *Electrical eye diagram for CSRZ after propagation through DCM-*
 $BER = 10^{-12}$.

From Figure 28, it can be observed that the effect of pulse broadening is effectively compensated by the DCF model. The BER is improved to 10^{-12} . The effects are in accordance to the negative dispersion characteristics of DCF. Further optical power attenuation is observed due to the power loss characteristics of 16 km of DCF.

The eye diagrams for RZ modulation format is not included in this report as it demonstrates essentially the same findings and principles as the NRZ and CSRZ modulation formats.

Some difficulty is expected when integrating SMFTF model with modulation models which involves phase shifting as part of its transmission due to the optical phase shift of the dispersion. As the optical phase is shifted during propagation the demodulation block may face difficulty in decoding the transmitted PRBS. The power losses in the fiber model also pose some difficulty to the demodulator, as it often does manipulation at a predetermined level. This issue is easily resolved by implementing an optical amplifier before the demodulator. However, the clashes in phase shifting are not as simple. To successfully demodulate the signals, it would necessitate a 100% compensation for the phase shifting. Implementing such a scheme would be highly redundant as the received signal would be essentially the same as the transmitted signal.

7 Concluding Remarks

Modelling the lightwaves propagation through optical fibers is a challenge especially in a simple and accurate platform. Simulink in this report has demonstrated a graphical and logical model of different types of optical fibers in advanced optical communications systems for ultra-high speed and ultra-long haul transmission. Linear transfer function based on the low pass filter characteristics of a single mode optical fiber is represented and the nonlinearly γ -induced dispersion model is also implemented in Matlab Simulink. When modelling optical fiber systems in Simulink, the high frequency involved renders the use of filters in time domain almost impossible. The model is proven with propagation of a number of modulation formats imposed on lightwave carriers.

Simulink models of EDFAs and other photonic components would be integrated in near future works to demonstrate ultra-long haul ultra-high speed optical transmission systems.

8 References

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